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STATISTICAL MODEL CALCULATIONS OF NUCLEAR LEVEL DENSITY WITH DIFFERENT SYSTEMATICS FOR THE LEVEL DENSITY PARAMETER

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The main systematics for the level density parameter are reviewed. The nuclear level density and fissility of different compound nuclei are analysed in the framework of the statistical model by using different systematics for the level density parameter, shell and pairing corrections in order to identify their reliability and range of applicability for statistical calculations of pre-equilibrium and evaporation cascades. It is shown that the Malyshev systematics for $a(Z,N)$ provides a good description of experimental data only for low excitation energies. In a larger range of excitation energies systematics of Cherepanov and Iljinov and that of Iljinov, Mebel *et al.* systematics for $a(Z,N,E^*)$ reproduce very close results and they seem to describe the data better than the popular systematics of Ignatyuk *et al.*

I. INTRODUCTION

The most important quantity in statistical pre-equilibrium and evaporation models is the nuclear level density. Usually, one uses $a = a_0 A$ for the level density parameter with $a_0 = const$ independent of the neutron (N) and proton (Z) numbers and of the excitation energy E^* of residual nuclei (see, e.g., [1, 2]) in calculations of nuclear reactions. This approach is well-grounded in the following cases: A) Not very high excitation energies of residual nuclei; B) Not very high bombarding energies (in the case when one uses the pre-equilibrium and/or evaporation models after the first, cascade stage of the reaction), when we know well in advance the neutron and proton numbers of residual nuclei; C) Residual nuclei have neutron and proton numbers lying in the middle of nucleon shells and their level density parameter does not change much with successive emission of several particles at pre-equilibrium and/or evaporative stages of the reaction; D) For high excitation energies, when we are interested in good description of only high energy parts of spectra of the ejectiles. In these cases, the corresponding experimental value for a_0

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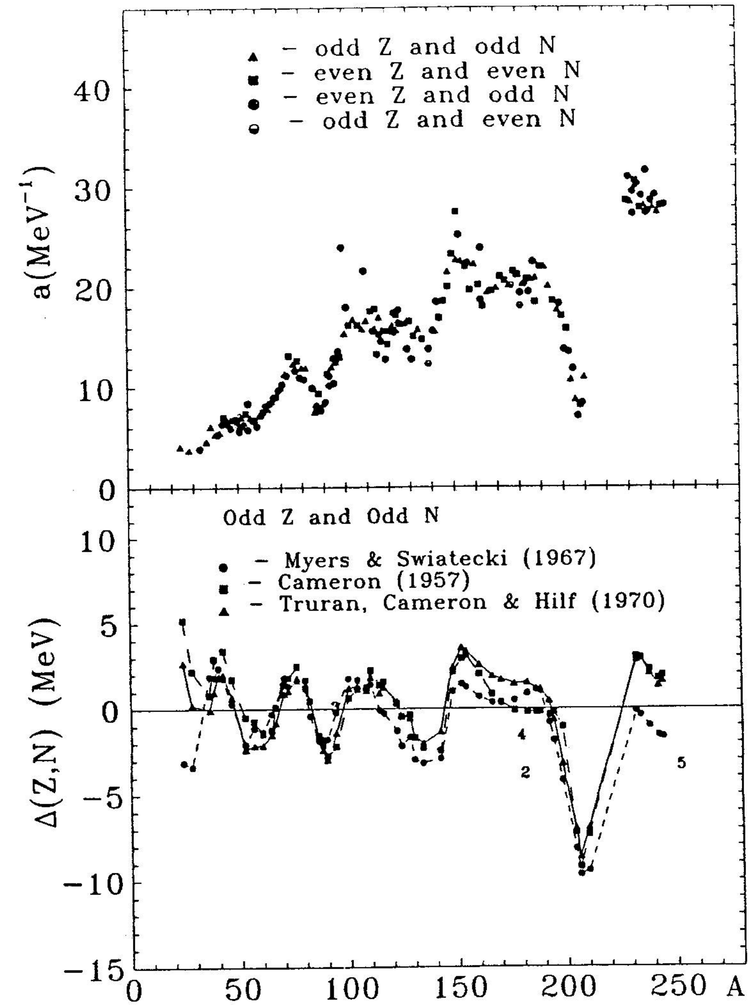

Fig. 1. Experimental values of the level density parameter obtained from measurements of neutron s-resonance spacing [5] and Cameron [6], Truran, Cameron and Hilf [7] and Myers and Swiatecki [8] for a set of odd-odd nuclei.

(or, in the case D, the asymptotic Fermi-gas value $\tilde{a}$ of the level density parameter at high excitation energies) may be used and the approach $a_0 = const$ allows one to obtain reliable results.

On the contrary, residual nuclei have a wide distribution over the neutron and proton numbers and over the excitation energy (see, e.g., [1]) at high energies. In these cases, residual nuclei may have neutron and proton numbers lying even in different nucleon shells within the frame of the same reaction. It is well known that the level density parameter a is strongly influenced by shell effects at low excitation energies (see, e.g., the monographs [3–5]).As one can see from Fig. 1, clear structures of experimental values of the level density parameter a are observed. These structures correlate unambiguously with similar structures in the A-dependence of the shell correction in the nuclear mass $\delta W_{gs}(Z,N)$.

Different phenomenological approaches were developed to describe the observed anomalies in the A-dependence of the level density parameter in connection with the value of the shell correction, or with the filling of nucleon shells with increasing A (see [3–5, 9–11] for reviews). We consider Malyshev's phenomenological approximation for $a = a(Z,N)$ fitted for $24 \leq A \leq 247$ [5] in the form proposed by Newton [10] as an example in the present paper,

$$a(Z,N) = \alpha \cdot 2(\bar{j}_Z + \bar{j}_N + 1)A^{2/3}, \qquad (1)$$

where

$$\alpha = \alpha_0 - \beta \sin\left\{\frac{\pi}{20}\frac{A}{1+\gamma(A-A_0)/2}\right\} \cdot \cos\left\{\frac{\pi}{20}\frac{(1-\gamma A_0/2)(N-Z)}{[1+\gamma(A-A_0)/2]^2}\right\};$$

$$\alpha_0 = 0.038; \quad \beta = 0.0125;$$

$$\gamma = \begin{cases} 6.7\cdot 10^{-3}, & \text{for } A \geq A_0 = 80\,; \\ 0, & \text{for } A < A_0\,. \end{cases}$$

The values of average proton $\bar{j}_Z$ and neutron $\bar{j}_N$ spins for $Z \leq 83$ and $N \leq 127$ are given in Table 2 of Ref. [5].

The results of calculation of the level density parameter by using the approximation (1) are compared with the experimental data obtained [5] from measurements of the neutron s-resonance spacing (Fig. 2). Let us recall that the approximation (1) was obtained for excitation energies of compound nuclei formed after thermal neutron capture $E^* \approx B_n$ (B_n is the neutron binding energy). One can see that Malyshev's systematics (1) reproduces very well the shape and the absolute value of the experimental level density parameters. This enables us to incorporate the approximation (1) in statistical pre-equilibrium and/or evaporation models and to use it confidently for $24 \leq A \leq 247$ and low excitation energies without knowing the corresponding experimental values of $a(Z,N)$.

But the situation changes at high excitation energies. The use of approximations like (1) at high excitation energies means that shell effects are assumed to manifest themselves in the level density in the same manner as at low energies. This contradicts the well-known fact of thermal damping of the shell effects in nuclei: different authors have shown that the strongest shell effects are at low excitation energies, and that they disappear at $E^* > 50 - 100$ MeV (see, e.g., [3, 12] and the references given therein). Moreover, let us recall that the level density parameter depends only on the mass number in the Fermi-gas model, $a = a_0 A$ with $a_0 = const$ [3, 12, 13].

