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SOME RECENT RESULTS IN NONLINEAR ACOUSTIC EFFECTS INVESTIGATION [1]

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The highly nonlinear acoustic properties of inhomogeneous solids are connected with defect structure. The possibility of nondestructive acoustic nonlinear strength control is discussed. Some peculiarities of measurements of high orders nonlinear parameters are considered.

I. INTRODUCTION

In this paper we consider the extremely high nonlinearity of some solids. There are interesting aspects of the problem, e.g highly nonlinear properties allow us to improve the efficiency of different types of acoustoelectronic devices. However, we concentrate our attention on the another feature: extremely high nonlinearity of structural inhomogeneous solids. Recently, this problem has attracted attention of some scientific groups and it developes sufficiently fast because it is perspective in microstructure diagnostics and consequently in strength control.

II. MOLECULAR, MIXED AND STRUCTURAL NONLINEARITIES.

As it is well known, the molecular nonlinearities (nonlinearities of ideal crystals) are caused by attractive and repulsive potential of ions in Born's model of dielectric solids. The values of the third order elastic modules of different crystals are known. For the longitudinal wave the nondimensional quadratic parameters $X_1 = b/a$, where b is the effective modulus of third order (linear combination of proper third order modules) and a is its second order analog. The value of X_1 does not significantly exceed 10 for the majority of crystals.[2]

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[2] For the Coulomb potential of attraction $\sim r^{-1}$, this parameter has an order n (the degree of repulsive potential decrease $\sim r^{-n}$).

Mixed nonlinearities are inherent in the solids with not only phonon subsystem, i.e. in such crystals as piezoelectrics, piezosemiconductors, magnetics etc. The subsystems connection cause sometimes very high nonlinearity on the elastic side. Giant magnetoelastic nonlinearity has been observed in magnetics, e.g. in antiferromagnetics (hematit [1]) and in ferrodielectrics (ferro-yttrium garnet [2]). Here, the high nonlinearity is caused by the spin-system. Under the definite conditions the spin-system has effective connection with the phonon nonlinearity and the magnetoelastic one can exceed that of the lattice by the 3-5 orders. These materials are very perspective to use in acoustoelectronic devices.

It was known long ago that the different scale defects and inhomogenities raise the middle nonlinearity. The interactions forbidden in ideal solids can be observed in real ones. For example the symmetry of shear deformation in isotropic solids does not yeld the generation of the second harmonic of shear wave. However, the prohibition is weak and weak second harmonic was observed. Its amplitude strongly depends on nonuniform external tension [3]. Very important results were obtained in [4], where it was shown that the nonlinear parameter was by 2-3 orders higher in porous media than in material without pores. Inhomogeneous materials have also a high value of X_1, for instance rocks ($|X_1| \cong 10^2$) [5], custed iron ($|X_1| \cong 3 \cdot 10^2$) and especially different types of concretes ($|X_1|$ exceed 10^3) [6]..

III. STRUCTURAL NONLINEARITY. SOME QUESTIONS OF STATISTICAL THEORY OF CRUSHING. NONLINEAR PARAMETERS - STRENGTH CORRELATION (SIMPLEST MODEL). ABSOLUTE AND RELATIVE STRENGTH.

The different scale defects, from the point to the macrocraks violate locally the Hook law and, consequently, they cause the local nonlinearities. As a rule, sound wavelength is much less than the typical size of the defect. An important conception of nonlinear scattering, namely the scattering with harmonics generation, was introduced by Sutin [7] from the point of general nonlinear wave theory. If the defect concentration is high, then interactions of secondary fields make full picture extremely complicated. Nevertheless, we can suppose effective nonlinear parameters to be proportional to defect concentration for small concentrations.

Different types of nonlinear scatterers are possible. Let us consider some of them quantitatively.

One of them is the Hertz contact. The contact problem of two balls was solved first by Hertz. The strainstress distribution inside balls has a complicated form but one can use ball centres relative reapproachment as a strain in the low-frequency approximation and the tension on unit surface represents a stress for cubic packing of balls of equal size. Then in case of one-dimensional longitudinal tension for such grain media we obtain

$$\sigma = G_0\, \varepsilon^{3/2}, \tag{1}$$

where $G_0 = \rho_0 c_0^{2/3}$, ρ_0 is the density of ball material, and c_0 is the sound velocity in solid media of the same material. If the initial deformation of grain media is ε_0,

then the expansion in the neighbourhood of ε_0 reads

$$\sigma/G = (\varepsilon - \varepsilon_0) + \frac{1}{4\varepsilon_0}(\varepsilon - \varepsilon_0)^2 + \frac{1}{24\varepsilon_0^2}(\varepsilon - \varepsilon_0)^3, \tag{2}$$

