

DAMPING OF THE $1h_{11/2}$ NEUTRON HOLE STATE OF THE $^{145-149}\text{Nd}$

R. Majumdar

*Nuclear Physics Study Centre,
30/41 Attapara Lane, Calcutta - 700050, India.*

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The distribution of the shell-model hole strength of the $1h_{11/2}$ state of the $^{145-149}\text{Nd}$ has been obtained within the framework of the Rotation-Particle Coupling model and the Quasi-Particle Phonon Coupling model schemes. The electric quadrupole and magnetic dipole moments as well as the $B(E2)$ and $B(M1)$ transition rates have been calculated from the wave functions derived within the Quasi-Particle Phonon Coupling model calculations. The theoretical results have been compared with the existing experimental results on the above nuclei.

I. INTRODUCTION

During the past few years various experiments based on the stripping and pick-up reactions have been performed to know the structures of odd- A nuclei in the vicinity of $A=145$ [1-6]. The study of the structures of these nuclei merits special importance because the nuclei lie in the transitional region. Recently the level systematics of $^{145-149}\text{Nd}$ have been reproduced by Lovhoiden et al [7]. The distribution pattern of the $1h_{11/2}$ neutron hole state of these three nuclei have been obtained at first within the Rotation-Particle coupling model calculation. Here we have taken coriolis effect on the band mixing calculation. Then the spreading of the hole strength of the $1h_{11/2}$ states have been calculated within the Quasi-Particle Phonon coupling model scheme. The applicability of both the models seems to be optimistic because the nuclei lie near the rare earth region.

II. ROTATION-PARTICLE COUPLING MODEL

The Hamiltonian for an odd nucleon moving in a deformed potential has a particle-rotation coupling term,

$$H_{\text{RPC}} = \hbar^2/2\mathcal{J}[I_+j_- + I_-j_+] \quad (1)$$

Here, j is the total angular momentum of a Nilsson orbital whose projection along the nuclear symmetry axis is Ω . The rotational nucleus has spin angular momentum $I = j$. The rotational angular momentum R couples with j and vectorially,

$$\vec{R} + \vec{j} = \vec{I} \quad (2)$$

The cross-section for a rotational state $I = j$ depends on the chance of finding the odd nucleon in a Nilsson state having total angular momentum j . The matrix elements of the operator $I_+ j_- + I_- j_+$ are

$$\begin{aligned} &< IMK | I_+ j_- + I_- j_+ | IM(K+1) \rangle \\ &= (\hbar^2/2J)(UU' + VV')A_{K,K+1}[(I-K)(I+K+1)]^{1/2}. \end{aligned} \quad (3)$$

$A_{K,K+1}$ represents the matrix elements of the j_+ operator between the single particle orbitals characterised by K and $K+1$. U and V are the non-occupation and occupation probabilities of the shell-model hole orbital.

The state $|K = \Omega\rangle$ is the intrinsic state of an odd- A nucleus. The state gives a rotational band that consists of levels $I = K, K+1$ etc. The off-diagonal matrix elements of H_{RPC} are non zero only when $\Delta K = 1$ or when $K = K' = 1/2$. For $K = 1/2$,

$$H_{RPC} = -(\hbar^2/2J)(UU' + VV')A_{\frac{1}{2},\frac{1}{2}}(-1)^{I-1/2}(I+1/2). \quad (4)$$

The Hamiltonian of the physical system is diagonalised by writing the wavefunction for $K = \Omega$ state as

$$\chi_K = \sum_j C_{ji}^K \phi_{jR}. \quad (5)$$

Here K represents band quantum number, j is the total angular momentum of a Nilsson orbital whose projection along the nuclear symmetry axis is Ω . The matrix elements of $A_{K,K+1}$ and $A_{\frac{1}{2},\frac{1}{2}}$ are

$$\begin{aligned} A_{K+1,K} &= A_{K,K+1} = C_{ji}^K C_{ji}^{K+1} [(j-K)(j+K+1)]^{1/2} \\ A_{\frac{1}{2},\frac{1}{2}} &= C_{ji}^{1/2} j_i^{1/2} (-1)^{j-1/2} (j+1/2). \end{aligned} \quad (6)$$