By now, different phenomenological methods taking into account the damping of shell effects with increasing excitation energy have been developed [14–22] to calculate the level density parameter $a(Z,N,E^*)$. In the present work, we compare

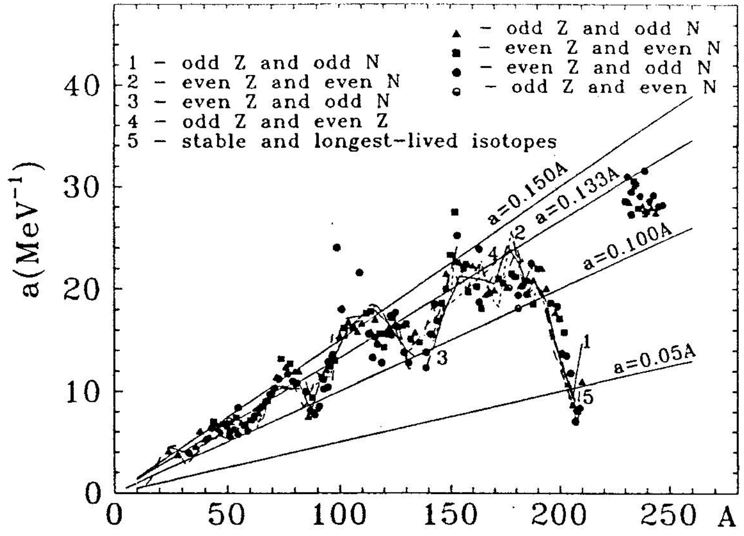

Fig. 2. Experimental values of the level density parameter from Ref. [5] and the results of calculation by using Malyshev's approximation (1).

different easy-computing approaches for calculation of the level density parameter to find out their applicability for statistical pre-equilibrium and evaporation models.

II. COMPARISON OF PHENOMENOLOGICAL SYSTEMATICS WITH DATA ON THE LEVEL DENSITY PARAMETER

The first semiempirical systematics for description of the level density parameter by taking into account the thermal damping of shell effects, i.e. the excitation energy E^* dependence of the parameter a, has been performed by Ignatyuk *et al.* [14]. In this approach, the function which describes the thermal damping of shell effects was found from microscopic calculations. The Ignatyuk *et al.* formula for $a(Z,N,E^*)$ [14] has the following form:

$$a(Z,N,E^*) = \tilde{a}(A)\left\{1 + \delta W_{gs}(Z,N)\frac{f(E^*-\Delta)}{E^*-\Delta}\right\}, \qquad (2)$$

where

$$\tilde{a}(A) = (\alpha + \beta A)A \qquad (3)$$

is the asymptotic Fermi-gas value of the level density parameter at high excitation energy and

$$f(E^*) = 1 - \exp(-\gamma E^*)\,. \qquad (4)$$

The parameters α, β and γ were fitted to the experimental resonance spacing, and therefore, include collective effects in a non-explicit, phenomenological way. It was found that

$$\alpha = 0.154; \quad \beta = -6.3 \cdot 10^{-5}; \quad \gamma = 0.054 \text{ MeV}^{-1} . \tag{5}$$

(We will refer to these values as to the "first" set of parameters of Ignatyuk *et al.* in the following.) Shell effects are included in the term $\delta W_{gs}(Z,N)$. In the present paper, we will use three different approximations for $\delta W_{gs}(Z,N)$, namely, the one of Cameron [6], that of Truran, Cameron and Hilf [7] and the set of Myers and Swiatecki [8].

In the subsequent paper [15], Ignatyuk *et al.* proposed to use the following, "second" form for $\tilde{a}(A)$

$$\tilde{a}(A) = \alpha A + \beta A^{2/3} b_s , \tag{6}$$

where b_s is the surface area of the nucleus in units of the surface for the sphere of equal volume (for the ground state of nucleus $b_s \approx 1$), and

$$\alpha = 0.114; \quad \beta = 0.162; \quad \gamma = 0.054 \text{ MeV}^{-1} . \tag{7}$$

(we will call these values as the "second" set of Ignatyuk *et al.*). As the Ignatyuk systematics are very simple and suitable for using in the pre-equilibrium and evaporation calculations, they are well known, cited and probably the most frequently used ones by now in the literature.

Later on, Cherepanov and Iljinov [16] performed a systematics analogous to that of Ignatyuk *et al.* using not only the neutron resonance data to fit the parameters, but also the data at higher excitation energies E^*. In addition, the systematics of Cherepanov and Iljinov takes into account an explicit form of the contribution of collective (i.e. rotational and vibrational) states to level densities. Cherepanov and Iljinov used the functional form of Ignatyuk *et al.* for parametrization (2-4) and obtained, in the case when the collective states were not explicitly taken into account, the following values for the parameters:

$$\alpha = 0.148; \quad \beta = -1.39 \cdot 10^{-4}; \quad \gamma = 6 \cdot 10^{-2} \text{ MeV}^{-1} . \tag{8}$$

(Further on, we will call these values to be the "first" set of parameters of Cherepanov and Iljinov.) When collective states were taken explicitely into account, Cherepanov and Iljinov obtained

$$\alpha = 0.134; \quad \beta = -1.21 \cdot 10^{-4}; \quad \gamma = 6.1 \cdot 10^{-2} \text{ MeV}^{-1} . \tag{9}$$

(This will be referred to as the "second" set of parameters of Cherepanov and Iljinov.)

Recently, Iljinov, Mebel *et al.* [17] performed a new systematics of all known existing data on level densities. The authors of [17] used again the functional form of $a(Z,N,E^*)$ with the asymptotic Fermi-gas value of the level density parameter at high energy in the form (6) suggested by Ignatyuk *et al.* [17]. Iljinov, Mebel *et al.* used two sets of "empirical" shell corrections in their fitting procedure (namely, those of Ref. [7] and Ref. [8], respectively); they performed the fits with and without taking into account of collective effects explicitly; and, in addition, they

Table 1
Iljinov, Mebel's *et al.* results of level density analysis for different variants of the phenomenological systematics [17]

No. of fit	α	β	γ [MeV^{-1}]	f-factor	Shell corrections
Without collective effects ($K_{rot} = 1$, $K_{vib} = 1$)					
1	0.114	0.098	0.051	1.68	Myers, Swiatecki [8]
2	0.111	0.107	$\tilde{a}/0.46A^{4/3}$	1.71	Myers, Swiatecki [8]
3	0.072	0.257	0.059	2.31	Cameron *et al.* [7]
4	0.077	0.229	$\tilde{a}/0.37A^{4/3}$	2.48	Cameron *et al.* [7]
With collective effects ($K_{rot} \neq 1$, $K_{vib} \neq 1$)					
5	0.090	-0.040	0.070	1.63	Myers, Swiatecki [8]
6	0.034	0.312	0.011	5.00 (*)	Myers, Swiatecki [8]
7	0.052	0.113	0.086	2.20	Cameron *et al.* [7]
8	0.029	0.332	0.012	5.47 (*)	Cameron *et al.* [7]

(*) Nuclides with deformation less than 0.2 were assumed to be spherical ($K_{rot} = 1$).

presented two different sets of fits: A) with the energy dependence $f(E^*)$ in a universal form (4) for all the nuclei, and B) following Schmidt *et al.* [18] assuming the parameter γ to be A-dependent,

$$\gamma = \frac{\tilde{a}}{\epsilon A^{4/3}} , \tag{10}$$

where ϵ is a phenomenological parameter.

The eight sets of parameters values obtained by Iljinov, Mebel *et al.* [17] are shown in Table 1.

