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LONGITUDINAL SURFACE WAVE PROPAGATION IN A PRESTRESSED GENERALIZED THERMO PIEZOELECTRIC HALF SPACE OF MONOCLINIC SYMMETRY

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Propagation of a longitudinal surface wave on the surface of a monoclinic piezoelectric half space which is initially under stress has been considered, taking into account a generalized thermal coupling as in the Lord and Shulman model. Variations of the piezoelectric potential, particle displacement, temperature etc. with depth into the medium, piezoelectric Poynting vector along with the power flow components and the angle between the group velocity and the phase velocity have been derived assuming the existence of a shorting plane at a finite distance above the free surface of the half space.

I. INTRODUCTION

The problem of surface wave propagation in an isotropic elastic half space was first investigated by Lord Rayleigh [1] in the year 1885. Similar studies, in the case of anisotropic piezoelectric solids have been attempted by researchers like Coquin and Tiersten [2], White and Tseng [3], Kaliski [4] etc. due to their various applications in electronics.

Propagation of surface acoustic waves in a prestressed piezoelectric medium by incorporating the initial stresses directly in the equations of motion was first investigated by Nalamwar and Epstein [5]. In the year 1979 Pal [6] extended propagation of such longitudinal waves of the Rayleigh type to a thermopiezoelectric medium, where a triple coupling between mechanical, electrical and thermal fields takes place. The thermal coupling considered by him was of classical nature. The theory of thermoelasticity which takes into account the time needed for the acceleration of heat flow has aroused much interest in recent years. This theory is a generalization of the classical coupled thermoelasticity. Several authors, for example, Lord and Shulman [7], Green and Lindsay [8] etc., have derived the field equations of this theory on different grounds taking into account one or two thermal relaxation parameters respectively. After their pioneering attempts [7], [8] several other researchers have recently devoted their attention to the study of thermoelastic problems from the standpoint of generalized thermal coupling. In this connection, the works done by Agarwal [9], Chandrasekhariah [10], [11] etc. should be mentioned. Similar studies in case of anisotropic piezoelectric solids have been carried out by Pal [12], Chandrasekhariah [13], Nandy [14], Bassiouny et al. [15] to name only a few.

In the present paper an attempt has been made to consider the propagation of a longitudinal surface wave of the Rayleigh type in a prestressed generalized thermopiezoelectric half space of monoclinic symmetry following Nalamwar and Epstein [5]. The generalized thermal coupling has been considered as in the Lord and Shulman model [7]. Ultimately, wave parameters like variations of piezoelectric potential, particle displacement, temperature etc. with depth into the medium, piezoelectric Poynting vector along with the power flow components and the deviation of the direction of group velocity from the phase velocity have also been determined, assuming the existence of a shorting plate at a finite distance 'h' above the free surface of the half space.

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II. FUNDAMENTAL EQUATIONS OF THE PROBLEM

Newton's vibration equation, Gauss's divergence equation and the equations of state of the piezoelectric material constitute the governing equations of the problem. Since we consider the substrate to be a prestressed monoclinic piezoelectric half space, the vibration equation can be taken as

$$\frac{\partial}{\partial x_i}\left(\sigma_{ik}\frac{\partial u_j}{\partial x_k}\right)+\frac{\partial \tau_{ij}}{\partial x_i}=\rho\frac{\partial^2 u_j}{\partial t^2} \tag{2.1}$$

see Bolotin [16], Nalamwar and Epstein [5] etc., where σ_{ik} are the initial stress components. The divergence equation due to Gauss is the following

$$D_{i,i}=0 \tag{2.2}$$

The constitutive equations of the material on which surface waves are assumed to propagate are

$$\begin{aligned}\tau_{ij}&=c_{ijkl}-e_{mij}E_m-\lambda_{ij}\Theta\\ D_j&=e_{jkl}S_{kl}+\epsilon_{ij}E_m+p_j\Theta\\ \sigma&=\lambda_{kl}S_{kl}+p_iE_i+a^{sE}\Theta\end{aligned} \tag{2.3}$$

see Mindlin [17] where τ_{ij} are the components of stress, S_{kl} are the strain components, D_i are the electric displacement components, E_i are the electric field components, σ is the entropy, u_i are the displacement components and Θ is the temperature. c_{ijkl}, e_{mij} and ϵ_{ij} are the elastic stiffness, piezoelectric constants and dielectric constants of the material. λ_{ij} and p_j are the thermoelectric and pyroelectric constants of the material, a^{sE} is some coupling constant and ρ is the density of the medium. Here the summation convention for repeated tensor indices is employed and an index preceded by a comma denotes differentiation with respect to some space co-ordinate. Dot notation signifies time derivative.