The diagonal matrix elements of the Hamiltonian are

$$E(I, K) = E_K + (\hbar^2/2J)[I(I+1) + \delta_{K,1/2} a(-1)^{I+1/2}], \quad (7)$$

where a is the decoupling parameter. Six Nilsson states $11/2^-$ (505), $9/2^-$ (514), $7/2^-$ (523), $5/2^-$ (532), $3/2^-$ (541) and $1/2^-$ (550), originating from the $1h_{11/2}$ shell model state, have been mixed within the framework of the rotation-particle coupling model to set up the Hamiltonian matrices for the $11/2^-$ spin states of $^{145-149}\text{Nd}$ nuclei. The decoupling parameter a and the band mixing parameters $A_{K,K+1}$ for the particular Nilsson state have been taken from Ref. 8. The input parameter E can be calculated from the eigen value of the Hamiltonian H for a deformed potential well. This can be written in units of $\hbar\omega$ as

$$H = \frac{1}{2}(-\nabla^2 + \rho^2) - \beta\rho^2 Y_{20} - 2k\vec{l} \cdot \vec{s} - \mu k\vec{l} \cdot \vec{l}. \quad (8)$$

Here,

$$\rho^2 = m\omega_0/\hbar r^2 \text{ and } \nabla^2 = \frac{\hbar}{m\omega_0} \sum_{x=1}^3 \partial^2/\partial x^2. \quad (9)$$

The value of E_K has been taken from Ref. 9. We have taken $k = 0.5$, $\mu = 0.63$ and $\beta = 0.1, 0.2, 0.3$ respectively for $^{145-149}\text{Nd}$. E_K and $\hbar^2/2J$ have been optimised for the diagonalisation of the $11/2^-$ matrix.

III. QUASI-PARTICLE PHONON COUPLING MODEL

The Hamiltonian of physical system within the framework of this model can be written as

$$H = H_n + H_{vib} + H_{int}. \quad (10)$$

Here $\langle H_n \rangle$ is the energy of the neutron state in the average shell-model potential. $\langle H_{vib} \rangle$ is the vibrational energy of the core nucleus. $\langle H_{int} \rangle$ represents the core-particle interaction Hamiltonian which is given by

$$H_{int} = -K(\tau) \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi). \quad (11)$$

Here $K(\tau)$ is the radial part of the potential well.

In order to know the eigen values and eigen vectors for H we write for the wave function for the J^π spin state with its projection M along the Z -axis at an energy $E^{(\alpha)}$ as

$$\psi_J = |E^{(\alpha)}, JM\rangle = \sum_{R_i} a_{R_i}^{(\alpha)} \{ (N_2 R_2, N_3 R_3) R_i; (n1/2) \}^{J^\pi} \quad (12)$$

where j is the angular momentum of the shell-model state, R_2 and R_3 are the phonon angular momenta for the quadrupole phonon number N_2 and the octupole phonon number N_3 and R is the coupled angular momentum of R_2 and R_3 . In the present work $N_2 = 1, 2, 3$ and $N_3 = 1$. The J is equal to $R+J$. Writing $\alpha_{\lambda\mu}$ in terms of the creation and annihilation operators $b_{\lambda\mu}^+$ and $b_{\lambda\mu}$ we have for $\langle H_{int} \rangle$

$$\begin{aligned} &< N_2 R_2, N_3 R_3; R, j; J | H_{int} | N_2' R_2', N_3' R_3'; R' j'; J \rangle = \\ &\sum_{\lambda} \chi_{\lambda} \hbar \omega_{\lambda} (-1)^{R+J} [2(2R+1)(2j+1)(2R'+1)(2R_{\lambda}+1)]^{1/2} W(R' R_j j; \lambda J) \\ &\begin{pmatrix} j & \lambda & j' \\ 1/2 & 0 & 1/2 \end{pmatrix} [\delta_{\lambda 2} W(R' R_{\lambda} R_{\lambda}; \lambda R_2') (-1)^{R_{\lambda}' - R - R_2' - \lambda} \\ &+ \delta_{\lambda 3} W(R' R R_{\lambda} R_{\lambda}; \lambda R_2') (-1)^{R' + R_{\lambda} - R_2' - \lambda} \delta_{N_2, N_2'} \delta_{N_3, N_3'}] \delta_{N_2, N_2'} \delta_{N_3, N_3'} \\ &< N_{\lambda} R_{\lambda} \| b_{\lambda}^+ \| N_{\lambda}' R_{\lambda}' \rangle + \delta_{N_{\lambda}, N_{\lambda}'} \delta_{N_{\lambda} + p, N_{\lambda}'} (-1)^{R_{\lambda} - R_{\lambda}'} \\ &< N_{\lambda}' R_{\lambda}' \| b_{\lambda}^- \| N_{\lambda} R_{\lambda} \rangle [(U_j U_j - V_j V_j)], \end{aligned} \quad (13)$$

where [10]

$$\chi_{\lambda} = \left(\frac{2\lambda + 1}{4\pi \hbar \omega_{\lambda} C_{\lambda}} \right) K(\tau), \quad (14)$$