The values of the f-factor, the averaged ratio of the level densities (see Ref. [17]), serve us to give a quantitative overall estimation of the agreement between the calculated and experimental data on the level density

$$f \equiv < \frac{\rho_{calc}}{\rho_{exp}} > = \exp \left[\frac{1}{n} \sum_{i=1}^{n} \left(\ln \frac{\rho^i_{calc}}{\rho^i_{exp}} \right)^2 \right]^{1/2}$$

Here, n is the number of the considered experimental points.

In the present paper, we test both systematics of Ignatyuk *et al.* [14, 15], both sets of parameters of Cherepanov and Iljinov [16], and the first four sets of Iljinov *et al.* [17] obtained without taking explicitly the collective effects into account. For every systematics, we use three approaches for shell corrections, namely, the Cameron one [6], that of Truran *et al.* [7], and the one of Myers and Swiatecki [8]. The results of our calculations for a collection of odd-odd nuclei for excitation energies $E^* = 5$, 50, 100, and 300 MeV together with the experimental values

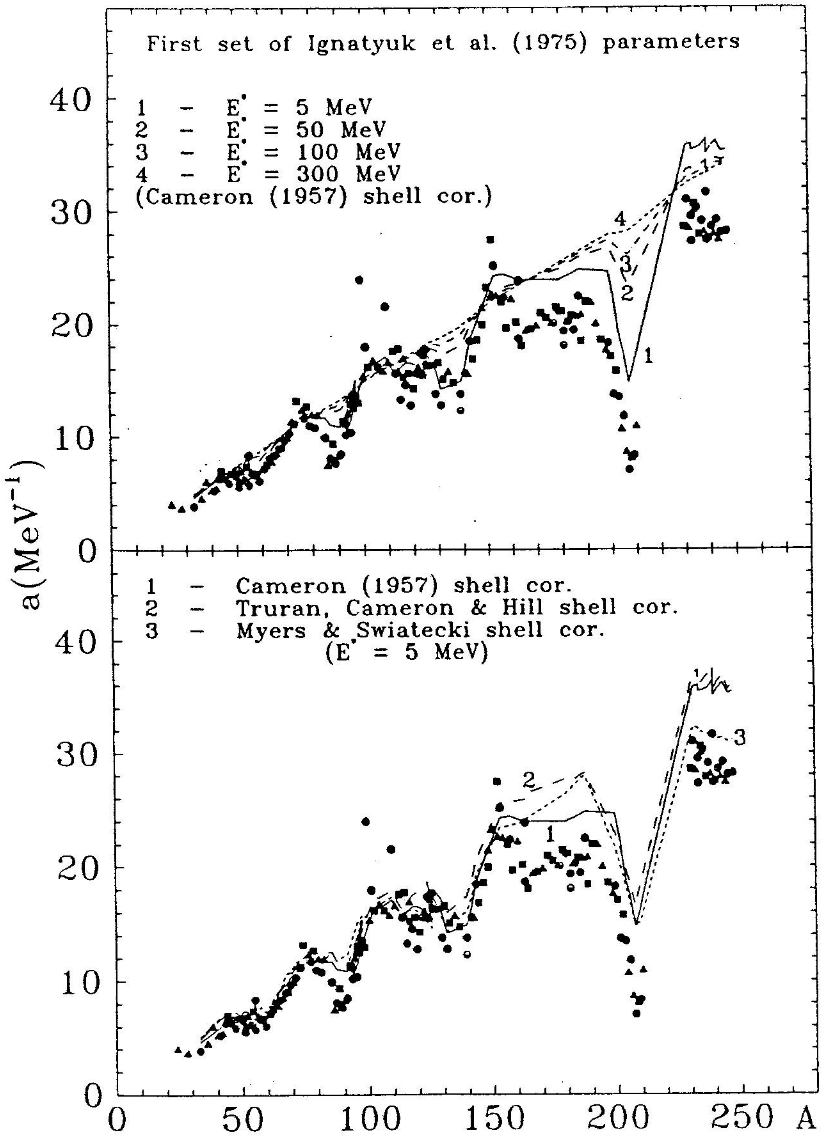

Fig. 3a. Experimental values of the level density parameter obtained at the neutron binding energy (Ref. [5]) and the results of calculation with the first systematics (2–5) of Ignatyuk *et al.* [14]. The broken lines in the upper part of the picture show the level density parameters obtained with Cameron's shell corrections at four different values of excitation energy, whereas the lower part compares three types of shell corrections at $E^* = 5$ MeV.

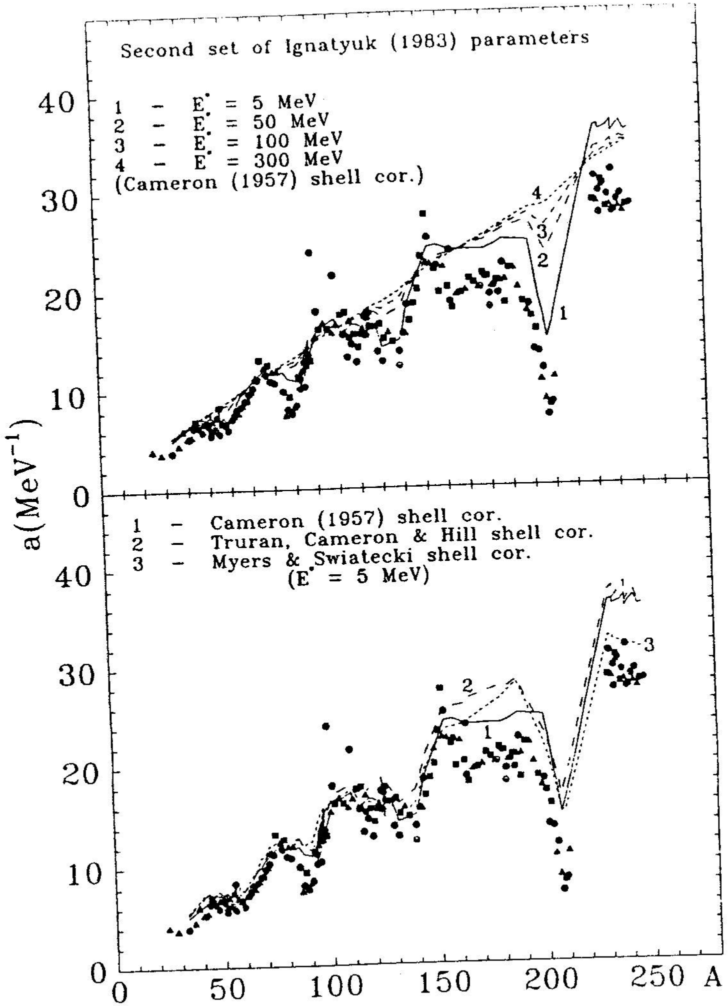

Fig. 3b. The same as in Fig. 3a, but for the second systematics of Ignatyuk *et al.* [15], i.e., by using formulae (2), (4) and (6-7). The layout of the picture is similar to that in Fig. 3a.

of the level density parameter are shown in Figs. 3a–3h. One can see that all systematics regarded here provide very close rates of thermal damping of shell effects with increasing excitation energies of nuclei. The shell effects disappear practically completely in all systematics for $E^* > 100$ MeV.

