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PROPAGATION OF ELECTROMAGNETIC WAVES IN BELOW CUT-OFF WAVEGUIDE WITH FERRIMAGNETIC SLAB

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Propagation of electromagnetic waves in below cut-off waveguide containing YIG slab has been studied as a function of the slab parameters and position. The reported measurements here were made on 88 μm thick YIG on GGG substrate placed in a below cut-off rectangular waveguide at X band frequencies. Rich spectrum of transmitted power has been observed. The shape and the amplitude of the observed transmitted power through the waveguide section containing a ferrimagnetic slab was strongly dependent on the position of the slab within the waveguide. This dependence can be attributed to the different types of excitations within the ferrimagnetic slab which contribute to the power transmission. A simple theoretical approach of the longitudinal impedance was introduced to account for the observed experimental results. The calculations are in good agreement with the experimental observations.

I. INTRODUCTION

Propagation of electromagnetic waves in rectangular waveguides containing ferromagnetic slabs is textbook topics since late 50-th [1,2]. A new possibilities for the investigation of these old structures emerged due to the preparation of high quality yttrium iron garnet (YIG) slabs on gallium gadolinium garnet (GGG) substrate grown by liquid phase epitaxy, as well as due to the experimentally discovered window of transparence in the below cut-off waveguide containing a ferrimagnetic slab [3,4]. This discovery opened new possibilities for the investigation of ferrite materials as well as new device applications.

The objective of this work was to study in detail the basic properties of a below cut-off rectangular waveguide containing a ferrimagnetic slab. A method of

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the "longitudinal impedance" was introduced for this purpose and the experimental investigation was performed to test the capability of this theoretical approach.

The analysis of the below cut-off waveguide section containing a single in plane magnetized ferrite slab was treated in the books [1,2]. The more general case of the electromagnetic wave propagation of in a rectangular waveguide loaded by an anisotropic slabs has been studied by F. Gardiol [5].

Due to the practical importance of the multilayered structures, we present a modified approach of calculation of such structures. The approach is based on the assumption that the only modes propagating in by ferrimagnetic slab loaded below cut-off rectangular waveguide are the transverse electromagnetic TE_{n0} modes. The restriction to the TE_{n0} modes only, is justified because, the investigated waveguide structure is below cut-off already for the principal TE_{10} mode cut-off waveguide.

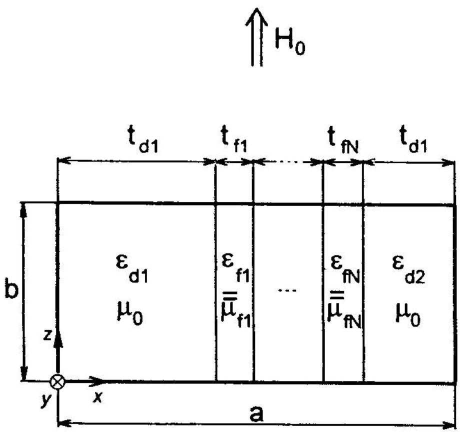

Fig. 1. Geometry of the waveguide section with multilayered anisotropic structure.

II. THEORY

The geometry of the waveguide section is indicated in the Fig.1. Its dimensions, the width a, high b and length l are chosen in such a way, that for the given principal frequency ω of the propagating electromagnetic wave, they represent, as already mentioned, a below cut-off waveguide section with reasonable attenuation in transmission. Indicated dimensions t_{di} and t_{fi} represent the width's of the dielectric and ferrite slabs respectively. The slabs are supposed to extend through the whole high of the waveguide. The i-th ferrimagnetic slab in the waveguide section is magnetized in the z-direction by an external magnetic field H_0 laying in the plane of the slab. The properties of the externally magnetized ferrimagnetic slab are described by a tensor of permeability in the form

$$\bar{\bar{\mu}}_i = \begin{pmatrix} \mu_i & j.\mu_{ai} & 0 \\ -j.\mu_{ai} & \mu_i & 0 \\ 0 & 0 & \mu_0 \end{pmatrix} \qquad (1)$$

where the diagonal and nondiagonal components can be expressed as

$$\mu_i = \mu_0 \left(1 + \frac{\omega_0 \omega_{Mi}}{\omega_0^2 - \omega^2} \right) \quad \text{and} \quad \mu_{ai} = \mu_0 \frac{\omega \omega_{Mi}}{\omega_0^2 - \omega^2} \qquad (2)$$

with $\omega_0 = \gamma.H_0, \omega_{Mi} = \gamma.M_{si}, \gamma$ the gyromagnetic ratio equals $3.522\pi 10^4\ s^{-1}A^{-1}m$ and μ_0 the permeability of the vacuum. In the first approximation it is assumed that the walls of the waveguide are ideal conductors.

