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CONDITIONAL MEASUREMENTS IN MICROMASERS: FOCK STATES VIA TRAPPPING CONDITIONAL MEASUREMENTS

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We propose a new method for production of Fock states of a cavity field in a micromaser via conditional measurements.

We consider a lossless micromaser driven by a stream of two-level Rydberg atoms initially prepared in the upper maser level [1]. The Rydberg atoms are injected into the micromaser cavity on a very slow rate such that only *one* atom is in the cavity at a given time. It is assumed that a particular Rydberg atom interacts with only one cavity mode. The interaction between a single two-level atom and a single-mode cavity field can be described in a framework of the Jaynes-Cummings model [2] with the Hamiltonian in the dipole and the rotating-wave approximation (we adopt units such that $\hbar = 1$):

$$\hat{H} = \omega_F \left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right) + \frac{1}{2}\omega_A \hat{\sigma}_3 + \lambda \left(\hat{a}^\dagger \hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right), \quad (1)$$

where ω_A is the atomic transition frequency; ω_F is the frequency of the cavity mode; λ is the atom-field coupling constant; $\hat{a}$ and $\hat{a}^\dagger$ are the field annihilation and creation operators, respectively ($[\hat{a}, \hat{a}^\dagger] = 1$); $\hat{\sigma}_3$ is the atomic inversion operator, and $\sigma_\pm$ are the atomic "spin-flip" operators, which can be expressed as the atomic projection operators, i.e. $\hat{\sigma}_3 = |e\rangle\langle e| - |g\rangle\langle g|$, $\hat{\sigma}_+ = |e\rangle\langle g|$, and $\hat{\sigma}_- = |g\rangle\langle e|$, where $|e\rangle$ and $|g\rangle$ describe atomic upper and lower states, respectively. In the interaction picture in the two-dimensional atomic basis the evolution operator corresponding to the Hamiltonian (1) can be written in the form [3]:

$$\hat{U}(t) = \begin{pmatrix} \hat{U}_{11}(t) & \hat{U}_{12}(t) \\ \hat{U}_{21}(t) & \hat{U}_{22}(t) \end{pmatrix}. \quad (2)$$

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In the case of the exact resonance between the atom and the field ($\omega_A = \omega_F$) the operators $\hat{U}_{ij}(t)$ are given by expressions [3]:

$$\hat{U}_{11}(t) = \cos\hat{\Omega}_{n+1}t; \qquad \hat{U}_{22}(t) = \cos\hat{\Omega}_n t;$$

$$\hat{U}_{21}(t) = -i\lambda\hat{a}^\dagger \frac{\sin\hat{\Omega}_{n+1}t}{\hat{\Omega}_{n+1}}; \qquad \hat{U}_{12}(t) = -i\lambda\hat{a}\frac{\sin\hat{\Omega}_n t}{\hat{\Omega}_n}, \tag{3}$$

where

$$\hat{\Omega}_n = \lambda(\hat{n})^{1/2}. \tag{4}$$

In an ideal situation one can consider that at the initial moment ($t=0$) the atom-field system is in a pure state while the atom and the field are uncorrelated. Providing both the atom and the field are at $t=0$ in pure states $|\Psi_A^{(0)}\rangle$ and $|\Psi_F^{(0)}\rangle$, respectively, then the initial atom-field states vector can be written in a factorized form:

$$|\Psi_{A-F}^{(0)}\rangle = |\Psi_A^{(0)}\rangle \otimes |\Psi_F^{(0)}\rangle. \tag{5}$$

The pure initial state of the cavity field in the Fock basis can be described as a superposition of number (Fock) states:

$$|\Psi_F^{(0)}\rangle = \sum_{n=0}^{\infty} C^{(0)}(n)|n\rangle; \qquad \sum_{n=0}^{\infty}|C^{(0)}|^2 = 1. \tag{6}$$

The atom is initially prepared in the upper level $|e\rangle$, i.e.