where $G = 3G_0\varepsilon_0^{1/2}/2$ is the elastic modulus of grain media. Here, the sound velocity $c = c_0\,(9\varepsilon_0/(\pi^2))^{1/4}$ may be much less than in solid ones. It is known for example that the sound velocity in the sand near the earth surface (at the depth of approximately two meters) is close to 500 m/s. As the sound velocity in solid quartz is 5000 m/s the initial deformation is 10^{-4}at that depth ε_0. From (2) we obtain the nonlinear parameter $|X_1| \cong 2.5 \times \cdot 10^3$. The value $\sim 10^3$ for underground layer of earth was obtained many times in experimental conditions in nonlinear coherent seismology [8]. We shall use later the nonlinear cubic parameter $X_1 X_2$. From (2) we can see:

$$X_1 = \frac{1}{4\varepsilon_0}; \qquad X_2 = -\frac{1}{6\varepsilon_0} \tag{3}$$

Hertz media nonlinear parameters at small ε_0 can be very high. Contact nonlinearity may be observed not only in sandy media, as it was discovered in polycrystalline metal [9]. There is a lot of papers in which artificial Hertz media are used to obtaine high nonlinearity. The above results are applied strictly only to the media of equal radius of grain. An interesting investigation of contact surface nonlinearity with random Hertz contacts distribution (the random radii are partly free) was done in [10]. The correlation between the nonlinear parameters and the medial initial deformation ε_0 was approximately governed by (3) therein. This allows us to generalize (3) with some part of confidence on media with random contact distribution.

Another type of high local nonlinearity arises near the microcraks. If crack thickness in equilibrium is less than the displacement amplitude then the crack is a different modulus object. The local modulus becomes equal to a continuous media one after the crack slams. But in the phase of stretching the effective modulus is much less. Such modulus difference causes high values of X_1 and X_2. Sometimes this nonlinearity is called clapping.

One should say some words about bubbles media. Experimental investigation [4] showed the growth of the nonlinear parameter of such media especially in the region of low bubble concentration. Liquid bubble media have been investigated more accurately than the solid ones. The increase of the nonlinear parameter by two-three orders was obtained in liquid-air system. Nonlinearity of porous solids was theoretically investigated in [11], where it was shown that high nonlinearity may be expected in rubberlike porous solids (with small shear modulus).

The dislocation structure also causes an additional nonlinearity [3,12], see well known effect of amplitude dependent internal friction for instance .

The above types of nonlinearities contribute to full nonlinearity. The matrix nonlinearity is much less than that one caused by defects in solids with a lot of defects. From the other point of view the above types of defects are the origin of destroying processes. This qualitative assumption may be the starting point to a solution of the very important technical problem of nonlinear acoustic strength prediction [13,14].

The strength problem has laid long ago in the focus of attention of mechanics, physicists and other specialists of corresponding technical disciplines. The problem is extremely complicated because the crushing process depends on a great number of factors and conditions. We hope that the methods of nonlinear acoustics could contribute to its solution. Of course even in future the nonlinear methods will not be able to classify defects as dangerous and non-dangerous ones. However, we hope that it will be possible to obtain an information about the average concentration of defects from acoustic data. The statistical theory of strength can give us a correlation between that concentration and an expected strength. A lot of papers concerning the statistical theory of strength was written. Here, we consider a new treatment, developed in [15] in which the probability of local dangerous defects concentration was calculated in another correct way.

Let us consider the following problem: there are N equal noninteracting defects in volume V. Their average concentration is $n = N/V$. We have to find the probability that m from N defects are located in a little volume $\Delta \ll V$, so that the local concentration of defects $n_{cr} = m/\Delta$ becomes critical for a given external stress p. Connection of critical concentration with rupture stress demands rupture mechanics notions. It was shown by Griffits [16] from energetic consideration that $p\tau^{1/2}$ = const (p is rupture stress) for the crack of dimension τ. By the way, $pa^{3/2}$ = const for liquid rupture on the bubble (a is the initial bubble radius). Suppose $p\tau^{s}$ = const ($s \cong 1/2 \div 3/2$). Then by modelling of the critical defects concentration by microcracks or micropores we can obtain the rupture probability in the form

$$W(p) = \frac{n_{cr}V}{\sqrt{2\pi}} \left(\frac{p_0}{p}\right)^{3/s} \exp\{-\beta(p_0/p)^{2/s}\} \qquad (4)$$

where $p \propto (n_{cr}h)^{5/2}$; $\beta = \ln(n_{cr}/n)^{-1}$; h is the characteristic dimension of the defect. The function $W(p)$ is defined only for those p, where $W(p) \leq 1$ or $p \leq Rp_0$, where R is the nondimensional rupture limit obtained from the equation $W(p) = 1$. The approximate solution of the last equation is

$$R \cong \left[\frac{\ln(n_{cr}/n) - 1}{\ln(n_{cr}V/\sqrt{2\pi})}\right]^{s/2} \qquad (5)$$