The classical approach for solving such a structure as described in [1] uses the so called partial wave solution and the corresponding dispersion relations are obtained from the imposed boundary conditions. In the TE_{n0} approximation the dispersion characteristics of the investigated structure is obtained by the introduction of an artificial parameter the "longitudinal impedance" defined for each section shown in Fig.1 as

$$Z_i^L(x) = E_{zi}(x)/H_{yi}(x) \qquad (3)$$

where $E_{zi}(x)$ and $H_{yi}(x)$ are the corresponding field components of the propagating TE_{n0} mode within the waveguide.
Due to the assumed field distribution within the investigated structure the parameter introduced by equation (3) does fulfill automatically the electromagnetic boundary conditions at the slab interfaces. Taking into account that for the TE_{n0} mode the only nonzero electromagnetic field components are E_z, H_x and H_y the Maxwell's equations retain the form

$$\frac{\partial E_{zi}}{\partial y} = -j\omega(\mu_i H_{xi} + j\mu_{ai} H_{yi}) \qquad (4)$$

$$\frac{\partial E_{zi}}{\partial x} = j\omega(-j\mu_{ai} H_{xi} + \mu_i H_{yi}) \qquad (5)$$

$$\frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y} = j\omega\varepsilon_i E_{zi} \qquad (6)$$

where $\mathbf{B} = \bar{\bar{\mu}}.\mathbf{H}$ with permeability tensor in the form of (1) has been taken into account.

Let the field distribution within the i-th slab is given by

$$E_{zi} = [A_i \cos(k_{xi}x) + B_i \sin(k_{xi}x)]e^{-jsk_y y} \qquad (7)$$

where $s = \pm 1$ describes the direction of propagation of the electromagnetic wave in the $\pm y$ axis and k_{xi}, k_y are up to now unknown wavevector components within each slab. From (4) and (5) the magnetic field component H_{yi} can be expressed through the electric field component E_{zi} in the form

$$H_{yi} = \frac{j}{\omega \mu_{pi}} \left[sk_y \frac{\mu_{ai}}{\mu_i} E_{zi} - \frac{\partial E_{zi}}{\partial x} \right] \qquad (8)$$

where

$$\mu_{pi} = (\mu_i^2 - \mu_{ai}^2)/\mu_i, \tag{9}$$

the so called perpendicular permeability was introduced.

A same approach can be used to obtain a similar relation between the magnetic field component H_{xi} and the electric field component E_{zi}. The substitution of expressions for magnetic field components into (6) gives the relation between the wavevector components within the i-th ferrimagnetic slab in the form

$$k_{xi}^2 = \omega^2 \varepsilon_{fi} \mu_{pi} - k_y^2 \tag{10}$$

where ε_{fi} is the dielectric permitivity of the ferrite slab. The other symbols have been already introduced above.

The longitudinal impedance as defined in (3) within the i-th ferrite layer therefore can be expressed as

$$Z_i^L(x) = \frac{(-j\omega.\mu_{pi})[1 + D_{fi}\tan(k_{xi}x)]}{\left(sk_y\frac{\mu_{ai}}{\mu_i} - k_{xi}D_{fi}\right) + \left(sk_y\frac{\mu_{ai}}{\mu_i}D_{fi} + k_{xi}\right)\tan(k_{xi}x)} \tag{11}$$

where $D_{fi} = A_i/B_i$ is an unknown integration constant, which has to be obtained from the boundary conditions and i is the imaginary unit. Similarly the longitudinal impedance within the dielectric slab can be expressed as

$$Z_{di}^L(x) = \frac{(-j\omega.\mu_0)[1 + D_{di}.\tan(k_{xi}x)]}{k_{xi}(-D_{di} + \tan(k_{xi}x))} \tag{12}$$

where $k_{xi} = \sqrt{\omega^2\mu_0\varepsilon_{di} - k_y^2}$, and ε_{di} is the permitivity of the dielectric slab and again D_{di} an unknown integration constant. k_y is the common for each section wavevector component in the direction of propagation.

As already mentioned the continuity of longitudinal impedance Z^L at all interfaces fulfills automatically the necessary electromagnetic boundary conditions. As assumed the waveguide is ideal what means that on the walls $E_z = 0$ and therefore also $Z^L(wall) = 0$. This approximation is not necessary but we use it for convenience.