$$|\Psi_A^{(0)}\rangle = |e\rangle. \tag{7}$$

At time $t > 0$ the state vector $|\Psi_{A-F}^{(0)}(t)\rangle$ of the atom-field system is given by the expression

$$|\Psi_{A-F}^{(0)}(t)\rangle = |\Phi_e^{(0)}(t)\rangle \otimes |e\rangle + |\Phi_g^{(0)}(t)\rangle \otimes |g\rangle, \tag{8}$$

where

$$|\Phi_e^{(0)}(t)\rangle = \hat{U}_{11}(t)|\Psi_F^{(0)}\rangle = \sum_{n=0}^{\infty} C^{(0)}(n)\cos\Omega_{(n+1)}t|n\rangle; \tag{9a}$$

$$|\Phi_g^{(0)}(t)\rangle = \hat{U}_{21}(t)|\Psi_F^{(0)}\rangle = -i\sum_{n=0}^{\infty} C^{(0)}(n)\sin\Omega_{(n+1)}t|n+1\rangle. \tag{9b}$$

The operators $\hat{U}_{ij}(t)$ are given by Eq.(3) and $\Omega_{(n)} = \lambda\sqrt{n}$.

After the first atom-field interaction which lasts for a time Δt the outgoing atom is probed by a static electric field when it lefts the cavity. Simultaneously we impose the condition on the result of the measurement; we will be interested only in those measurements where the state of the atom after the interaction is the same as at the beginning. In other words we are interested only in those sequences of measurements where no energy is transferred from the field to the atom and vice versa.

According to the measurement postulate of the quantum mechanics [4-6] the state *immediately* after the measurement of the physical quantity $\hat{A}$ on the system $|\Psi\rangle$ is

the normalized projection of $|\Psi\rangle$ onto the eigenspace associated with the outcome of the measurement. In particular, if we assume the initial state of the atom-field system to be given by Eq.(5) with $|\Psi_F^{(0)}\rangle$ and $|\Psi_A^{(0)}\rangle$ given by Eqs.(6) and (7), respectively, then at time Δt the atom-field system is described by the state vector [Eq.(8)] with $t = \Delta t$.

If at time Δt the atom-field state vector (8) collapses due to the measurement of the atom in the state $|\Psi_A^{(0)}\rangle$, then the state vector of the cavity field after the measurement has the form:

$$|\Psi_F^{(1)}\rangle = \frac{\langle\Psi_A^{(0)}|\Psi_{A-F}^{(0)}(\Delta t)\rangle}{\sqrt{|\langle\Psi_A^{(0)}|\Psi_{A-F}^{(0)}(\Delta t)\rangle|^2}}, \tag{10a}$$

or, alternatively,

$$|\Psi_F^{(1)}\rangle = \frac{|\Phi_e^{(0)}(\Delta t)\rangle}{\langle\Phi_e^{(0)}(\Delta t)|\Phi_e^{(0)}(\Delta t)\rangle^{1/2}} \equiv \sum_{n=0}^{\infty} C^{(1)}(n)|n\rangle, \tag{10b}$$

where

$$C^{(1)}(n) = \frac{C^{(0)}(n)\cos\Omega_{(n+1)}\Delta t}{\left[\sum_{m=0}^{\infty}|C^{(0)}(m)|^2\cos^2\Omega_{(m+1)}\Delta t\right]^{1/2}}. \tag{11}$$

