acta phys. slov. 43 (1993), No. 2, 74–85

COLLAPSES AND REVIVALS OF QUANTUM PHASE FLUCTUATIONS IN THE NONDEGENERATE DOWN-CONVERSION WITH QUANTUM PUMP

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Received 26 January 1993; in final form 9 February 1993
Accepted 15 February 1993

Quantum phase properties of the field generated in the nondegenerate down-conversion with quantum pump are studied within the framework of the Pegg-Barnett Hermitian phase formalism. It is shown that, unlike the ideal two-mode squeezed vacuum, the solutions are oscillatory, and the sum phase locking observed for the two-mode squeezed vacuum is not perfect when the quantum fluctuations of the pump mode are taken into account. The long-time evolution of the quantum phase fluctuations for the phase sum of the signal and idler modes as well as for the pump mode exhibits collapses and revivals of phase properties of the field.

I. INTRODUCTION

Two-mode squeezed states that can be generated in the nondegenerate down-conversion process have been a subject of considerable interest in quantum optics [1]-[6]

The two-mode squeezed vacuum was in fact the first squeezed state obtained experimentally [7]. The two-mode squeezed states have very interesting properties because of the strong correlations between the modes. Quantum phase properties of the two-mode squeezed vacuum have been shortly discussed by Fan and Zaidi [8] within the framework of the Susskind and Glogower [9] phase formalism. Recently, Barnett and Pegg [10], and Gantsog and Tanaś [11] have discussed the quantum phase properties of the two-mode squeezed vacuum using the Hermitian phase formalism introduced earlier by Pegg and Barnett [12]-[14]. It has been shown [10, 11] that the joint phase distribution for the two-mode squeezed vacuum depends only on the sum of the phases of the two modes, and that the sum of the two phases is locked to a certain value as the squeezing parameter r increases, while the individual phases as well as the phase difference remain random.

The two-mode squeezed vacuum can be generated in the nondegenerate down-conversion process in which the pump mode is assumed to be classical and non-depleted. Within this parametric approximation the resulting state , i.e. the two-mode squeezed vacuum, has relatively simple analytical form, and asymptotically as r tends to infinity the sum of the phases of the two modes becomes well defined. However, the parametric approximation breaks down when a considerable amount of power is transferred from the pump mode to the signal and idler modes. In such situations the pump mode must be treated dynamically and its quantum mechanical evolution must be taken into account. Owing to the energy conservation the intensity of the signal mode cannot grow infinitely, and the solutions become oscillatory. The resulting field state is no longer the ideal two-mode squeezed vacuum, and its quantum phase properties are also different. This effect is known for the case of degenerate down-conversion [17]. Recently, Drobný and Jex [18] and Bužek et al. [19, 20] have discussed photon statistics, mode entanglement and squeezing of the fields generated during the quantum evolution with the trilinear interaction Hamiltonian drawing attention, in particularly, to the problem of the collapses and revivals in such a model.

In this paper we address the problem of collapses and revivals of quantum phase fluctuations in the nondegenerate down-conversion with quantum pump. The Hermitian phase formalism of Pegg and Barnett [12]-[14] is applied to describe the phases of the interacting modes. The quantum evolution is found with the numerical diagonalization of the interaction Hamiltonian. The quantum phase fluctuations of the phase sum of the signal and idler modes for both short as well as long times are studied showing the collapses and revivals of the phase properties in the long time evolution. A comparison is made to the results for the ideal two-mode squeezed vacuum showing essential differences in the long-time behaviour.