$h_{\omega\lambda}$ is the energy of the phonon state for the λ -mode vibration of the collective core nucleus. W is the Racah coefficient and U_j, V_j are the non occupation and value 2 when $\lambda = 3$ and vice versa. Here $p = \Delta N_{\lambda} \text{ and } N_{\lambda}$ is the phonon number for the λ -mode vibration. The spectroscopic factor of the J -hole state is the squared amplitude of the zero phonon coupled state weighted with V_j^2 and is given by

$$S_j = V_j^2 |a_{0j}|^2 \quad (15)$$

We have coupled $1h_{11/2}, 2f_{7/2}, 1h_{9/2}, 3p_{3/2}, 3p_{1/2}, 2f_{5/2}, 1i_{3/2}, 3s_{1/2}$ and $2d_{3/2}$ states of the $148\text{--}150\text{Nd}$ to set up the $11/2^-$ Hamiltonian matrices of the same nuclei. The energy levels of the vibrational states of the core nuclei have been taken from the refs. [11-13]. The quasi particle energies E_j have been calculated from the pairing gap Δ_n and is given by

$$E_j = [(e_j - \lambda)^2 + \Delta_n^2]^{1/2}, \quad (16)$$

Δ_n is calculated from the neutron separation energies S_n [14] and is [15]

$$\Delta_n = 1/4[S_n(N-1, Z) - 2S_n(N, Z) + S_n(N+1, Z)]. \quad (17)$$

The V_j^2 is calculated from the relation

$$V_j^2 = 1/2 \left[1 + \frac{\epsilon - \lambda}{E_j} \right] \quad (18)$$

From the wave functions of the J^π nuclear levels magnetic dipole moments, electric quadrupole moments and the reduced transition probabilities $B(E2)$ and $B(M1)$ between the relevant nuclear J^π states have been calculated. The electric quadrupole operator is given by

$$m(E2, \mu) = \frac{3}{4\pi} ZeR_0^2 \left(\frac{h\omega_2}{2C} \right)^{1/2} [b_{2\mu} + (-1)^\mu b_{2\mu}^*] + e_{eff} \left(1 + \frac{Z}{A} \right) r^2 Y_{2\mu} \quad (19)$$

where e_{eff} denotes the effective charge for the neutron to account for the polarization effect of the core. The magnetic dipole operator is given by

$$m(M1, \mu) = (3/4\pi)^{1/2} (g_l l + g_s S + g_R R) NM, \quad (20)$$

where NM denotes the nuclear magneton. g_l, g_s and g_R are the g factors for the orbital angular momentum, spin and core respectively. For the calculation of the electric quadrupole moment we have

$$\begin{aligned} Q &= \langle E^\alpha; J, M = J | \left(\frac{16}{5} \pi \right)^{1/2} m(E2, \mu = 0) | E^{(\alpha)}; J, M = J \rangle \\ &= \left(\frac{J(2J-1)(2J+1)}{(2J+3)(J+1)} \right)^{1/2} \sum_{R_1 R_2} a_{R_1 R_2}^{(\alpha)} a_{R_1' R_2'}^{(\alpha)} \\ &\quad \left[\frac{3}{\sqrt{5} \pi} ZeR_0^2 \left(\frac{h\omega_2}{2C_2} \right)^{1/2} Z_1 + 2Z_2 \right], \quad (21) \end{aligned}$$

where

$$\begin{aligned} Z_1 &= (-1)^{R_2 - R_3 + J - j'} [(2R_2' + 1)(2R + 1)(2J' + 1)(2R' + 1)(2R_2 + 1)]^{1/2} \\ &\quad [\delta_{N_2, N_2' + p} < N_2 R_2 \parallel b_2^2 \parallel N_2' R_2' > + \delta_{N_2', N_2 + p} \\ &\quad (-1)^{R_2 - R_2'} < N_2' R_2' \parallel b_2^2 \parallel N_2 R_2 >] \\ &\quad W(j' R' J_2; j' R) W(R_3' R_2' R_2; R' R_2) \delta_{N_3, N_3'} \delta_{R_3, R_3'} \delta_{j, j'} \\ Z_2 &= [(2J' + 1)(2j + 1)]^{1/2} W(R' j' 2; j' j) < l' j' | r^2 | l j > < j' | Y_2 | j > \\ &\quad (U_j U_{j'} - V_j V_{j'}) \delta_{R, R'} \end{aligned}$$

and $R_0 = 1.2A^{1/3}$ fm is the nuclear radius.