The Myers and Swiatecki shell corrections [8] are very popular in literature and are widely used for the description of nuclear fission. Though they are easy-

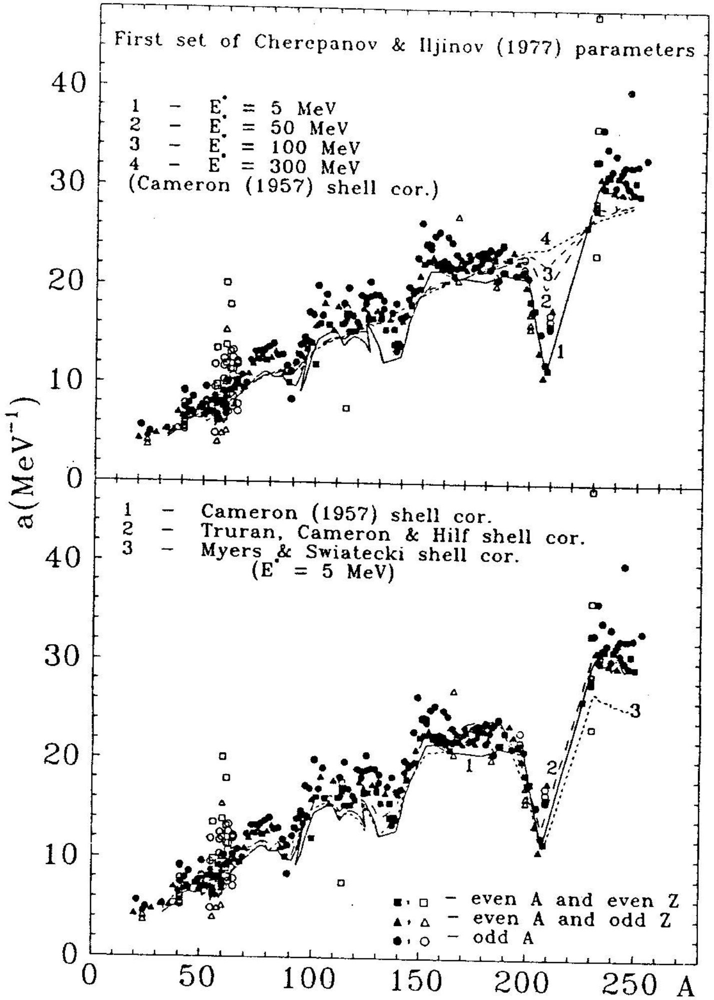

Fig. 3c. Experimental values of the level density parameter obtained by Cherepanov and Iljinov [16] from measurements of neutron resonance spacing (full symbols), for a larger interval of excitation energy from data counting low-lying levels and from level spacing data from different reactions (open symbols) as well as the results of calculation under Cherepanov and Iljinov's systematics with the first set of parameters (8). The layout of the picture is similar to that in Fig. 3a.

computing; their use e.g. in Monte Carlo simulations may need too much computer time to yield satisfactory statistics. On the contrary, the Cameron shell corrections [6, 7] are published in a tabulated form, do not need any time for their calculation

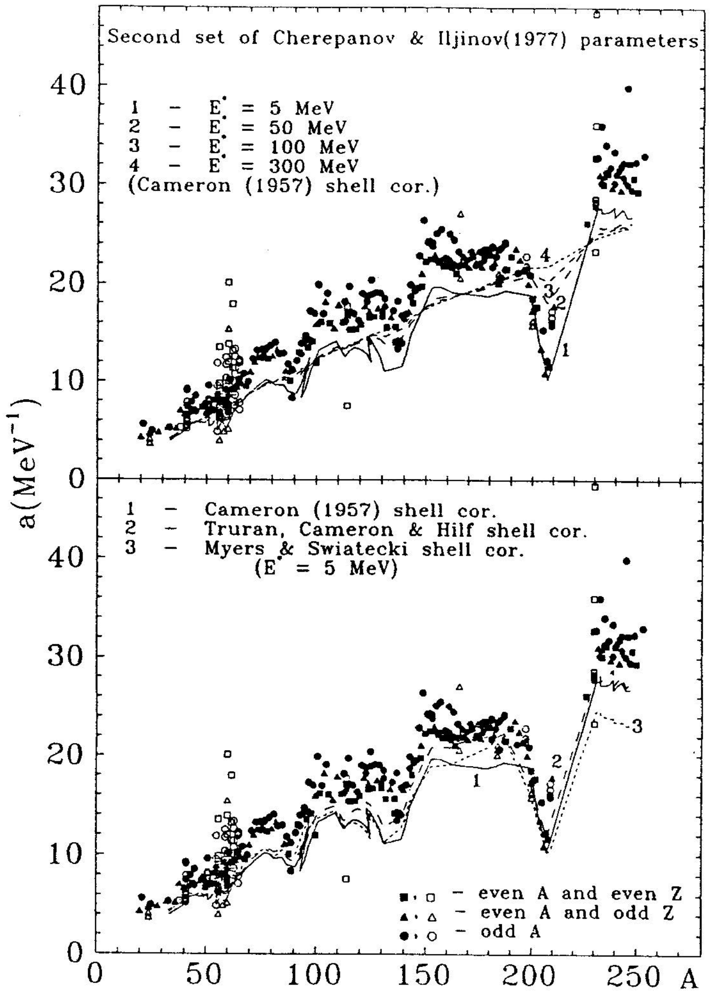

Fig. 3d. The same as in Fig. 3c, but for calculations performed with the second set of Cherepanov and Iljinov's parameters (9). The layout of the picture is similar to that in Fig. 3a.

and, therefore, they are more convenient for Monte Carlo calculations. As one can see from Figs. 3a–3h, the use of shell corrections of Cameron [6] or Cameron *et al.* [7] allows one to describe the level density parameters practically as well as the calculations with Myers and Swiatecki's shell corrections. To have a more reliable conclusion about what shell corrections may be used in the chained calculations of pre-equilibrium and/or evaporation cascades, it is desirable to compare

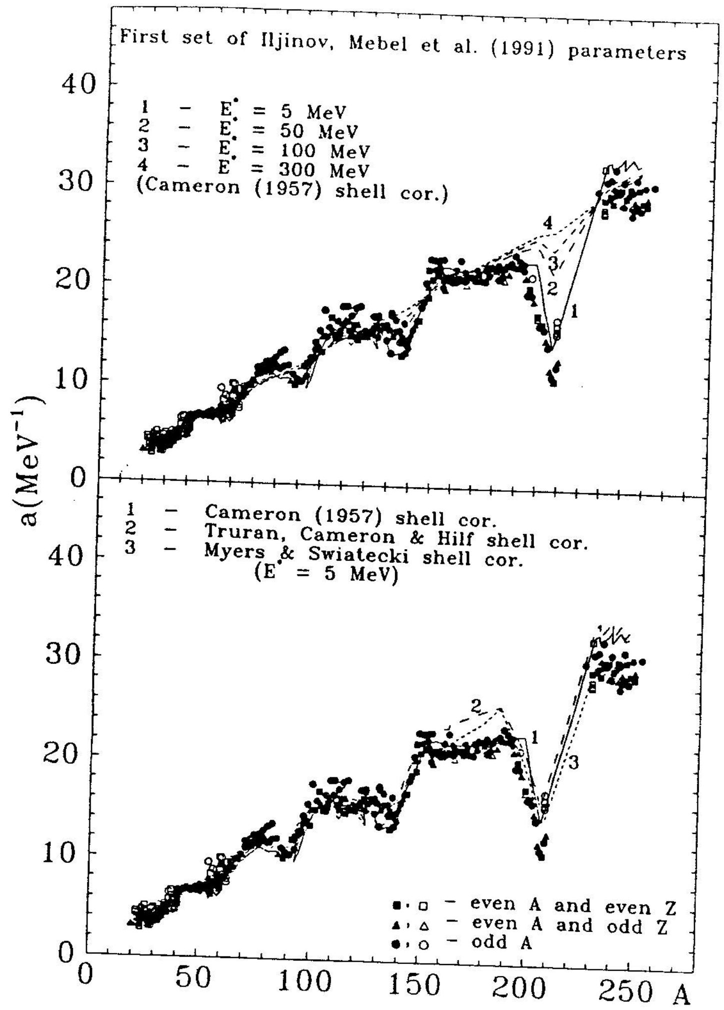

Fig. 3e. Experimental values of the level density parameter obtained by Iljinov, Mebel *et al.* [17] from measurements of neutron resonance spacing (full symbols), for a larger interval of excitation energy from data counting low-lying levels and from level spacing data from several reactions $[(\gamma,n),(p,\gamma),(p,p'),(\alpha,\gamma),(\alpha,p),(^3He,d)$, and $(^3He,\alpha)]$ (open symbols) as well as present calculation under Iljinov, Mebel's *et al.* systematics with the first set of parameters (the first line of Table 1). The layout of the picture is similar to that in Fig. 3a.