II. QUANTUM EVOLUTION OF THE FIELD STATE

The nondegenerate down-conversion process is described by the following model Hamiltonian

$$H = H_0 + H_I = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_c c^\dagger c + \hbar g(a^\dagger b^\dagger c + abc^\dagger), \quad (1)$$

where a ($a^\dagger$), b ($b^\dagger$) and c ($c^\dagger$) are the annihilation (creation) operators of the signal mode at frequency ω_a, the idler mode at frequency ω_b and the pump mode at frequency ω_c, respectively. The coupling constant g, which is assumed real, describes the coupling between the three modes. Under conditions of perfect energy

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conservation, i.e., $\omega_c = \omega_a + \omega_b$, the free field Hamiltonian H_0 can be written in the form

$$H_0 = H_0^{(1)} + H_0^{(2)} \tag{2}$$

where

$$H_0^{(1)} = \frac{\hbar}{2}(\omega_a + \omega_b)(a^\dagger a + b^\dagger b + 2c^\dagger c), \tag{3}$$

and

$$H_0^{(2)} = \frac{\hbar}{2}(\omega_a - \omega_b)(a^\dagger a - b^\dagger b), \tag{4}$$

and the Hamiltonians commute with each other

$$[H_0^{(1)}, H_0^{(2)}] = [H_0^{(1)}, H_I] = [H_0^{(2)}, H_I] = 0. \tag{5}$$

Thus, there are three constants of motion, $H_0^{(1)}$, $H_0^{(2)}$ and H_I. H_0 determines the total energy stored in all modes, which is conserved by the interaction H_I. This allows us to factor out $\exp(-iH_0 t/\hbar)$ from the evolution operator and, in fact, to drop it altogether. In effect, the resulting state of the field can be written as

$$|\Psi(t)\rangle = \exp(-iH_I t/\hbar)|\Psi(0)\rangle, \tag{6}$$

where $|\Psi(0)\rangle$ is the initial state of the field. If the Fock states are used as basis states, the interaction Hamiltonian H_I is not diagonal in such a basis. To find the state evolution, we apply the numerical method of diagonalization of H_I [21].

Let us assume that initially there are n photons in the pump mode (c) and no photons in the signal (a) and idler (b) modes, i.e., the initial state of the field is $|0,0,n\rangle = |0\rangle_a|0\rangle_b|n\rangle_c$. Since $H_0^{(1)}$ and $H_0^{(2)}$ are constants of motion, we have the relations

$$\frac{1}{2}(\langle a^\dagger a\rangle + \langle b^\dagger b\rangle) + \langle c^\dagger c\rangle = \text{const} = n, \tag{7}$$

and

$$\langle a^\dagger a\rangle - \langle b^\dagger b\rangle = \text{const} = 0, \tag{8}$$

which implies that the annihilation of k photons of the pump mode requires creation of k photons of each the signal and the idler modes, simultaneously. Thus, for given n, we can introduce the states

$$|\psi_k^{(n)}\rangle = |k,k,n-k\rangle, \qquad k = 0,1,...,n\ , \tag{9}$$

which form a complete basis of states of the field for given n. We have

$$\langle\psi_{k'}^{(n')}|\psi_k^{(n)}\rangle = \delta_{nn'}\delta_{kk'}, \tag{10}$$

which means that the constant of motion H_0 splits the field space into orthogonal subspaces, which for given n have the number of components equal to $n+1$. The basis states $|\psi_k^{(n)}\rangle$ given by (9) are numbered by the total energy (in units of $\hbar\omega_c$) which is n and by the number of photons in the pump mode which is $n-k$.

The matrix elements of the interaction Hamiltonian are given by

$$\begin{aligned}\langle\psi_{k+1}^{(n)}|H_I|\psi_k^{(n)}\rangle &= \langle\psi_k^{(n)}|H_I|\psi_{k+1}^{(n)}\rangle \\ &= \hbar g(k+1)\sqrt{n-k}.\end{aligned} \tag{11}$$

This is a tridiagonal matrix which can be diagonalized efficiently allowing for the numerical evaluation of the matrix elements of the evolution operator.