In addition to the equations presented above, the following are also important for the problem. The strain components

$$S_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) \tag{2.4}$$

and if ϕ is some potential function, then the electric field components are given by

$$E_i = -\phi_{,i}. \tag{2.5}$$

Moreover, since we consider the Lord and Shulman model [7] of thermal coupling, the necessary heat equation can be obtained by eliminating the heat flux q_i from the following two equations.

$$q_i + \tau_0 \dot{q}_i = -\kappa_{ij}\Theta_{,j} \tag{2.6}$$

$$\Theta_0 \dot{\sigma} = -q_{i,i}, \tag{2.7}$$

where τ_0 and Θ_0 are the relaxation parameter and some reference temperature, respectively. The elastic stiffnesses l_{ijkl} and the piezoelectric constants e_{mij} appearing in the constitutive equations are with four and three indices, respectively. These constants can be expressed in two index notation, see Mason [18].

We consider the direction of propagation of the surface wave to be along the x_1 axis and the x_2 axis is taken perpendicular to the free surface drawn towards the interior of the half space. x_3 axis is taken tangential to the free surface in a direction perpendicular to the $(x_1 - x_2)$ plane. On substituting the first equation of

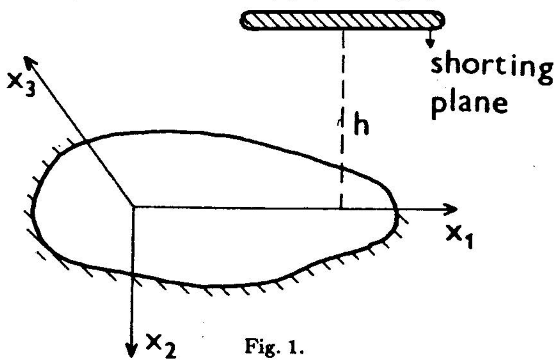

Fig. 1.

(2.3) in the equation (2.1) and using the relevant matrices for elastic, piezoelectric, dielectric, thermoelectric and pyroelectric constants corresponding to monoclinic symmetry, see Tiersten [19] and then rejecting derivatives with respect to the x_3 co-ordinate, we find the following three equations

$$\begin{aligned}
\rho\ddot{u}_1 =& \sigma_{11}u_{1,11} + \sigma_{12}u_{1,21} + \sigma_{21}u_{1,12} + \sigma_{22}u_{1,22} + c_{11}u_{1,11} + c_{12}u_{2,21} + \\
&+ c_{14}u_{3,21} + c_{66}(u_{1,22} + u_{2,12}) + c_{56}u_{3,12} + \\
&+ e_{11}\Phi_{,11} + e_{26}\Phi_{,22} - \lambda_{11}\Theta_{,1}, \\
\rho\ddot{u}_2 =& \sigma_{11}u_{2,11} + \sigma_{12}u_{2,21} + \sigma_{21}u_{2,12} + \sigma_{22}u_{2,22} + c_{56}u_{3,11} + \\
&+ c_{66}(u_{1,21} + u_{2,11}) + c_{12}u_{1,12} + c_{22}u_{2,22} + c_{24}u_{3,22} + \\
&+ e_{26}\Phi_{,21} + e_{12}\Phi_{,12} - \lambda_{22}\Theta_{,2}, \\
\rho\ddot{u}_3 =& \sigma_{11}u_{3,11} + \sigma_{12}u_{3,21} + \sigma_{21}u_{3,12} + \sigma_{22}u_{3,22} + c_{55}u_{3,11} + \\
&+ c_{56}(u_{1,21} + u_{2,11}) + c_{14}u_{1,12} + c_{24}u_{2,22} + \\
&+ c_{44}u_{3,22} + e_{25}\Phi_{,21} + e_{14}\Phi_{,12} - \lambda_{23}\Theta_{,2}.
\end{aligned} \tag{2.8}$$