not only calculated level density parameters but also the proper level densities of excited nuclei and various concrete characteristics of nuclear reactions calculated

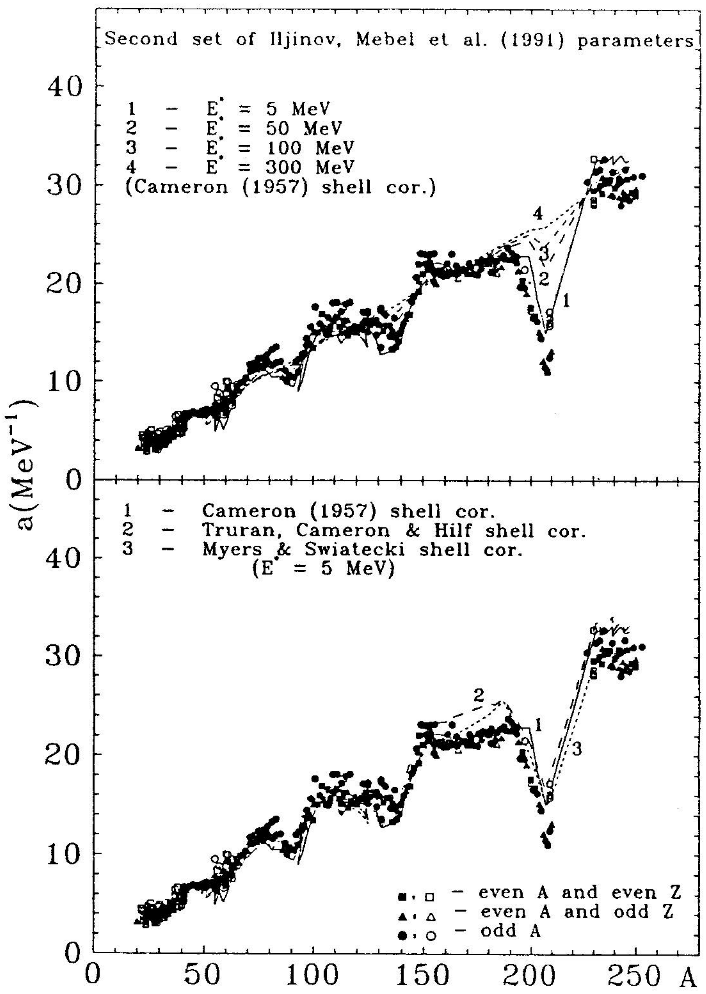

Fig. 3f. The same as in Fig. 3e, but for the second set of Iljinov, Mebel's *et al.* parameters (second line of Table 1).

with different shell corrections.

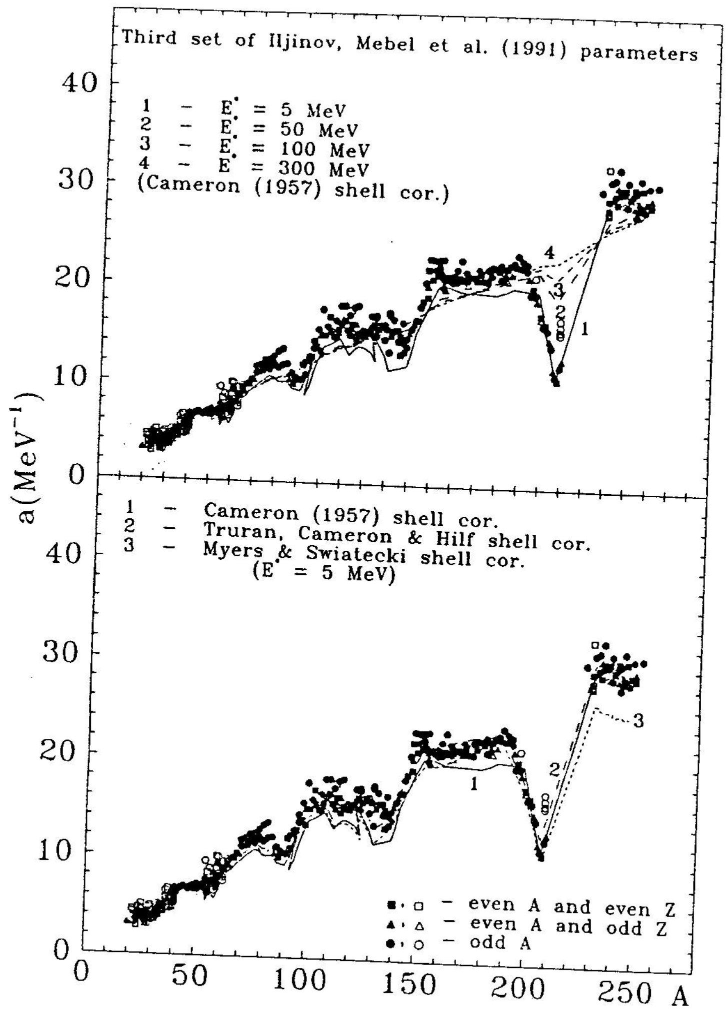

Fig. 3g. The same as in Fig. 3e, but for the third set of Iljinov, Mebel's *et al.* parameters (third line of Table 1).

III. CALCULATION OF NUCLEAR LEVEL DENSITIES WITH DIFFERENT SYSTEMATICS FOR LEVEL DENSITY PARAMETERS

In this section, we will calculate level densities of nuclei using different systematics for the level density parameter and different shell corrections following the scheme used by Iljinov, Mebel *et al.* [17]. In the adiabatic approximation for the selection between rotational and vibrational modes, the nuclear level density

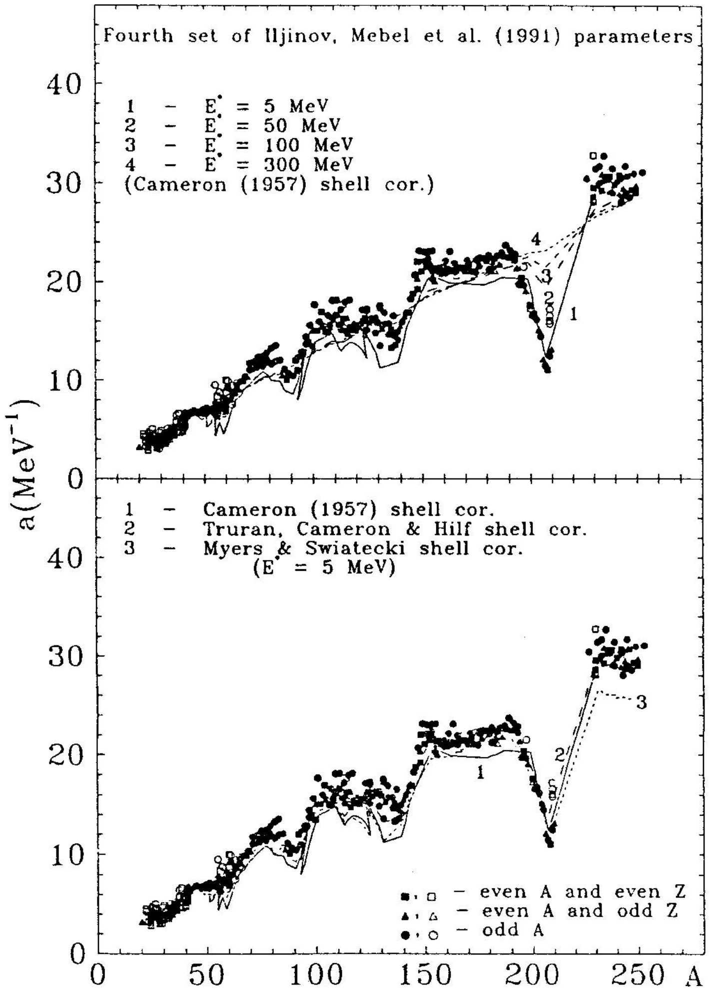

Fig. 3h. The same as in Fig. 3e, but for the fourth set of Iljinov, Mebel's *et al.* parameters (fourth line of Table 1).