If we assume that initially the pump mode is in a coherent state with the mean number of photons $|\alpha_c|^2$, and both the signal and idler modes are in the vacuum, the resulting state of the field can be written as

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} b_n e^{in\varphi_c} \sum_{k=0}^{n} c_{n,k}(t)|k,k,n-k\rangle, \tag{12}$$

where

$$b_n = \exp(-|\alpha_c|^2/2)|\alpha_c|^n/\sqrt{n!} \tag{13}$$

is the Poissonian weighting factor of the coherent state $|\alpha_c\rangle$ of the pump mode represented as a superposition of n-photon states, and the coefficients $c_{n,k}(t)$ are the matrix elements of the evolution operator

$$c_{n,k}(t) = \langle\psi_k^{(n)}|\exp(-iH_I t/\hbar)|\psi_0^{(n)}\rangle \tag{14}$$

that are calculated numerically. This allows us to find the evolution of the state (12).

III. PHASE PROPERTIES OF THE FIELD

To study phase properties of the field produced in the down-conversion process, we use the Pegg-Barnett [12]–[14] Hermitian phase formalism. According to Pegg and Barnett, the Hermitian phase operator can be constructed in a finite $(s+1)$-dimensional state space Ψ spanned either by the number states, $|n\rangle$, or , $(s+1)$ orthonormal phase states, $|\theta_m\rangle$. The phase states can be expanded in terms of the number states as

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}}\sum_{n=0}^{s}\exp(in\theta_m)|n\rangle, \qquad (m = 0,1,...,s) \tag{15}$$

where

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1}. \tag{16}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The Hermitian phase operator is defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|, \qquad (17)$$

where the subscript θ indicates the dependence on the choice of θ_0. The phase states (15) are eigenstates of the phase operator (17) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0 + 2\pi$. Recently Bužek et al. [15] have analyzed algebraic properties of the phase operator (17) and introduced interesting concept of the annihilation and creation operator of phase quanta. Lukš and Peřinová [16] have proposed another approach to the quantum phase problem based on the mapping and operator ordering.

Physical results are obtained in the limit $s \to \infty$, and according to Pegg and Barnett prescription this limit has to be taken only after c numbers, such as the expectation value and variance of the phase, have been calculated in the finite basis (15). Here we are interested in phase fluctuations, so we need the phase distribution function. Projecting the field state (12) onto the phase states (15), defined for each mode entering the process, we find a probability amplitude for the field state being in a definite phase state. In our case of field produced in the down-conversion process with quantum pump, the state of the field (12) is a three-mode state, and the Pegg-Barnett phase formalism generalized to the three-mode case gives

$$\langle\theta_{m_a}|\langle\theta_{m_b}|\langle\theta_{m_c}|\psi(t)\rangle = (s_a+1)^{-1/2}(s_b+1)^{-1/2}(s_c+1)^{-1/2}$$
$$\times \sum_{n=0}^{s_a} b_n e^{in\varphi_c} \sum_{k=0}^{n} \exp\{-i[k(\theta_{m_a}+\theta_{m_b}) + (n-k)\theta_{m_c}]\} c_{n,k}(t). \qquad (18)$$

We use the indices a, b and c to distinguish between the signal (a), idler (b) and pump (c) modes. There is still a freedom of choice in (18) of the values of $\theta_0^{a,b,c}$, which define the phase values window. We can choose these values at will, so we take them as

$$\theta_0^{a,b,c} = \varphi_{a,b,c} - \frac{\pi s_{a,b,c}}{s_{a,b,c}+1} \qquad (19)$$

and we introduce the new phase values

$$\theta_{\mu_{a,b,c}} = \theta_{m_{a,b,c}} - \varphi_{a,b,c}, \qquad (20)$$

where the new phase labels $\mu_{a,b,c}$ run in unit step between the values $-s_{a,b,c}/2$ and $s_{a,b,c}/2$. This means that we symmetrize the phase windows for the signal, idler and pump modes with respect to the phases φ_a, φ_b, and φ_c respectively. We are free to choose the parameters $s_{a,b,c}$ as large as they are needed, and for real physical states it is always possible to choose $s_{a,b,c}$ much larger than the contributing number states. So, the parameters $s_{a,b,c}$ in the sum of Eq. (18) can be replaced to any desired degree of accuracy by the infinity.