From the second equation of (2.3) and Gauss's divergence equation (2.2), together with the corresponding coefficient matrices and then rejecting the derivatives with respect to the x_3 co-ordinates we find

$$\begin{aligned}
&e_{11}u_{1,11} + e_{12}u_{2,21} + e_{14}u_{3,21} + e_{26}(u_{1,22} + u_{2,12}) + \\
&+ e_{25}u_{3,12} - \epsilon_{11}\Phi_{,11} - \epsilon_{22}\Phi_{,22} + p_2\Theta_{,2} = 0.
\end{aligned} \tag{2.9}$$

To obtain the other necessary equation we eliminate the entropy σ from the last equation of (2.3) and the relation obtained by eliminating the heat flux vector q_i from equations (2.6) and (2.7). Ultimately, we find the generalized heat equation in the form

$$\begin{aligned}
K_{11}\Theta_{,11} + K_{22}\Theta_{,22} =& \Theta_0\{\lambda_{11}(\dot{u}_{1,1} + \tau_0\ddot{u}_{1,1}) + \lambda_{22}(\dot{u}_{2,2} + \tau_0\ddot{u}_{2,2}) \\
&+ \lambda_{23}(\dot{u}_{3,2} + \tau_0\ddot{u}_{3,2}) - p_2(\dot{\Phi}_{,2} + \tau_0\ddot{\Phi}_{,2}) + a^{sE}(\dot{\Theta} + \tau_0\ddot{\Theta}).
\end{aligned} \tag{2.10}$$

These five equations (2.8), (2.9) and (2.10) are the basic equations governing the problem.

III. PARTIAL WAVE SOLUTIONS OF THE PROBLEM

Let us seek the solution to the five fundamental equations (2.8) to (2.10) de scribed above in the form

$$\{u_1, u_2, u_3, \Phi, \Theta\} = \{A_1, A_2, A_3, A_4, A_5\}\exp(-\xi_2 x_2)\exp\{i(\xi_1 x_1 - \omega t)\}.$$

Substituting the above expression in the system of equations (2.8), (2,9) and (2.10) we find the following characteristics determinantal equation for the existence of nontrivial solutions of the problem.

$$\begin{vmatrix} \rho v_s^2 - c_{11} + c_{66}\alpha^2 + A & -(c_{12}+c_{66})i\alpha & -(c_{14}+c_{56})i\alpha & e_{26}\alpha^2 - e_{11} & -\lambda_{11}i/\xi_1 \\ -(c_{66}+c_{12})i\alpha & \rho v_s^2 + c_{22}\alpha^2 - c_{66} + A & c_{24}\alpha^2 - c_{56} & -(e_{12}+e_{26})i\alpha & \lambda_{22}\alpha/\xi_1 \\ -(c_{56}+c_{14})i\alpha & c_{24}\alpha^2 - c_{56} & \rho v_s^2 - c_{55} + c_{44}\alpha^2 + A & -(e_{25}+e_{14})i\alpha & \lambda_{23}\alpha/\xi_1 \\ e_{26}\alpha^2 - e_{11} & -(e_{26}+e_{12})i\alpha & -(e_{14}+e_{25})i\alpha & \epsilon_{11} - \epsilon_{22}\alpha^2 & -p_2\alpha/\xi_1 \\ \lambda_{11}\Theta_0 v_s - \tau_0(\lambda_{11}\Theta_0 i v_s^2\xi_1) & \lambda_{22}\Theta_0 i\alpha v_s + \tau_0(\lambda_{22}\Theta_0\alpha v_s^2\xi_1) & \lambda_{23}\Theta_0 i\alpha v_s + \tau_0(\lambda_{23}\Theta_0\alpha v_s^2\xi_1) & -p_2\Theta_0 i\alpha v_s - \tau_0(p_2\Theta_0\alpha\xi_1 v_s^2) & k_{11} - k_{22}\alpha^2 - a^{sE}\Theta_0 i v_s/\xi_1 - \tau_0(a^{sE}\Theta_0 v_s^2) \end{vmatrix}$$