$\rho(E^*)$ is generally described by the following expression [3, 21]:

$$\rho(E^*) = K_{rot} K_{vib} \rho_{intr}(E^*) \,, \tag{11}$$

where K_{rot} and K_{vib} are the coefficients for rotational and vibrational enhancement of the noncollective intrinsic excitations $\rho_{intr}(E^*)$. To describe this quantity, one often uses the Fermi-gas expression [3, 12, 13]

$$\rho_{intr}(E^*) = \frac{\sqrt{\pi}}{12a^{1/4}(E^* - \Delta)^{5/4}} \exp\left(2\sqrt{a(E^* - \Delta)}\right), \qquad (12)$$

where a is the level density parameter, and

$$\Delta = \chi \frac{12}{\sqrt{A}} \text{ [MeV]} \qquad (13)$$

is the pairing energy ($\chi = 0$, 1, or 2, for odd-odd, odd-even, or even-even nuclei, respectively).

The observed level density $\rho_{exp}(E^*)$ is connected with the total level density (state density) by the relation

$$\rho_{exp}(E^*) = \sum_L \rho(E^*, L) \approx \frac{\rho(E^*)}{\sqrt{2\pi}\sigma}. \qquad (14)$$

Here, $\rho(E^*, L)$ is the level density of nucleus having angular momentum L and excitation energy E^* and is connected with the total level density $\rho(E^*)$ by the relation [12, 13]

$$\rho(E^*, L) = \frac{2L+1}{2\sqrt{2\pi}\sigma^3} \exp\left[-\frac{(L+1/2)^2}{2\sigma^2}\right] \rho(E^*). \qquad (15)$$

The spin-cutoff parameter σ is usually calculated by the formula

$$\sigma^2 = \frac{T\mathcal{J}_{rb}}{\hbar^2}, \qquad (16)$$

where $T = \sqrt{(E^* - \Delta)/a}$ is the nuclear temperature and $\mathcal{J}_{rb} = 0.4 M_n r_0^2 A^{5/3}$ is the rigid body momentum of inertia, M_n is the nucleon mass; for the nuclear radius $R = r_0 A^{1/3}$ we use $r_0 = 1.2$ fm.

The collective enhancement of the level density is especially large in the case of deformed nuclei. The coefficient of rotational increase of the level density K_{rot} in (11) is defined by the expression [3, 21]

$$K_{rot} = \begin{cases} 1, & \text{for spherical nuclei;} \\ \mathcal{J}_\perp T, & \text{for deformed nuclei,} \end{cases} \qquad (17)$$

where $\mathcal{J}_\perp = \mathcal{J}_{rb} f(\beta_2, \beta_4)$ is the perpendicular moment of inertia;

$$f(\beta_2, \beta_4) = 1 + \sqrt{\frac{5}{16}\pi\beta_2 + \frac{45}{28\pi}\beta_2^2 + \frac{15}{7\pi\sqrt{5}}\beta_2\beta_4}\,; \qquad (18)$$

β_2 and β_4 are the parameters of quadrupole and octupole deformations of the nucleus [22]. The liquid drop model estimation for the vibrational coefficient K_{vib} is [3]

$$K_{vib} \approx \exp(0.0555 A^{2/3} T^{4/3}). \qquad (19)$$

The rotational enhancement of the level density of deformed nuclei $K_{rot} \simeq (10-100)$ is considerably larger than the vibrational enhancement $K_{vib} \approx 3$ at energy $E^* \approx B_n$ [3, 17].

To calculate the collective enhancement of the nuclear level density in accordance with (11-19), it is necessary to know the values of the parameters β_2 and β_4 of nuclear deformation. This is not always possible in calculations of pre-equilibrium and evaporation cascades, because a residual nucleus after successive emission of several particles may have such proton and neutron numbers for which there are no available data for β_2 and β_4. Besides that, let us remind that statistical pre-equilibrium and evaporation models deal not directly with the nuclear level density but with their ratios only. At last, it should be noted that systematics for the description of the level density parameters fitted to experimental resonance spacings without an explicit taking into account of collective effects (i.e., with $K_{rot} = 1$ and $K_{vib} = 1$) also include collective effects in a phenomenological, nonexplicit way. On the whole, a question of a redefinition of the level density parameter $a(Z, N, E^*)$ arises (see [17]). This leads us to use here the systematics obtained without an explicit taking into account of collective effects.

The results of calculations of level densities according formulae (11-16) for $K_{rot} = 1$ and $K_{vib} = 1$ with different systematics for the level density parameter and by using different shell corrections are shown in Figs. 4-6. One can see that in general Malyshev's systematics for $a(Z, N)$ without excitation energy dependence allows one to describe satisfactorily the experimental data only at low excitation energies E^*. Independently of the concrete shell corrections used in the calculation, all systematics used here with excitation energy dependence of the level density parameter permit one to reproduce correctly (within a factor of 3) the absolute values of the measured level density up to $E^* \approx 20 - 25$ MeV for medium (Fig. 4) and heavy spherical or weakly deformed nuclei (Fig. 5), and a little worse for light deformed nuclei (Fig. 6). To describe the data at higher energies or for strongly deformed nuclei better, the systematics with collective effects has to be used [16, 17]. One can see that the systematics of Cherepanov and Iljinov [16] and Iljinov, Mebel *et al.* [17] reproduce results rather well and seem to describe the data better than the systematics of Ignatyuk *et al.* [14].

However, it is desirable to analyze other characteristics of the decay of excited nuclei before drawing a more definite conclusion about the advantage of a concrete systematics for $a(Z, N, E^*)$.

IV. FISSIBILITY OF EXCITED NUCLEI

In this section, we will use the systematics of $a(Z, N, E^*)$ regarded above to analyze the energy dependence of nuclear fissibility. The fissibility is the ratio of the fission cross-section to the inelastic cross-section $P_f = \sigma_f/\sigma_{in}$. It may be estimated as the ratio of partial widths Γ_f/Γ_{tot} for a given composite nucleus. Here, $\Gamma_{tot} = \Gamma_f + \sum_j \Gamma_j$ is the total decay width of the composite nucleus, equal to the fission partial width Γ_f plus the sum of the emission widths Γ_j of the jth-type particles.