On inserting (19) and (20) into (18), taking the modulus squared of (18), and performing the continuum limit transition we arrive at the continuous joint probability distribution for the continuous variables θ_a, θ_b and θ_c, which has the form

$$P(\theta_a,\theta_b,\theta_c) = \frac{1}{(2\pi)^3}\left|\sum_{n=0}^{\infty} b_n \sum_{k=0}^{n} c_{n,k}(t) \times \exp\{-i[k(\theta_a+\theta_b) + (n-k)\theta_c + k(\varphi_a+\varphi_b-\varphi_c)]\}\right|^2 . \qquad (21)$$

The distribution (21) is normalized so as

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} P(\theta_a,\theta_b,\theta_c)d\theta_a d\theta_b d\theta_c = 1. \qquad (22)$$

To fix the phase windows for θ_a, θ_b and θ_c we have to assign to φ_a, φ_b and φ_c particular values. It is interesting to note that the distribution $P(\theta_a,\theta_b,\theta_c)$ given by (21) depends on the phase combination $\varphi_a+\varphi_b-\varphi_c$ only. This reproduces the classical phase relation for the parametric amplifier, and classically this quantity should be equal to $-\pi/2$ to get the amplification of the signal mode (if the coupling constant g is positive). Such choice means that a peak should appear in the phase distribution at $\theta_a+\theta_b=0$.

Marginal phase distributions of the a and b modes are

$$P(\theta_a) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} P(\theta_a,\theta_b,\theta_c)d\theta_b d\theta_c = \frac{1}{2\pi}, \qquad P(\theta_b) = \frac{1}{2\pi}. \qquad (23)$$

We introduce new variables

$$\theta_+ = \theta_a + \theta_b, \qquad (24)$$
$$\theta_- = \theta_a - \theta_b \qquad (25)$$

and get the new joint probability distribution $P(\theta_+,\theta_-,\theta_c)$ for the phase sum θ_+ and difference θ_- for the signal and idler modes, and the phase θ_c of the pump mode. The Jacobean for this transformation is 2. However, for this new probability distribution the ranges of values that θ_+ and θ_- can take are $-2\pi \le \theta_\pm < 2\pi$. While physically distinct values of the phase sum and difference exist only in a 2π range. Therefore it is desirable to reduce the possible values of phase sum and difference into a 2π interval. This is achieved by means of the casting procedure proposed by Barnett and Pegg [10]. We select central 2π ranges, from $-\pi$ to π, for $\theta_\pm$, by adding or subtracting 2π as necessary to values of θ_+ and θ_- outside these 2π ranges. As a result of this procedure we obtain the joint mod(2π) probability distribution

$$P_{2\pi}(\theta_+,\theta_-,\theta_c) = \frac{1}{(2\pi)^3}\left|\sum_{n=0}^{\infty} b_n \sum_{k=0}^{n} c_{n,k}(t) \times \exp\{-i[k\theta_+ + (n-k)\theta_c + k(\varphi_a+\varphi_b-\varphi_c)]\}\right|^2 . \qquad (26)$$

where now $-\pi \leq \theta_\pm < \pi$ and $-\pi \leq \theta_c < \pi$.

Since $P_{2\pi}(\theta_+, \theta_-, \theta_c)$ is independent of θ_-, the integral of the distribution over θ_+ and θ_c will also be independent of θ_-, so we have for the 2π range marginal phase-difference distribution

$$P_{2\pi}(\theta_-) = \frac{1}{2\pi}. \tag{27}$$

This means that the phase-difference for the signal and idler modes is random. Again, this effect is known from the case of the ideal squeezed vacuum [10, 11], and it is not affected by the quantum fluctuations of the pump mode.