It is obvious that solution (5) is correct for large N if $nV \gg 1$ and if there is a sufficiently large difference between the critical concentration and the average one. The functions $R(n)$ are shown in Fig. 1 for $s = 1/2$ and $s = 3/2$. The dotted line shows the intervals where the solution (5) is not fulfilled. Logarithmical dependence of strength on the average concentration of defects gives us the slow strength decrease, too. Probably it is due to the leaving out of the collective defects interaction, the reproduction of which rises in the process of the tension. This statistical model needs further improvement.

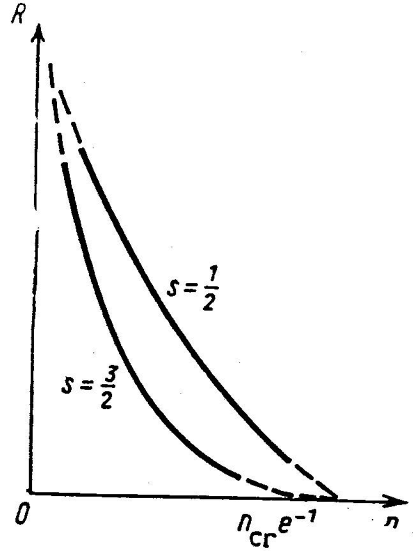

Fig. 1. Dependence on nondimensional rupture strength from middle concentration defects.

One can suppose the nonlinear parameters X_1 and X_2 to be proportional to n in the first approximation and for low initial defect concentration. Another type of strength decrease with defects concentration can be shown within the simplest model of nonlinear solids in terms of nonlinear parameters. Further considerations do not claim to be rigorous [13,14], but they give us a simple connection between the strength and nonlinear parameters. Let us suppose the stress-strain connection $\sigma(\varepsilon)$ has the form

$$\sigma(\varepsilon)/M = \varepsilon + X_1\varepsilon^2 + X_1X_2\varepsilon^3. \qquad (6)$$

Here M is the linear elastic modulus for the strain-compression deformation ε. It is the simplest model of a nonlinear elastic solid with square and cubic nonlinearities. In this model the critical deformations can be obtained from the square equation $\sigma'(\varepsilon) = 0$ for brittle crushing (and the plasticity limits in other cases). Because $X_1 < 0$ and X_2 is possitive, the quadratic equation has two roots, $\varepsilon_1 < 0$ and $\varepsilon_2 > 0$. The compressional limit is $R_1(X_1, X_2) = \sigma(\varepsilon_1)/M < 0$ and the strain one is $R_2(X_1, X_2) = \sigma(\varepsilon_2)/M > 0$. It can be shown that $|R_1| > R_2$. The general solution is too bulky. In the limit case of "steady nonlinear solids" $|X_1| \cong X_2$

$$R_1 \cong -1/|X_{1,2}|; \qquad R_2 \cong 5/27|X_{1,2}|. \qquad (7)$$

For the "strongly cubic solids" $X_2 \gg |X_1|$ and

$$R_1 \cong \frac{2}{3\sqrt{3|X_1|X_2}} - \frac{1}{3X_2};$$

$$R_2 \cong \frac{2}{3\sqrt{3|X_1|X_2^{1/2}}} - \frac{1}{3X_2}. \qquad (8)$$

Surface topographies $R_1(X_1, X_2)$ and $R_2(X_1, X_2)$ are shown in Fig. 2. One can see that the growth of $|X_1|$ and X_2 causes a decrease of the limits of strength. The small raise of compressional limit $|R_1|$ has probably not taken place in reality for "strongly quadratic solid" ($|X_1| \gg X_2$) for $|X_1| > 10^2$. No real solid is known in this region. One can obtain the physical meaning of X_2 from limits $X_2 \to 0$, $R_1 \to -\infty$ and $X_2 \to 0(\infty)$, $|R_1| \to R_2$. The first one means that the solid has a strong side support (nonlinear Poisson coefficient ~ 0), the second one - that the support is weak (nonlinear Poisson coefficient $\sim 0,5$). The conclusion about the strength decrease with nonlinearity parameters growth is correlated with the above results of the statistical theory of strength.