In the case of the lossless micromaser the field state $|\Psi_F^{(1)}\rangle$ represents an initial state for the second atom-field interaction. The second atom is again supposed to be prepared in the state $|\Psi_A^{(0)}\rangle$. After the interaction time Δt the second atom is conditionally measured in its initial state and the cavity field is now in the state:

$$|\Psi_F^{(2)}\rangle = \frac{\langle\Psi_A^{(0)}|\Psi_{A-F}^{(1)}(\Delta t)\rangle}{\sqrt{|\langle\Psi_A^{(0)}|\Psi_{A-F}^{(1)}(\Delta t)\rangle|^2}} \equiv \sum_{n=0}^{\infty} C^{(2)}(n)|n\rangle, \tag{12}$$

where $|\Psi_{A-F}^{(1)}(\Delta t)\rangle$ is the atom-field state vector which evolves from the "second" initial state $|\Psi_A^{(0)}\rangle \otimes |\Psi_F^{(1)}\rangle$. In an analogous way we can obtain an expression for the field state vector $|\Psi_F^{(K)}\rangle$ after the sequence of K atoms are measured in their initial state. The conditional measurement process as described here has been recently studied by Sherman and Kurizki [7] in connection with production of quantum-mechanical superposition states of light in micromaser cavities (see also paper by Brune et al. [8]). The recurrent expression for amplitudes $C^{(K)}(n)$ reads:

$$C^{(K)}(n) = \frac{C^{(K-1)}(n)\cos\Omega_{(n+1)}\Delta t}{\left[\sum_{m=0}^{\infty}|C^{(K-1)}(m)|^2\cos^2\Omega_{(m+1)}\Delta t\right]^{1/2}}. \tag{13}$$

At this stage of our analysis we make an additional assumption. Namely, we impose a condition on the length of the time interval during which each atom interacts with the cavity field. We will assume that Δt is such, that for *one* value of n (let say for $n = N$) the condition

$$\left|\cos\Omega_{(N+1)}\Delta t\right| = 1 \tag{14a}$$

is fulfilled, while for any other n (i.e., for all $n \neq N$) we have

$$\left|\cos \Omega_{(n+1)}\Delta t\right| < 1. \qquad (14b)$$

The condition (14) is *equal* to the trapping condition introduced by Meystre [9] which reads

$$\lambda\sqrt{(N+1)}\Delta t = q\pi, \qquad (15)$$

where q is an integer. Nevertheless we have to stress here the difference between the present approach based on the conditional measurement process (therefore, we can call our approach as the *trapping conditional measurement*) and the method discussed by Meystre [9] who have analyzed the standard micromaser dynamics [2] with no conditional measurements.

If we rewrite the expression (13) for the probability amplitudes $C^{(K)}(n)$ in the form:

$$C^{(K)}(n) = \frac{C^{(0)}(n)\cos^K \Omega_{(n+1)}\Delta t}{\prod_{j=0}^{K-1}\langle\Phi_e^{(j)}(\Delta t)|\Phi_e^{(j)}(\Delta t)\rangle^{1/2}}, \qquad (16)$$

then with the use of the condition (14) we can easily find that in the limit of large K the probability amplitudes $C^{(K)}(n)$ take the following form

$$\lim_{K\to\infty} C^{(K)}(n) = \delta_{n,N}, \qquad (17)$$

which describes the fact that after the trapping conditional measurement the initial field state collapses into the Fock state $|N\rangle$. Changing the interaction time Δt we can prepare in the single mode high-Q cavity any Fock state with an apriori given value of N. In Fig.1a we plot the photon number distribution $P(n)$ of the field mode as a function of the number (K) of atoms conditionally measured after their interaction with the cavity field. The transformation of $P(n)$ from the initial Poissonian distribution (corresponding to the coherent state $|\alpha\rangle$)

$$P(n) = |C^{(0)}(n)|^2; \qquad C^{(0)}(n) = e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}, \qquad (18)$$

with $\alpha = 5$, to the Kronecker-delta function describing the Fock state $|25\rangle$ is clearly seen. In this case we choose the interaction time Δt as $\lambda\Delta t_1 = \pi/\sqrt{26}$.

The probability to observe a sequence of K atoms in the upper state $|e\rangle$ is given by the expression

$$P^{(K)} = \sum_{n=0}^{\infty}|C^{(0)}(n)|^2\cos^{2K}\Omega_{(n+1)}\Delta t. \qquad (19a)$$

With the condition (14) the probability (19a) reads

$$P^{(K)} \simeq |C^{(0)}(N)|^2, \qquad (19b)$$

which means that if initially the Fock state $|N\rangle$ contributed significantly into the superposition (6), then the probability to observe a sequence of K atoms in their upper levels is rather high.