$$= 0. \tag{3.1}$$

The determinantal equation (3.1) can also be written in the following form

$$\Big\{\rho v_s^2 - c_{11} + c_{66}\alpha^2 + A,\quad \rho v_s^2 - c_{66} + c_{22}\alpha^2 + A,\quad \rho v_s^2 - c_{55} + c_{44}\alpha^2 + A,$$

$$\epsilon_{11} - \epsilon_{22}\alpha^2,\quad k_{11} - k_{22}\alpha^2 - \frac{a^{sE}\Theta_0 v_s \mathrm{i}}{\xi_1} - \tau_0 a^{sE}\Theta_0 v_s^2\Big\} = 0,$$

where the elements within the second bracket are the diagonal elements of the above determinant. The equation (3.1) can again be written as

$$\Delta^* + \tau_0\xi_1 v_s\Delta^{**} = 0, \tag{3.2}$$

where

$$\Delta^* = \Big\{\rho v_s^2 - c_{11} + c_{66}\alpha^2 + A,\quad \rho v_s^2 - c_{66} + c_{22}\alpha^2 + A,\quad \rho v_s^2 - c_{55} +$$

$$+ c_{44}\alpha^2 + A,\quad \epsilon_{11} - \epsilon_{22}\alpha^2,\quad \kappa_{11} - \kappa_{22}\alpha^2 - \frac{a^{sE}\mathrm{i}\Theta_0 v_s}{\xi_1}\Big\} \tag{3.3}$$

and

$$\Delta^{**} = \Big\{\rho v_s^2 - c_{11} + c_{66}\alpha^2 + A,\quad \rho v_s^2 - c_{66} + c_{22}\alpha^2 + A,\quad \rho v_s^2 - c_{55} +$$

$$+ c_{44}\alpha^2 + A,\quad \epsilon_{11} - \epsilon_{22}\alpha^2 - \frac{a^{sE}\Theta_0 v_s}{\xi_1}.\Big\} \tag{3.4}$$

where

$$\frac{\xi_2}{\xi_1} = \alpha, \qquad \frac{\omega}{\xi_1} = v_s, \qquad A = \sigma_{22}\alpha^2 - \sigma_{11} - i\alpha(\sigma_{12} + \sigma_{21}),$$

α and v_s are the decay parameter and the phase velocity of the wave, A is some function of the prestress components. Equation (3.2) indicates clearly how the dispersion equation for the corresponding classical thermal coupling problem changes

due to the introduction of the relaxation parameter τ_0 in the case of generalized thermal coupling. Substituting the values of all the material constants and expanding the determinantal equation (3.1), we find a tenth degree equation in α in terms of the phase velocity v_s and wave number ξ_1. Putting the relaxation parameter τ_0 equal to zero, the determinantal dispersion equation (3.1) reduces to

$$\Delta^* = 0 \tag{3.5}$$

which has been obtained by Pal [6] assuming classical thermal coupling. In the absence of thermal coupling the dispersion equation (3.1) simplifies to a fourth order determinantal equation

$$\begin{vmatrix} \rho v_s^2 - c_{11} + c_{66}\alpha^2 + A & -(c_{12}+c_{66})i\alpha & -(c_{14}+c_{56})i\alpha & -e_{11} + e_{26}\alpha^2 \\ -(c_{66}+c_{12})i\alpha & \rho v_s^2 + c_{22}\alpha^2 - c_{66} + A & c_{24}\alpha^2 - c_{56} & -(e_{12}+e_{26})i\alpha \\ -(c_{56}+c_{14})i\alpha & c_{24}\alpha^2 - c_{56} & \rho v_s^2 - c_{55} + c_{44}\alpha^2 + A & -(e_{25}+e_{14})i\alpha \\ -e_{11} + e_{26}\alpha^2 & -(e_{12}+e_{26})i\alpha & -(e_{14}+e_{25})i\alpha & \epsilon_{11} - \epsilon_{22}\alpha^2 \end{vmatrix} = 0 \tag{3.6}$$

where the following amplitude ratios P, Q, R and T are introduced

$$P = \frac{A_2}{A_1}, \qquad Q = \frac{A_3}{A_1}, \qquad R = \frac{A_4}{A_1}, \qquad T = \frac{A_5}{A_1}. \tag{3.7}$$