In the Weisskopf statistical theory of particle emission [23] and Bohr and Wheeler theory of fission [24], the partial widths Γ_j for the emission of a particle j

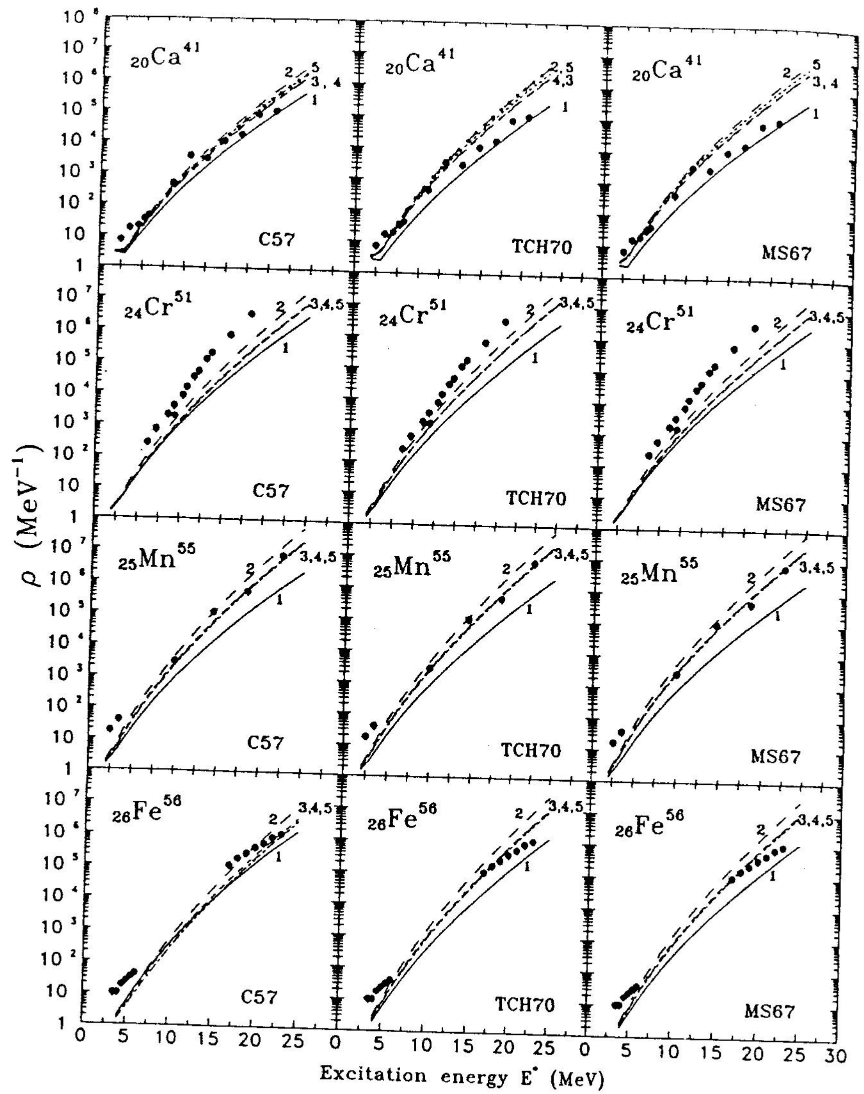

Fig. 4. Energy dependence of the level densities of nuclides ^{41}Ca, ^{51}Cr, ^{55}Mn, and ^{56}Fe. Points are the experimental data from the summary Table 2 of Ref. [16]. The curves denoted as 1, 2, 3, 4, and 5 are the results of calculation with Malyshev's [5], first Ignatyuk's *et al.* [14], first Cherepanov and Iljinov's [16], first and third Iljinov, Mebel's *et al.* [17] systematics for the level density parameter, respectively. In the left, center, and right parts of the figure, the results of calculations with Cameron's [6] (marked as *C57*), Truran, Cameron and Hilf's [7] (marked as *TCH70*), and Myers and Swiatecki's [8] (marked as *MS67*) shell corrections are shown, respectively.

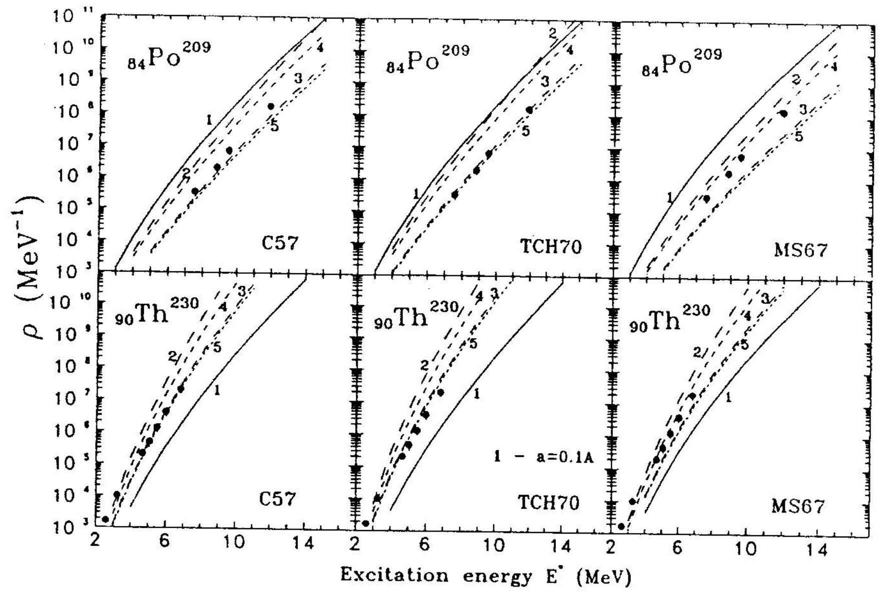

Fig. 5. The same as in Fig. 4, but for ^{209}Po and ^{230}Th nuclides.

($j = n, p, d, t, ^3He, \alpha$) and Γ_f for fission are expressed by the following approximate formulae (in accord with [17], these formulae are written using $\hbar = c = 1$)

$$\Gamma_j = \frac{(2s_j+1)m_j}{\pi^2 \rho_c(U_c)} \int_{V_j}^{U_j - B_j} \sigma^j_{inv}(E)\rho_j(U_j - B_j - E)E\,dE\,, \quad (20)$$

$$\Gamma_f = \frac{1}{2\pi\rho_c(U_c)} \int_0^{U_f - B_f} \rho_f(U_f - B_f - E)\,dE\,. \quad (21)$$

Here, ρ_c, ρ_j, and ρ_f are the level densities of the compound nucleus, the residual nucleus produced after the emission of the j-th particle, and for the fission saddle point, respectively; m_j, s_j and B_j are the mass, spin and the binding energy of the j-th particle, respectively; B_f is the fission barrier height. We calculate the binding energies of particles by the use of the Cameron formulae [6] in the present work. The inverse cross-section for absorption of j-th particle with kinetic energy E by the residual nucleus is $\sigma^j_{inv}(E)$. We approximate it according to Dostrovsky [25]. Further on,

$$U_c = E^* - \Delta_c\,; \quad U_j = E^* - \Delta_j\,; \quad U_f = E^* - \Delta_f\,, \quad \text{where}$$

$$\Delta_c = \chi \cdot 12\sqrt{A_c}\,; \quad \Delta_j = \chi \cdot 12\sqrt{A_{fj}}\,; \quad \text{and} \quad \Delta_f = \chi \cdot 14\sqrt{A_c} \quad \text{(in MeV)}$$