Integrating (26) over θ_- leads to

$$\begin{aligned} P_{2\pi}(\theta_+, \theta_c) &= \int_{-\pi}^{\pi} P_{2\pi}(\theta_+, \theta_-, \theta_c) d\theta_- \\ &= \frac{1}{(2\pi)^2} \Bigg| \sum_{n=0}^{\infty} b_n \sum_{k=0}^{n} c_{n,k}(t) \\ &\quad \times \exp\{-i[k\theta_+ + (n-k)\theta_c + k(\varphi_a + \varphi_b - \varphi_c)]\} \Bigg|^2 . \end{aligned} \tag{28}$$

The phase distribution $P_{2\pi}(\theta_+, \theta_c)$ is the mod(2π) joint phase distribution that depends on two variables θ_+ and θ_c, the two phase variables that describe the essential phase dynamics of the nondegenerate down-converter with quantum pump.

Integrating $P_{2\pi}(\theta_+, \theta_c)$ over one of the variables leads to the marginal phase distributions $P_{2\pi}(\theta_+)$ and $P(\theta_c)$ for the phase sum θ_+ and phase of the pump mode θ_c, respectively. Performing the integrations we have

$$\begin{aligned} P_{2\pi}(\theta_+) &= \int_{-\pi}^{\pi} P_{2\pi}(\theta_+, \theta_c) d\theta_c \\ &= \frac{1}{2\pi} \Bigg\{ 1 + 2\mathrm{Re} \sum_{n>m} b_n b_m \exp[-i(n-m)(\theta_+ - \pi/2)] \\ &\quad \times \sum_{k=0}^{m} c_{n,n-m+k}(t) c^*_{m,k}(t) \Bigg\}, \end{aligned} \tag{29}$$

and

$$\begin{aligned} P(\theta_c) &= \int_{-\pi}^{\pi} P_{2\pi}(\theta_+, \theta_c) d\theta_+ \\ &= \frac{1}{2\pi} \Bigg\{ 1 + 2\mathrm{Re} \sum_{n>m} b_n b_m \exp[-i(n-m)\theta_c] \\ &\quad \times \sum_{k=0}^{m} c_{n,k}(t) c^*_{m,k}(t) \Bigg\}. \end{aligned} \tag{30}$$

Since the phases θ_a and θ_b of the individual modes as well as the phase difference θ_- are all random, the quantum phase properties associated with them are well known. The nontrivial quantum phase properties that are related to the quantum evolution of the nondegenerate two-photon down-converter are described by the phase distributions (28), (29) and (30). Knowing these phase distributions we are able to calculate all necessary phase expectation values by simple integrations over the phase variables. This gives us for the mod(2π) phase variance of the phase sum θ_+ the following expression

$$\begin{aligned} \Delta_{2\pi}(\hat{\phi}_{\theta_a} + \hat{\phi}_{\theta_b})^2 &= \int_{-\pi}^{\pi} \theta_+^2 P_{2\pi}(\theta_+) d\theta_+ \\ &= \frac{\pi^2}{3} + 4\mathrm{Re} \sum_{n>m} b_n b_m \frac{(-1)^{n-m}}{(n-m)^2} \exp[i(n-m)\pi/2] \\ &\quad \times \sum_{k=0}^{m} c_{n,n-m+k}(t) c^*_{m,k}(t), \end{aligned} \tag{31}$$

and for the phase variance of the pump mode we have

$$\begin{aligned} (\Delta\hat{\phi}_{\theta_c})^2 &= \int_{-\pi}^{\pi} \theta_c^2 P(\theta_c) d\theta_c \\ &= \frac{\pi^2}{3} + 4\mathrm{Re} \sum_{n>m} b_n b_m \frac{(-1)^{n-m}}{(n-m)^2} \sum_{k=0}^{m} c_{n,k}(t) c^*_{m,k}(t), \end{aligned} \tag{32}$$

The 2π subscript of Δ in (31) denotes that the variance is calculated with the mod(2π) phase distribution according to the casting procedure of Barnett and Pegg [10]. This means that we study the phase properties of the phase sum being rather a single phase variable and not a sum of two phase variables. Since θ_c is a single phase variable from the outset and there is no need to cast it into a new 2π range.