The expressions for displacement components, piezoelectric potential and temperature become

$$\{u_1, u_2, u_3, \Phi, \Theta\} = \{A_1, PA_1, QA_1, RA_1, TA_1\}\exp(-\xi_2 x_2)\exp\{\mathrm{i}(\xi_1 x_1 - \omega t)\}. \tag{3.8}$$

Now substituting the above expressions in the equations (2.8) and (2.9) and then solving we find

$$-P = \frac{\Delta_1}{\Delta_0}, \qquad -Q = \frac{\Delta_2}{\Delta_0}, \qquad -R = \frac{\Delta_3}{\Delta_0}, \qquad -T = \frac{\Delta_4}{\Delta_0}. \tag{3.9}$$

where Δ_1, Δ_2, Δ_3, Δ_4 and Δ_0 are the following determinants.

$$\begin{aligned} \Delta_1 &= |C_1,\ C_3,\ C_4,\ C_5| \\ \Delta_2 &= |C_2,\ C_1,\ C_4,\ C_5| \\ \Delta_3 &= |C_2,\ C_3,\ C_1,\ C_5| \\ \Delta_4 &= |C_2,\ C_3,\ C_4,\ C_1| \\ \Delta_0 &= |C_2,\ C_3,\ C_4,\ C_5| \end{aligned} \tag{3.10}$$

where C_i $(i = 1, 2, 3, 4, 5)$ are the five columns of the determinant in equation (3.1) omitting the last row.

Though equation (3.1) is a tenth degree equation in the decay parameter α having ten complex roots α_j for each value of v_s and ξ_1, five of them can be eliminated by means of conditions at infinity as in Kaliski [4]. The remaining roots α_j ($j = 1, 2, 3, 4, 5$) are used to build up the general solution of the problem.

$$\{u_1, u_2, u_3, \Phi, \Theta\} = \left\{ \sum_{j=1}^{5} A_1^{(j)},\ \sum_{j=1}^{5} P_j A_1^{(j)},\ \sum_{j=1}^{5} Q_j A_1^{(j)},\ \sum_{j=1}^{5} R_j A_1^{(j)},\ \sum_{j=1}^{5} T_j A_1^{(j)} \right\} \exp(-\xi_1 \alpha_j x_2) \exp\{i(\xi_1 x_1 - \omega t)\} \tag{3.11}$$

where P_j, Q_j, R_j and T_j are the values of P, Q, R and T for $\alpha = \alpha_j$ to be obtained from equations (3.9) using (3.10). In the case of classical thermal coupling the roots α_j are to be determined from the equation (3.3). When thermal coupling is switched off completely, the general solution for the surface wave propagation involves only four partial waves corresponding to the four permissible roots α_j ($j = 1, 2, 3, 4$) of the equation (3.6).

IV. BOUNDARY CONDITIONS

Though the particle motion is limited to the inside of the crystal, the electric field is not confined to that restriction. Hence we have to consider the equations of the electric field in vacuum beyond the free surface of the half space. If $\hat{E}_i$ is the electric field in the vacuum, then $\hat{E}_{i,i} = 0$ and hence using the potential $\hat{E}_i = -\hat{\Phi}_{,i}$ we find

$$\nabla^2 \hat{\Phi} = 0. \tag{4.1}$$

Moreover, since we assume the existence of an electrical shorting plane at a certain height (say h) above the free surface,

$$[\hat{\Phi}]_{x_2 = -h} = 0. \tag{4.2}$$

Let us choose the electric potential $\hat{\Phi}$ in the vacuum above the free surface of the piezoelectric half space in the form