The expressions (11) and (12) up to now were independent on the chosen origin of the coordinate system. Let us now introduce the local primed coordinate system of each slab within the waveguide. Let further the i-th ferrite slab has the thickness t_{fi} and let the ferrite slab is at $x' = 0$ loaded with some impedance denoted Z_i^L and at $x' = t_{fi}$ by impedance Z_{i+1}^L. The boundary conditions require $Z_f^L(x' = 0) = Z_{i-1}^L$ and $Z_f^L(x' = t_{fi}) = Z_i^L$. Inserting for x into (11) and eliminating the unknown constant D_{fi} one obtains the expression

$$Z_{i+1}^L = \frac{(\omega\mu_{pi})\{k_{xi}Z_i^L + \left[sk_y\frac{\mu_{ai}}{\mu_i}Z_i^L + j\omega\mu_{pi}\right]\tan(k_{xi}t_{fi})\}}{\omega\mu_{pi}k_{xi} + \{jZ_i^L\left[k_{xi}^2 + k_y^2\left(\frac{\mu_{ai}}{\mu_i}\right)^2\right] - \omega\mu_{pi}sk_y\frac{\mu_{ai}}{\mu_i}\}\tan(k_{xi}t_{fi})} \tag{13}$$

which binds the longitudinal impedances on both sides of the ferrite slab. Similarly from (12) and the boundary conditions the expression connecting the impedances at the both sides of a dielectric slab of thickness t_{di} is

$$Z_{i+1}^L = \frac{(\omega\mu_0)\{k_{xi}Z_i^L + j\omega\mu_0\tan(k_{xi}t_{di})\}}{k_{xi}[\omega\mu_0 + jZ_i^L k_{xi}\tan(k_{xi}t_{di})]} \tag{14}$$

The expressions (13), (14) together with the conditions for Z^L at the walls of the waveguide allow to solve any dispersion problem in the TE_{n0} mode approximation by expressing longitudinal impedance at the boundaries of different slabs and matching them. This procedure represents a simple algorithm for obtaining dispersion relations for the desired structures. The procedure can be proved for simple structures such as a simple ferrite slab in the waveguide analytically. For more complicated structures the dispersion relation is obtained by the computer.

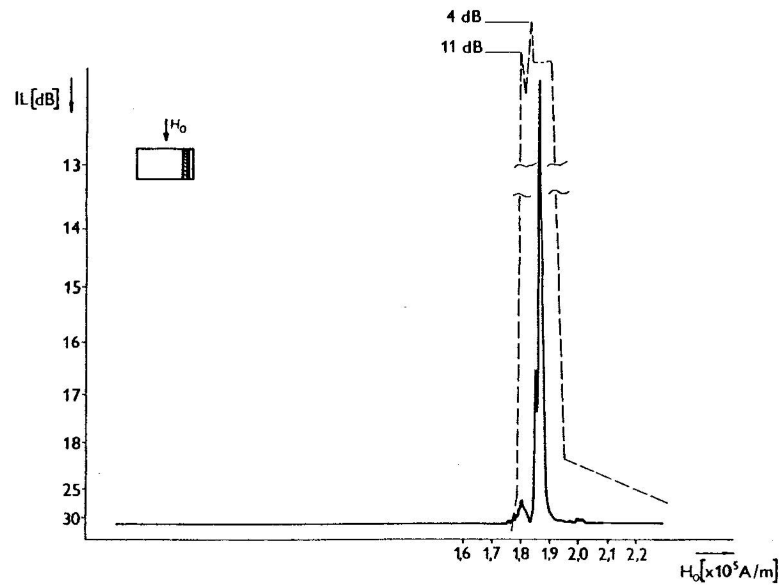

Fig. 2. AS recorded dependence of the insertion losses, as a function of the in plane external biasing magnetic field H_0. The theoretical results are shown by dashed line. ($a = 8$ mm, $b = 4$ mm, $l = 10$ mm, $t_f = 88$ μm, $t_{d2} = 0$ mm, $f = 8.5$ GHz)

III. RESULTS AND DISCUSSION

The experiments were performed at room temperature and X band frequencies using a rudimentary microwave equipment. The microwave power was provided by a klystron. A 3x12 mm^2 slab of 88 μm thick LPE grown YIG slab on GGG substrate has been placed in the 8x4 mm^2 rectangular, 10 mm long waveguide section. The saturation magnetization of the used YIG slab was M_s =140 kAm^{-1} and line width was $\Delta H \cong 120\ Am^{-1}$. The insertion loss of the waveguide section at 8.5 GHz was about 35 dB. The waveguide section has been placed between the poles of the electromagnet producing static magnetic field in the plane of the slab. The transmitted power was detected by a diode detector and recorded as a function of the external magnetic field H_0 by an $x-y$ plotter.