The results described by the formulae (28)-(32) are illustrated graphically in Figures 1-3. In Figure 1 the mod(2π) joint phase probability distribution given by formula (28) is plotted for several values of the scaled time gt. Initially, ($gt = 0$), the sum of the phases for the signal and idler modes $\theta_+ = \theta_a + \theta_b$ is uniformly distributed, and we see a completely flat distribution of this phase, while the phase θ_c of the pump mode shows a phase peak associated with the initial coherent state of this mode. As time elapses a peak of the phase sum appears for $\theta_+ = 0$, as expected for our choice of the phase windows. At the initial stages of the evolution the sum phase distribution becomes narrower, but at later times it starts to broaden back. One can also observe appearance of additional peaks for the pump mode phase θ_c. Such bifurcation of the phase distribution is known from the degenerate case of the down-conversion process [17]. In contrast to the the sum of the phases for the signal and idler modes $\theta_+ = \theta_a + \theta_b$ is uniformly distributed,

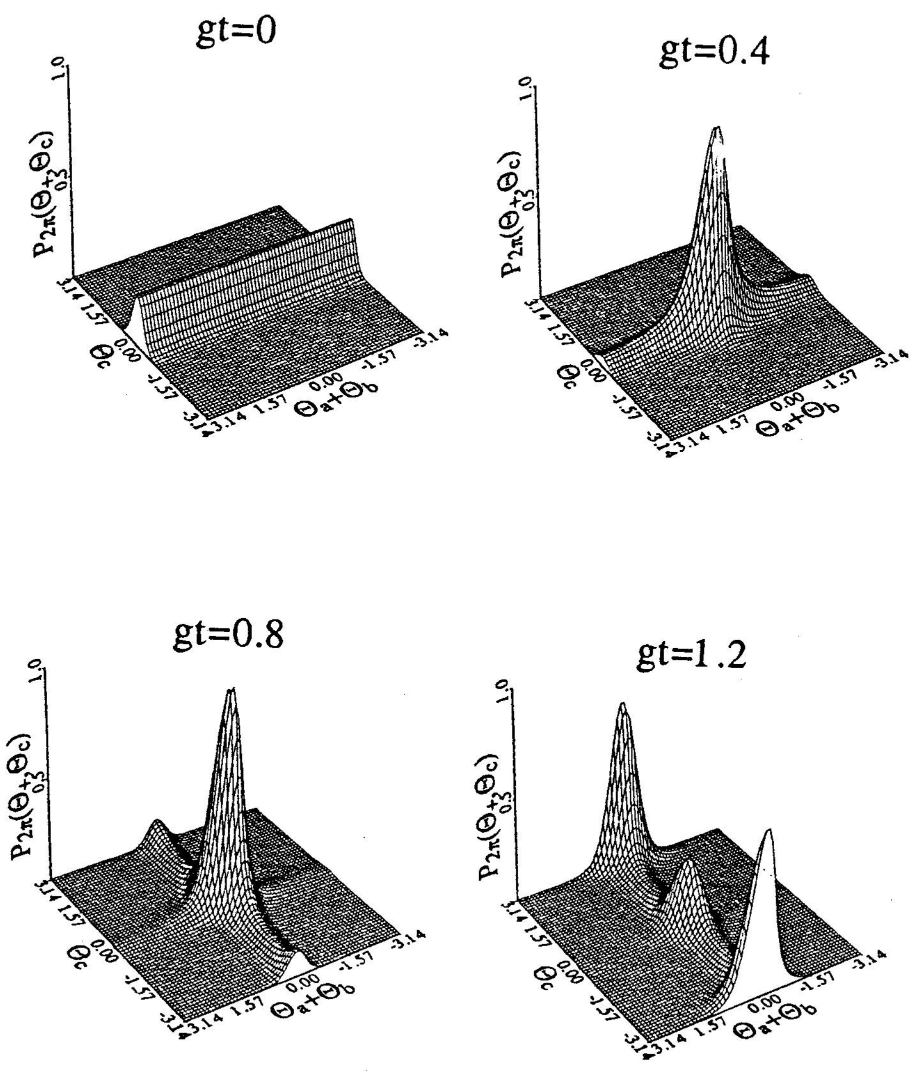

Fig. 1. Evolution of the mod(2π) joint phase probability distribution $P_{2\pi}(\theta_+, \theta_c)$. The initial mean number of photons of the pump mode $N_c = |\alpha_c|^2$ is equal to 4.

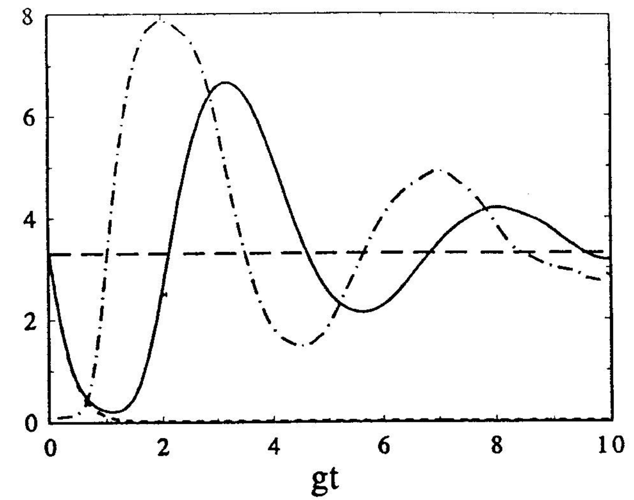

Fig. 2. Phase-sum variance $\Delta_{2\pi}(\hat{\phi}_{\theta_a}+\hat{\phi}_{\theta_b})^2$ (solid line), phase variance for the pump mode $(\Delta\hat{\phi}_{\theta_c})^2$ (dot-dashed line) and phase variance for the two-mode ideal squeezed vacuum (dashed line). The initial mean number of photons of the pump mode $N_c = |\alpha_c|^2$ is equal to 4.