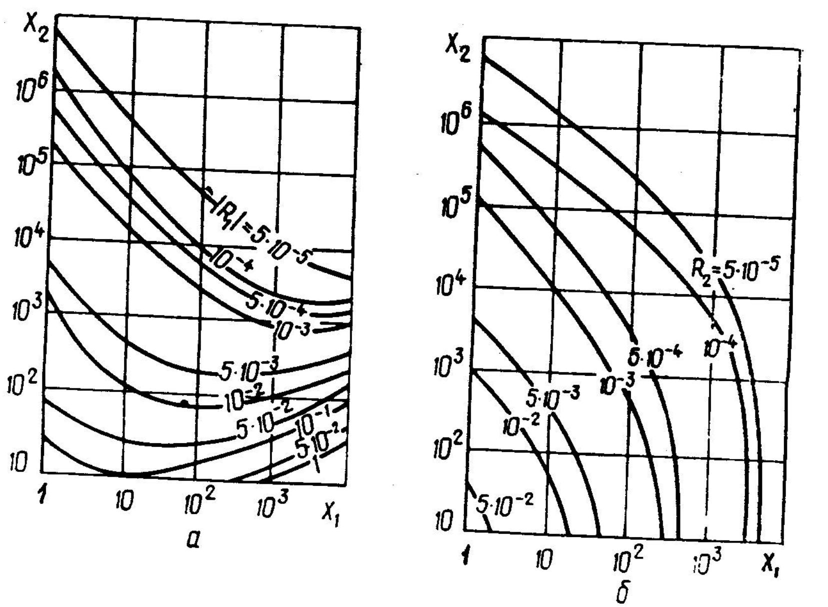

Fig. 2. Topography (the line of equal strength) of the surface a) tensile strength $|R_1(X_1X_2)|$ and b) rupture strength $R_2(X_1X2)$.

As an example let us apply Eq. (7) to the Hertz media (2). There are two roots for critical deformation and we obtain the strength limits

$$R_1 \cong (4 \div 6)\varepsilon_0; \quad R_2 = -5/27(4 \div 6)\varepsilon_0 \qquad (9)$$

The value of strain limit R_2 is near $-\varepsilon_0$. It is clear that such stretching of grain media (with initial deformation $+\,\varepsilon_0$) breaks it apart. The tension limit $R_1 \to 0$ for $\varepsilon_0 \to 0$; it means that the medium without any initial deformation has not any

tensile strength. As ε_0 increases, the strength increases too, but the limit R_1 is not definite because this model does not include the critical tension of each grain.

Essentialy the simplest nonlinear solid model (6) is far from real solids with their rheological and hysteresis properties. Acoustic investigations of mechanical properties up to the level of deformation $10^{-4} \div 10^{-5}$ can not reflect the processes developed near the crushing region. Acoustic emission results show that in this region a great number of new defects is generated, the dislocations move and coagulate, microcracks grow up and join together to form macrocracks and so on. All theses chaotic processes have only an indirect connection with original defect structure. From this point of view one can not hope to obtain the absolute value of strength from nonlinear data. An exceptional case perhaps is the solid with a very bad original structure of defects (e.g. a concrete in which the strength obtained by the nonlinear method is near the standard one [14,15]). In other cases, one may expect a too high limits.

However, using the fact that the high order moduli are connected with initial defects structure and that those are correlated with embryonic stucture of crushing processes one may expect to obtain relative strength of two equal samples from acoustic data. It is not necessary to emphasise that this problem is sometimes very important in microstructure defectoscopy and in material tiredness problems. Nonlinear acoustic control may be sometimes much more sensitive than usual methods of linear defectoscopy.

IV. SOME PECULIARITIES OF HIGH ORDER MODULUS MEASUREMENT

This communication would not be complete if we do not briefly mention the experimental methods of high orders modulus determination. As these methods have been developed long ago, it is not necessary to consider the problem in detail.

The first method consists of determination of static stress dependence on the sound velocity. Measured nonlinear parameter depends on the wave type, orientation and type of external tension. In the case of quadratic nonlinearity, velocity is a linear function of a tension. Deviation from linearity at upper range of tension is caused by higher orders of nonlinearity. The drawbacks of this method are in the low tension region, where dislocation and internal stresses reconstruction begins and where an irregular deviation from linearity and hysteresis effects may be observed. In the high tension region there are possible irreversible effects caused by destroying the original internal structure.