$$\hat{\Phi} = A_6 \sinh\{\xi_1(x_2 + h)\} \exp\{i(\xi_1 x_1 - \omega t)\} \tag{4.3}$$

such that it satisfies equations (4.1) and (4.2). The boundary conditions at the free surface can be taken as

i) Stress components

$$\tau_{2i} = 0 \quad (i = 1, 2, 3) \quad \text{at} \quad x_2 = 0. \tag{4.4}$$

ii) The tangential components of the electric field are continuous at the free surface

$$x_2 = 0 \quad \Phi_{,1} = \hat{\Phi}_{,1} \quad \text{at} \quad x_2 = 0. \tag{4.5}$$

iii) Normal components of the electric displacement are continuous at the free surface

$$x_2 = 0 \quad D_2 = \hat{D}_2 \quad \text{at} \quad x_2 = 0. \tag{4.6}$$

iv) The free surface of the half space is thermally insulated

$$\left(\frac{\partial \Theta}{\partial x_2}\right) = 0 \quad \text{at} \quad x_2 = 0. \tag{4.7}$$

Using the above mentioned boundary conditions we find a set of six equations in $A_1^{(j)}$ ($j = 1, 2, 3, 4, 5$) and A_6. From these six equations only the fourth and fifth equation involve in addition to $A_1^{(j)}$ ($j = 1, 2, 3, 4, 5$) the other amplitude A_6. Eliminating A_6 from these two equations, ultimately the above mentioned set of six equations can be reduced to a set of five equations in $A_1^{(j)}$. These five equations can be written in the form

$$d_{ij} A_1^{(j)} = 0 \quad (i, j = 1, 2, 3, 4, 5) \tag{4.8}$$

where d_{ij}'s are the following

$$\begin{aligned}
d_{ij} &= i c_{56} Q_j - c_{66} \alpha_j + i c_{66} P_j - e_{26} \alpha_j R_j, \\
d_{2j} &= i\xi_1 c_{12} - c_{22} \xi_1 P_j \alpha_j - \xi_1 c_{24} \alpha_j Q_j + i\xi_1 e_{12} R_j - \lambda_{22} T_j, \\
d_{3j} &= i c_{14} \xi_1 - c_{24} \xi_1 \alpha_j P_j - Q_j c_{44} \xi_1 \alpha_j + i e_{14} R_j \xi_1 - \lambda_{23} T_j, \\
d_{4j} &= \xi_1 \epsilon_{22}^{(0)} d_{4j}^{**} + \tanh(\xi, h) d_{4j}^{*}, \\
d_{5j} &= T_j \alpha_j, \\
d_{4j}^{*} &= i\xi_1 e_{25} Q_j - e_{26} \xi_1 \alpha_j + i\xi_1 e_{26} P_j + \xi_1 \epsilon_{22} \alpha_j R_j + \phi_2 T_j, \\
d_{4j}^{**} &= R_j.
\end{aligned} \tag{4.9}$$

In equations (4.9) the repeated suffix does not indicate summation over that suffix. Equations (4.8) would have nontrivial solution if

$$|d_{ij}| = 0. \tag{4.10}$$

This is a fifth order determinant and its elements are given by equations (4.9). Now let us divide each of (4.8) by $A_1^{(1)}$ and solving any four of these five equations we can find the ratios

$$\frac{A_1^{(2)}}{A_1^{(1)}}, \quad \frac{A_1^{(3)}}{A_1^{(1)}}, \quad \frac{A_1^{(4)}}{A_1^{(1)}}, \quad \frac{A_1^{(5)}}{A_1^{(1)}}$$