and we see a completely flat distribution of this phase, while the phase θ_c of the pump mode shows a phase peak associated with the initial coherent state of this mode. As time elapses a peak of the phase sum appears for $\theta_+ = 0$, as expected for our choice of the phase windows. At the initial stages of the evolution the sum phase distribution becomes narrower, but at later times it starts to broaden back. One can also observe appearance of additional peaks for the pump mode phase θ_c. Such bifurcation of the phase distribution is known from the degenerate case of the down-conversion process [17]. In contrast to the degenerate case for which there are two peaks in the phase distribution for the signal mode, here we have only one peak of the phase sum similarly as for the ideal two-mode squeezed vacuum [10, 11]. However, contrary to the two-mode squeezed vacuum, the phase sum distribution does not tend asymptotically to the δ function but retains finite width, and as time elapses it is even broadened meaning randomization of the phase. This effect is better seen from Figure 2, where the phase variances evolution is shown. We see the oscillatory behaviour of the phase variances in the case when the quantum fluctuations of the pump mode are taken into account. For reference we have also plotted the phase variance for the ideal squeezed vacuum, which asymptotically approaches zero as time increases (the squeezing parameter r is related to the scaled time gt by the equation $r = \sqrt{N_c}gt$). The horizontal line marks the value $\pi^2/3$, which is the phase variance for the randomly distributed phase. It is seen that both the phase sum and pump mode variances oscillate around the value for the randomly distributed phase approaching this value. This character of the oscillation resembles the collapse of some other properties of such a system like a mean number of photons, photon statistics, or squeezing [18]. Thus, a question