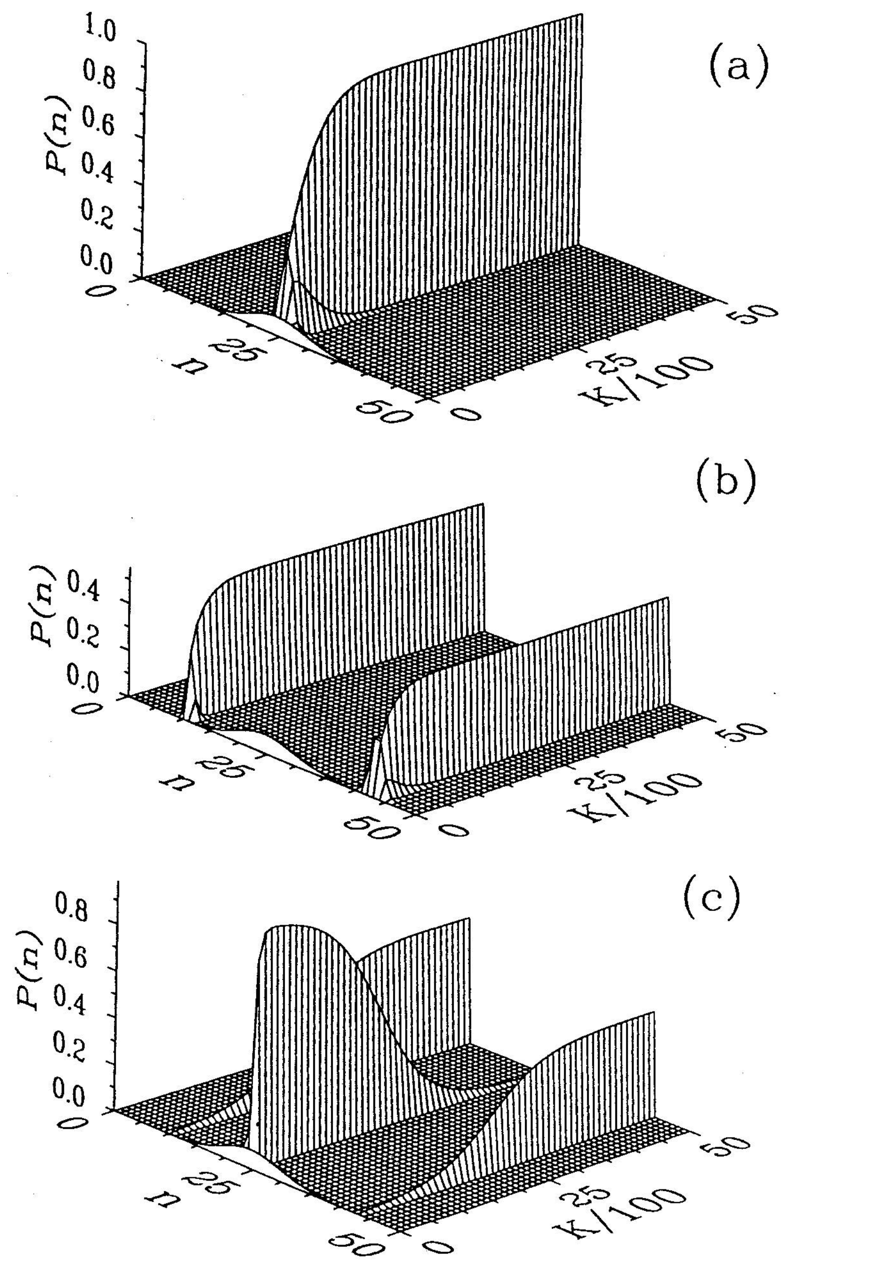

Fig 1. The photon number distribution $P(n)$ of the field mode as a function of number K of atoms conditionally measured after their interaction with the cavity field. The field is initially prepared in the coherent state with mean photon number equal to 25. The interaction time is taken to be $\lambda\Delta t_1 = \pi/\sqrt{26}$ (a); $\lambda\Delta t_2 = \pi/\sqrt{11}$ (b); and $\lambda\Delta t_3 = 2\pi/\sqrt{11}$ (c).