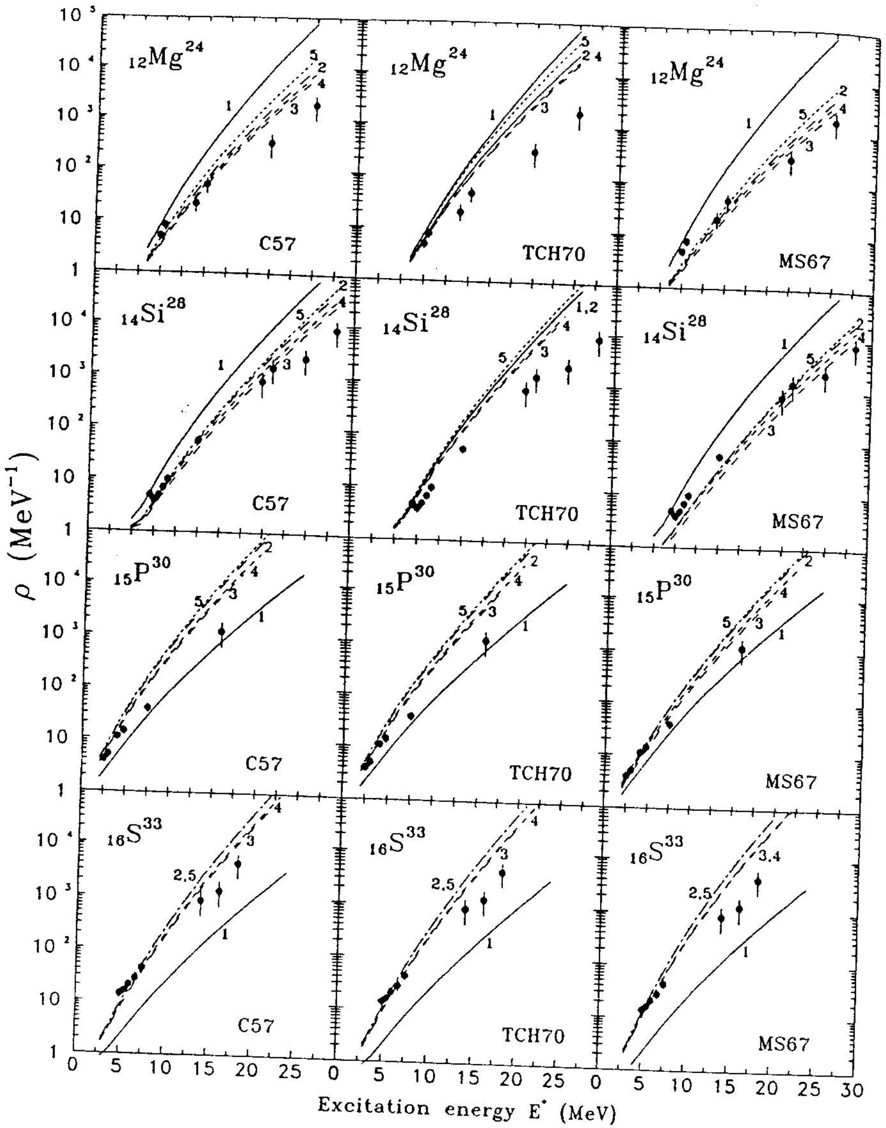

Fig. 6. The same as in Fig. 4, but for ^{24}Mg, ^{28}Si, ^{30}P, and ^{33}S nuclides. The experimental data are taken from the summary Table 2 of Ref. [17].

are the pairing energies for compound and residual nuclei, and of the fission saddle point, respectively; $A_{fj} = A_c - A_j$, where A_c and A_j are the mass numbers of the compound nucleus and of j-th particle, respectively.

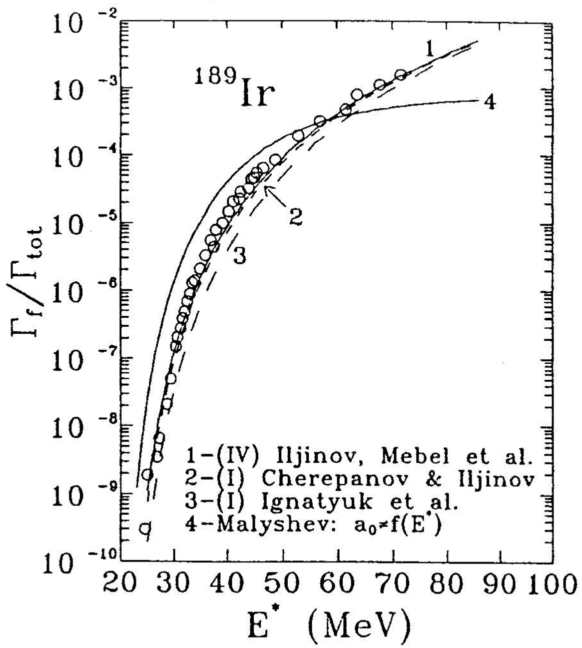

Fig. 7. Excitation energy dependence of the fissibility Γ_f/Γ_{tot} of the exited ^{189}I composite nucleus. Curves are our calculation results with fission barriers from Ref. [27], Cameron's [6] shell and pairing corrections, $a_f/a_n = 1.114$, as obtained for the third systematics of Iljinov *et al.* [17] of Cherepanov and Iljinov [16], the first systematics of Ignatyuk [14] and the Malyshev one [5] for the level density parameter. Experimental points were taken from the review [26].

We have analyzed a lot of experimental data on nuclear fissibility published in the review [26] by using the formulae (20-21) and the systematics for the level density parameter treated above. This analysis will be published in the following separate paper. Let us show here one such result for the illustration. The measured [26] and calculated fissibilities for ^{189}Ir nuclide are shown in Fig. 7. These calculations were performed by using the third Iljinov, Mebel *et al.* set of parameters [17], the first one of Cherepanov and Iljinov [16], the first one of Ignatyuk *et al.* [14], and the level density parameter systematics of Malyshev [5], with fission barriers from Ref. [27], Cameron's [6] shell and pairing corrections, and the value of the ratio $a_f/a_n = 1.114$. One can see that Malyshev's systematics [5] for $a(Z,N)$ provides a good description of the shape of the nuclear fissibility (and by fitting the ratio a_f/a_n also of the absolute value) as function of E^* for low values of E^* only. Systematics of $a(Z,N,E^*)$ due to Cherepanov and Iljinov [16] and of Iljinov *et al.* [17] allow one to obtain a good description of the data in larger interval of E^*, they reproduce very closely results and they seem to describe the data better than the systematics proposed in Ref. [14].

V. SUMMARY AND CONCLUSION

A review and comparative analysis of a number of systematics for the description of the level density parameter of excited nuclei are given. All systematics of $a(Z,N,E^*)$ regarded here provide rates very close to those of thermal damping of the shell effects due to increasing excitation energy of nuclei. The shell effects disappear practically completely in all systematics for $E^* > 100$ MeV.

Shell corrections of Myers and Swiatecki [8] are very popular in literature, and though they are easy-computing, their using in cascade-type calculations of pre-equilibrium and evaporative reactions may need too much computer time to give satisfactory statistics. On the contrary, Cameron's [6, 7] shell corrections are published in a very convenient tabulated form, they do not need any time for their calculation and, therefore, they are more convenient here. Our calculations have shown that Cameron's [6] or Cameron's *et al.* [7] shell corrections allow one to describe the experimental values of $a(Z,N,E^*)$, $\rho(E^*)$ and Γ_f/Γ_n practically as well as the calculations with that of Myers and Swiatecki [8].

It was shown that all systematics of $a(Z,N,E^*)$ regarded here permit to reproduce correctly (up to a factor of 3) the absolute value of the measured level density up to $E^* \sim 20-30$ MeV for medium and heavy spherical or weakly deformed nuclei without taking explicitly into account the collective effects. To describe the data in the high-energy region or for strongly deformed nuclei better, it is necessary to take into account the contribution from collective states to level density and to use the systematics for $a(Z,N,E^*)$ obtained with $K_{rot} \neq 1$ and $K_{vib} \neq 1$

The analysis of level densities and nuclear fissibility has shown that Malyshev's systematics [5] of $a(Z,N)$ provides a satisfactory description of the experimental data only for low values of excitation energies E^*. The systematics of Cherepanov and Iljinov [16] and of Iljinov *et al.* [17] for $a(Z,N,E^*)$ allow to obtain a good description of the data in larger interval of E^*; they reproduce results very closely and seem to describe the data better than the systematics of Ignatyuk *et al.* [14].

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