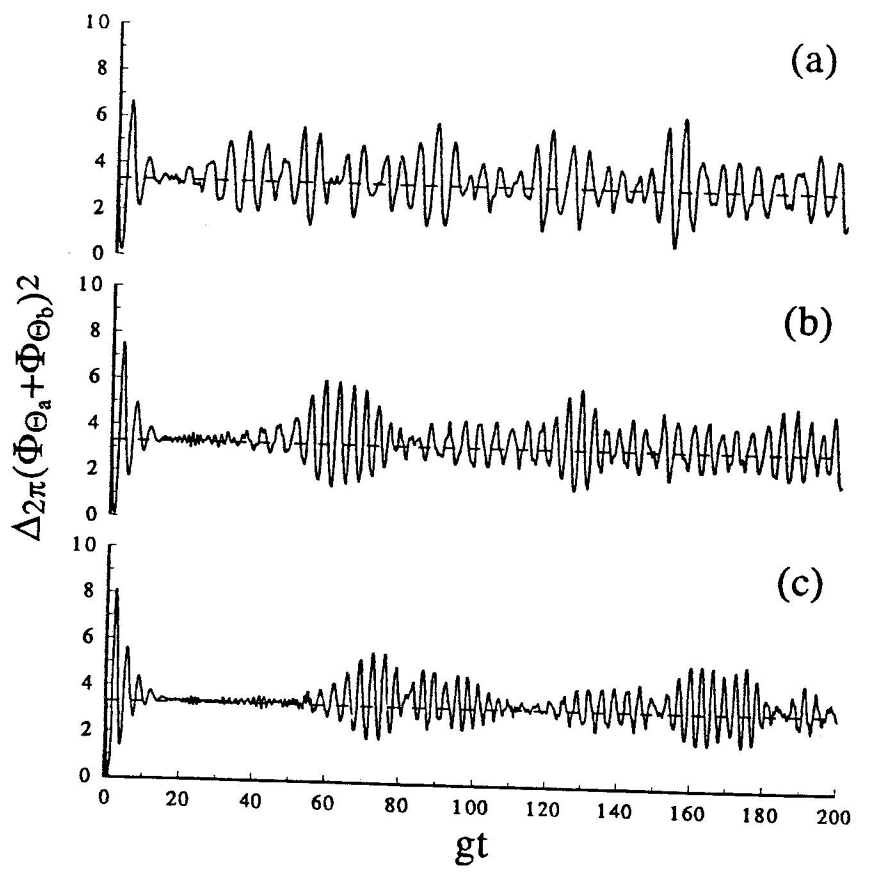

Fig. 3. Long-time evolution of the phase sum variance: (a) $N_c = 4$, (b) $N_c = 9$, (c) $N_c = 16$.

arises: Whether after a collapse of the phase properties one can expect also a revival of such properties? Drobný and Jex [18] have noticed revivals of other properties in a similar system under special conditions. In Figure 3 we have plotted the long time evolution of the phase sum variance for the initial values of the mean number of photons of the pump mode N_c equal to: 4 (a), 9 (b), and 16 (c). The collapse-revival character of the evolution is clearly visible, and it is more evident for larger mean numbers of photons. Similar behaviour of the quantum phase fluctuations for the signal as well as pump mode of the degenerate down-conversion has been found by Gantsog [22].

Since the phase of the pump mode is strongly correlated with the phase sum of the signal and idler modes, one can expect similar behaviour also for the phase

variance of the pump mode. We have really obtained similar behaviour for the pump mode too, but because the picture is very similar to Figure 3 we do not present it in this paper.

IV. CONCLUSION

We have shown that in the down-conversion with quantum pump phase properties of the phase sum are different from those of the ideal two-mode squeezed vacuum. They have oscillatory behaviour exhibiting collapses and revivals, which is in marked contrast to the asymptotically well defined phase sum in the ideal two-mode squeezed vacuum.

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