In conclusion, we have shown that if in a lossless micromaser we combine the idea of the conditional measurements with the trapping condition (14) then we can (at least in principle) produce any Fock state of the cavity field, providing this Fock contributes to the initial state of the field mode[1].

It is worth to note, that if there are several Fock states $|N_m\rangle$ $(m = 0, .., M)$ which fulfill the condition (14), then after the sequence of conditional measurements a pure state

$$|\Psi_F^{(K)}\rangle = \sum_{m=0}^{M} \frac{C^{(0)}(N_m)}{\left[\sum_{l=0}^{M} |C^{(0)}(N_l)|^2\right]^{1/2}} |N_m\rangle, \tag{20}$$

is produced in the cavity. The measurement sequence of K excited atoms is in this case realized with the probability

$$P^{(K)} \simeq \sum_{m=0}^{M} |C^{(0)}(N_m)|^2. \tag{21}$$

From Fig.1b we see how a pure superposition of two Fock states is produced from the initial coherent state $(\alpha = 5)$. The interaction time Δt in this case is chosen to be $\lambda\Delta t_2 = \pi/\sqrt{11}$. For this value of Δt there are two values of n for which $|\cos(\lambda\Delta t\sqrt{n+1})|$ is equal to unity, $n = 10$ and $n = 43$. From Fig.1b we see that in the limit of large K the superposition state

$$|\Psi_F^{(K)}\rangle = \frac{C^{(0)}(10)}{\left[|C^{(0)}(10)|^2 + |C^{(0)}(43)|^2\right]^{1/2}}|10\rangle + \frac{C^{(0)}(43)}{\left[|C^{(0)}(10)|^2 + |C^{(0)}(43)|^2\right]^{1/2}}|43\rangle, \tag{22}$$

is produced. If we double the interaction time Δt_2, i.e. we choose $\lambda\Delta t_3 = 2\pi/\sqrt{11} = 2\Delta t_2$, then obviously the condition $|\cos(\lambda\Delta t_3\sqrt{n+1})| = 1$ is fulfilled for $n = 11$ and $= 43$. Simultaneously one can find that $|\cos(\lambda\Delta t_3\sqrt{n+1})|$ *almost* equal to unity for $n = 24$ (namely, $|\cos(\lambda\Delta t_3\sqrt{n+1})| \simeq 0.99887$) Because of the fact, that for the Poissonian distribution with $\alpha = 5$ the relation $P(24) \gg P(10), P(43)$ is valid one has to expect that during the measurement process the Fock state $|25\rangle$ is transiently produced. For large values of K this Fock state is suppressed because $\cos^K(2\pi\sqrt{25/11}) \to 0$ and in the stationary limit the superposition state (18) is produced in the cavity. This behaviour is nicely seen in Fig.1c.

The scheme described above is valid not only for pure initial states of the cavity mode but can be applied for any statistical mixture state of the field mode.

There are three main restrictions which have to be taken into account if one wants to think about experimental realization of the proposed scheme: Firstly, it is detector efficiency of excited atoms is less than 100%. Secondly, velocity selection of the atoms has to be performed with an extreme accuracy. Thirdly, an environmental influence can change the dynamics of the process under consideration.

[1] If the cavity mode is influenced by an environment, then the Fock state $|N\rangle$ can appear after some time and then it can be selected more pronouncedly via the conditional measurement process as described above. Anyway, the role of decay and/or amplification of the cavity mode has to be discussed in more detail.

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