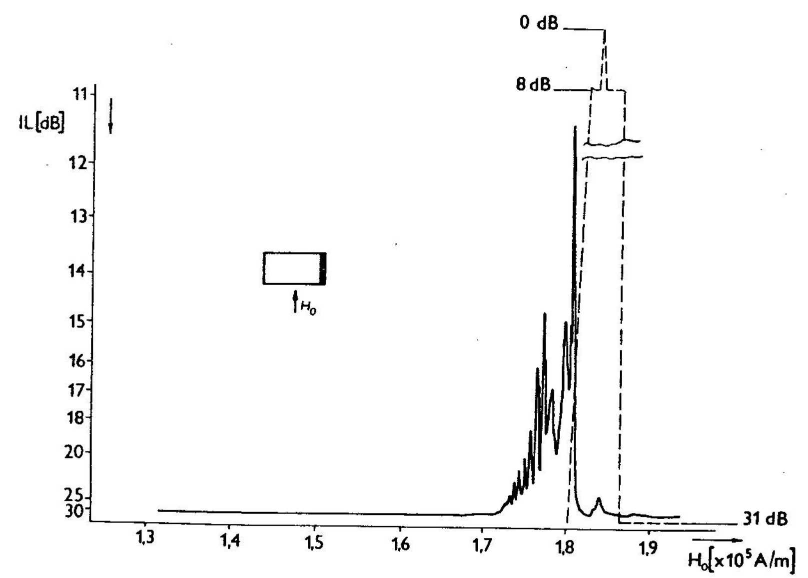

Fig. 3. As recorded dependence of the insertion losses for t_{d2} = 1mm, other parameters as in Fig.2.

The recorded dependence of the insertion loss for the slab placed at the side wall of the waveguide as a function of the external static magnetic field is shown in Fig.2. The magnetic field was swept through the region where $\mu < 0$, i.e. the conditions for the excitation of surface waves are fulfilled. Although the attenuation of the below cut-off waveguide section was on the order of 35 dB the recorded response in the Fig. 2 shows a dramatic increase in the transmitted power as different modes are excited in the slab as the magnetic field is swept. The change in the direction of the applied external magnetic field results in the change of the insertion loss dependence indicating nonreciprocal behavior of the investigated structure. The rich spectrum of excited modes can be attributed in this case to the excitation of the surface magnetostatic modes. Further measurements have shown, that slight change in the width or length of the waveguide has little effect on the observed surface wave spectrum.

Moving the slab away from the side wall the decrease in the amplitude of the excited surface waves is observed. The transmission of energy through the waveguide is for this position of the slab mainly due to the excitation of electromagnetic

waves in the narrow region of external magnetic fields corresponding to the so called perpendicular resonance (μ=0). Similar results can be obtained by changing the slab thickness. One of the typical recorded spectra for this case is shown in Fig.3. The drastic change in the recorded spectrum, especially in the region of the excitation of surface waves is obvious.

The theoretical results as calculated for the simple ferrite slab placed in the below cut-off waveguide are shown in Figs.2. and 3 by dashed lines. The overall agreement between calculated and experimental results is quite good. The observed differences are due to the fact that the presented calculation does not take into account (a) the difference in the coupling of different excited modes with the waveguide, (b) the fact that the ferrite slab of length l represents for the propagating magnetostatic modes a resonator, which behavior changes with the wavelength of the excited modes and (c) the influence of the dielectric slab and the losses in the walls of the waveguide. All this influences will be taken into account in the future work.

IV. CONCLUSION

The above results have shown that the positioning of a ferrimagnetic slab within the below cut-off waveguide section, can control the characteristics of the transmitted power through such structure and that it does support the transmission of the power through the variety of different excited modes. This modes are sensitive to the position of the slab as well as to the ferrite slab parameters. The presented modified theoretical approach offers a theoretical basis for the calculation of presented structures, but it needs some as mentioned refinements in course to be able to investigate the material parameters. The considered structure also shows promising properties for microwave device applications especially in the range of higher frequencies.

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