This method has been further developed into the modulation method, in which the static tension is changed by low-frequency sound. Sound-sound interaction causes an appearance of side-band components; index of the nonlinearity is proportional to the amplitude of these spectrum components at low modulation.

The quadratic nonlinear parameter can be determined from the amplitude of the second harmonic. The disadvantage of the modulation method is that the sound field absolute measurement is necessary to obtain the absolute value of X_1. This makes these experiments too complicated. To avoid the absolute measurement one should use a relative one.

It is necessary to say that nonlinear effects in solids are very small and correct measurements require a sufficient accuracy. As an example the amplitude of second harmonics is by two-four orders lower at the strain in region $\sim 10^{-6} \div 10^{-5}$. However, there is a possibility to increase the effect by using the resonant properties of the solid sample (as an acoustic detection of a modulated signal by one of the resonances of the rod [17]) for a weak signal.

The problem of determination of cubic nonlinearity is more complicated because the effects are very small and they are subjected to the influence of different interferring factors. This is a reason why only a few data of the forth order modulus are known till now. It is necessary to mention also the example, where the incorrectness of modulus measurement of the amplitude of the third harmonic plays its role. This harmonics in the cubic process, could be successively generated by two quadratic processes $\omega + \omega = 2\omega$; $2\omega + \omega = 3\omega$. The change the phase velocity with the amplitude is one of the "pure cubic" effects. In the case of acoustic resonator this effect has been observed a long time ago: The resonant frequency changes and the resonance curve becomes asymmetric at increase of the vibration amplitude. However, the change of resonant frequency takes place not only because there are four- order moduli, but also because powerful sound heats the resonator and the changes of the temperature cause the shift the resonance frequency. For the majority of solids nonlinear and temperature frequency changes occur on the same side of the linear resonant frequency. Separation of these two effects is possible thanks to the fact that thermal effects are much slower than the nonlinear ones. The nonlinear effect due to the selfheating is interesting in itself. For example it allows precise measurement of heat transfer through the surface of the resonator, including the nonlinear coefficient of heat transfer at the temperature difference of the order of some degrees of Celsius [18].

Phase methods are very useful for the measuring changes of the sound velocity amplitude. This method was used long ago [19] in the case of liquids, where one can obtain a travelling waves without any troubles. It is not difficult to modify this method for the simplest cases of standing wave: the increase of the wave amplitude is accompanied by the change of the received signal phase in respect to the input signal phase. This $\Delta\varphi$-method was used to determine the strength of high-quality concrete [13,14]. The dependence of the crushing limit from the phase change $\Delta\varphi$ is shown in Fig. 3. Real limits of different samples were determined by the tensional destroying of the samples in the machine. We can see a good correlation $R_1 \to \Delta\varphi$. $\Delta\varphi$ is proportional to the cubic nonlinear parameter X_1X_2. Increasing the last one decreases the strength, in qualitative agreement with (8). However $R_1 \sim 1/(\Delta\varphi)^{1/2}$ (8) and decreasing of R_1 must be sharper than the average experimental data. The scattering of experimental data is incidentally large. The $\Delta\varphi$-method is near the resonant one and has the same troubles in an idea sense.

To conclude, let us mention that the nonlinear acoustodiagnostics may give us useful integral data about the microstructure of a solid. The methods, how to obtain information about strength from these data are very perspective. I hope that the estimation of relative strength can be obtained. It is necessary to continue the investigation in this direction. We can expect to hear about the useful technical solutions of some important problems.

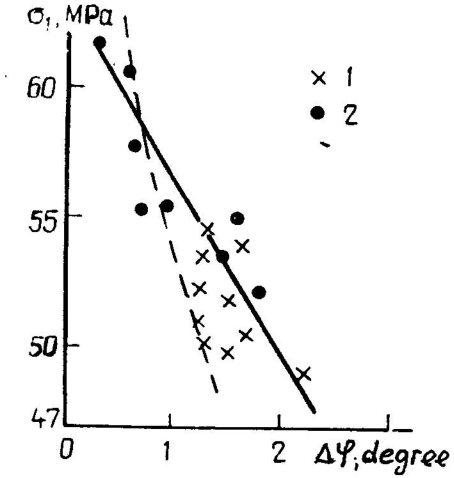

Fig. 3. Correlation of absolute value of tensile strength σ_1 of two concrete marks B40 (×) and B45 (·) (obtained by crushing of samples on the destroying machine) with the phase change $\Delta\varphi$ (obtained at the ultrasound of 40 kHz whose amplitude has been increased ten times). The dotted line is $\sigma_1 \sim 1/(\varphi)^{1/2}$.

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