The reduced transition probability is given by

$$B(E2; \alpha J \rightarrow \beta J') = (2J' + 1)$$

$$\left[\sum_{R_1 R_2} a_{R_1 R_2}^{(\alpha)} a_{R_1' R_2'}^{(\beta)} \left[\frac{3}{4\pi} ZeR_0^2 \left(\frac{h\omega_2}{2C_2} \right)^{1/2} Z_1 + \left(\frac{5}{4\pi} \right)^{1/2} e_{eff} \left(1 + \frac{Z}{A} \right) Z_2 \right] \right]^2 \quad (22)$$

The magnetic dipole moment is given by

$$\begin{aligned} \mu_d &= \langle E^{(\alpha)}; J, M = J | \left(\frac{4}{3} \pi \right)^{1/2} m(M1, \mu = 0) | E^{(\alpha)}; J, M = J \rangle \\ &= \left(\frac{J(2J+1)}{(J+1)} \right)^{1/2} \sum_{R_1 R_2} a_{R_1 R_2}^{(\alpha)} a_{R_1' R_2'}^{(\alpha)} (Z_3 g_l + Z_4 g_s + Z_5 g_R), \quad (23) \end{aligned}$$

where

$$\begin{aligned} Z_3 &= (-1)^{l' - l + j - j'} W(R' j' J_1; j' J) W(1/2 l' j_1; j' 1/2) \\ &\quad [(2J' + 1)(2j + 1)(2j' + 1)(2l + 1)(l + 1)]^{1/2} \\ &\quad (U_j U_{j'} + V_j V_{j'}) \delta_{N_2, N_2'} \delta_{R_2, R_2'} \delta_{N_3, N_3'} \delta_{R_3, R_3'} \delta_{j, j'} \\ Z_4 &= (-1)^{l' - l + j - j'} W(R' j' J_1; j' j) W(l' 1/2 j_1; j' 1/2) \\ &\quad [3/2(2j' + 1)(2J' + 1)(2j + 1)]^{1/2} \\ &\quad (U_j U_{j'} + V_j V_{j'}) \delta_{N_2, N_2'} \delta_{R_2, R_2'} \delta_{N_3, N_3'} \delta_{R_3, R_3'} \delta_{j, j'} \end{aligned}$$

and

$$Z_5 = (-1)^{R' - R + J - J'} W(J' R' J_1; j' R) [R(2J' + 1)(2R + 1)(R + 1)]^{1/2} \delta_{j, j'} \delta_{R, R'}$$

The corresponding $B(M1)$ transition rate is given by

$$B(M1; \alpha J \rightarrow \beta J') = \frac{3(2J' + 1)}{4\pi} \left| \sum_{R_1 R_2} a_{R_1 R_2}^{(\alpha)} a_{R_1' R_2'}^{(\beta)} (Z_3 g_l + Z_4 g_s + Z_5 g_R) \right|^2. \quad (24)$$

IV. RESULTS AND DISCUSSION

The experimental results for the fragmentation of the $1h_{11/2}$ hole states of $^{145-149}\text{Nd}$ have been shown in the Table 1. The theoretical results for the same have been depicted in the Tables 2 and 3. The fragmentation of the hole states of $^{145-149}\text{Nd}$. The band mixing calculations (Table 2) show the three distinct fragmented levels of ^{149}Nd . The major shell model strength of the two fragments at 1.141 and 0.944 MeV respectively have been carried away by the $f_{5/2}$ and $p_{3/2}$ Nilsson orbitals of ^{149}Nd . The dilution of the single particle strength of the $11/2^-$ state in $^{145,147}\text{Nd}$ is more pronounced in comparison to that of ^{149}Nd (Table 1).

Table 1
The experimental fragmentation of the $1h_{11/2}$ neutron hole states in $^{145-149}\text{Nd}$. E is the energy of the fragment in MeV and S is the spectroscopic factor.

$A = 145$	$A = 147$		$A = 149$	
	E	S	E	S
1.803	1.90	1.457	1.80	0.556
3.026	1.10	1.506	0.50	0.648
3.153	0.57	1.671	0.18	1.622
		2.430	0.29	0.65
		2.480	0.20	

But the theoretical results (Table 2) for the distribution of the $1h_{11/2}$ state in $^{145,147}\text{Nd}$ do not corroborate with the experimental ones (Table 1). Both the $1h_{11/2}$ states at 1.381 MeV and 1.729 MeV in $A = 145$ and $A = 147$ nuclei (Table 2) carry the main percentage of the total shell model hole strength. The other fragments of the $11/2^-$ states primarily originate from the $1h_{11/2}$, $2f_{5/2}$, $2f_{7/2}$, $2p_{1/2}$ and $2p_{3/2}$ Nilsson orbitals. This is the main drawback of the RPC model in $^{145,147}\text{Nd}$ nuclei. The basic reason for this discrepancy is due to less deformed nature of these two nuclei. The broad fragmented nature of the $11/2^-$ states in $^{145,147}\text{Nd}$ can be explained on the basis of the Quasi Particle Phonon Coupling model calculation. The hole strength distribution of the $11/2^-$ states in $^{145,147}\text{Nd}$ is spread over a broad excitation energy because of the proximity of the energies of the $1h_{11/2}$ shell model orbital with the several vibrational coupled hole state configurations. This is the prime reason for the loss of the shell model identities of the $11/2^-$ states in $^{145,147}\text{Nd}$. Also the nature of spreading of the shell-model strength of the $1h_{11/2}$ states as revealed from our theoretical works indicates the collectivities of the $^{146,148}\text{Nd}$ core nuclei. Quantitatively, there are three very weak fragments of the $11/2^-$ states in ^{145}Nd (Table 1) and they range from 1.8 to 3.2 MeV. Our calculated results on ^{145}Nd (Table 3) show that all the fragments of the $11/2^-$ states are weakly excited and they extend beyond the 3.0 MeV excitation energy. Also by virtue of their small shell model strengths, they are strongly collective in nature. In case of ^{147}Nd , we see five weak fragments of the $11/2^-$ states (Table 1)

Table 2
Energies E (MeV) and wavefunctions of the $11/2^-$ states of $^{145-149}\text{Nd}$ from the band-mixing calculation.

$E(\text{MeV})$	$CR_{11/2}$							
	$h_{11/2}$	$h_{9/2}$	$f_{7/2}$	$f_{5/2}$	$p_{3/2}$	$p_{1/2}$		
1.381	-0.855	-0.505	0.115	-0.027				
1.440	0.406	0.521	-0.632	0.374	-0.158			
1.384	0.308	0.625	0.300	-0.553	-0.345			
1.313	0.095	0.024	0.657	0.361	0.589	0.033		
1.203	0.012	0.052	0.258	0.650	0.711	-0.043		
1.729	0.940	-0.340	0.042					
1.504	0.335	0.890	-0.502	0.070	0.014			
1.365	0.067	0.296	0.846	-0.419	0.128			
1.239	0.012	0.072	0.421	0.746	-0.510	0.027		
1.090		0.013	0.121	0.512	0.849	0.049		
1.888	0.969	-0.245	0.020	0.261				
1.577	0.244	0.951	-0.188	0.281	0.054			
1.348	0.027	0.186	0.939	-0.281	0.378	0.018		
1.141		0.029	0.280	0.882	-0.378			
0.944			0.059	0.378	0.923	-0.510		

Table 3
Fragments of the $1h_{11/2}$ hole state from the quasi-particle phonon coupling model calculation. E is the energy in MeV and S is the spectroscopic factor weighted with $(2j+1)V^2$.

E	$A = 145$		$A = 147$		$A = 149$	
	E	S	E	S	E	S
2.076	0.034		1.877	0.064	0.615	0.046
2.716	0.093		1.993	0.080	0.875	0.043
3.073	0.115		2.185	0.218	0.922	0.130
3.121	0.108		2.735	0.342	0.941	0.592
3.186	0.074		2.818	0.915	0.967	0.446
3.879	0.063		2.871	0.117	1.016	0.092
3.907	0.152		2.932	0.058	1.092	2.553
4.343	0.089		3.030	0.270	1.178	2.703
			3.039	0.046	1.230	0.592
			3.087	0.237	1.339	2.596
					1.348	0.200
					1.365	0.742

V. CONCLUSION

The following points emerge from the present research works: 1. The RPC model can explain the neutron strength distribution of the $1h_{11/2}$ state of ^{149}Nd . 2. The Quasi-Particle Phonon Coupling model can be utilised to explain the extensive neutron strength distribution of the $1h_{11/2}$ state of the $^{145,147}\text{Nd}$. 3. The theoretical data obtained specially from the Quasi-Particle Phonon Coupling model might be of great help to extract the detailed experimental informations on these three Nd nuclei.

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