Denoting the above four ratios by l_{21}, l_{31}, l_{41} and l_{51} respectively, the displacement components, piezoelectric potential and temperature can be expressed as the sum of five partial waves. Hence

$$\begin{aligned} u_1 =& A_1^{(1)} \{\exp(-\xi_1\alpha_1 x_2) + l_{21}\exp(-\xi_1\alpha_2 x_2) + l_{31}\exp(-\xi_1\alpha_3 x_2) \\ &+ l_{41}\exp(-\xi_1\alpha_4 x_2) + l_{51}\exp(-\xi_1\alpha_5 x_2)\}\exp\{i(\xi_1 x_1 - \omega t)\}. \\ u_2 =& P_1 A_1^{(1)} \{\exp(-\xi_1\alpha_1 x_2) + P_{21}\exp(-\xi_1\alpha_2 x_2) + P_{31}\exp(-\xi_1\alpha_3 x_2) \\ &+ P_{41}\exp(-\xi_1\alpha_4 x_2) + P_{51}\exp(-\xi_1\alpha_5 x_2)\}\exp\{i(\xi_1 x_1 - \omega t)\}. \\ u_3 =& Q_1 A_1^{(1)} \{\exp(-\xi_1\alpha_1 x_2) + Q_{21}\exp(-\xi_1\alpha_2 x_2) + Q_{31}\exp(-\xi_1\alpha_3 x_2) + \\ &+ Q_{41}\exp(-\xi_1\alpha_4 x_2) + Q_{51}\exp(-\xi_1\alpha_5 x_2)\}\exp\{i(\xi_1 x_1 - \omega t)\}. \\ \Phi =& R_1 A_1^{(1)} \{\exp(-\xi_1\alpha_1 x_2) + R_{21}\exp(-\xi_1\alpha_2 x_2) + R_{31}\exp(-\xi_1\alpha_3 x_2) + \\ &+ R_{41}\exp(-\xi_1\alpha_4 x_2) + R_{51}\exp(-\xi_1\alpha_5 x_2)\}\exp\{i(\xi_1 x_1 - \omega t)\}. \\ \Theta =& T_1 A_1^{(1)} \{\exp(-\xi_1\alpha_1 x_2) + T_{21}\exp(-\xi_1\alpha_2 x_2) + T_{31}\exp(-\xi_1\alpha_3 x_2) + \\ &+ T_{41}\exp(-\xi_1\alpha_4 x_2) + T_{51}\exp(-\xi_1\alpha_5 x_2)\}\exp\{i(\xi_1 x_1 - \omega t)\} \end{aligned} \tag{4.11}$$

where

$$\{P_{ji}, Q_{ji}, R_{ji}, T_{ji}\}_{j=2,3,4,5} = \left\{\frac{P_j}{P_1}, \frac{Q_j}{Q_1}, \frac{R_j}{R_1}, \frac{T_j}{T_1}\right\} l_{j1}. \tag{4.12}$$

V. PIEZOELECTRIC POYNTING VECTOR AND POWER-FLOW COMPONENTS

The analysis of surface wave propagation remains incomplete if the piezoelectric Poynting vector and the associated power flow components is not considered.

Using Auld's [20] notation, piezoelectric Poynting vector P is given by the relation

$$P = \frac{v^* \cdot \tau}{2} + \frac{E \times H^*}{2}. \tag{5.1}$$

Neglecting the electro magnetic part $\frac{E \times H^*}{2}$ we find $P = \frac{v^* \cdot \tau}{2}$. (5.2)

Here $*$ indicates the complex conjugate of the corresponding factor. Hence the power flow components

$$P_{T,i} = \frac{1}{2}\,\text{real}\left[\int_0^\infty \left\{\tau_{ij}\frac{\partial u_j^*}{\partial t}\right\} \mathrm{d}x_2\right]; \tag{5.3}$$

see, Auld [20].

The magnitude of this vector gives the time average power crossing a strip of unit width and infinite depth oriented perpendicular to the vector.

Substituting the expressions for the stress component and velocity in equation (5.3) and then integrating, the components of power flow along the three directions of coordinate axes can be determined. The power flow component along the x_1 axis direction of propagation of the surface wave is the following:

$$P_{T,1} = \frac{1}{2}\,\text{real}\left[\sqrt{-1}\xi_1^2 v_s |A_1^{(1)}|^2 e_{ji}^* \Gamma_j\right]. \tag{5.4}$$

Similarly the component along the direction (x_3 axis) perpendicular to the Saggital plane is given by

$$P_{T,3} = \frac{1}{2}\,\text{real}\left[\sqrt{-1}\xi_1^2 v_s |A_1^{(1)}|^2 l_{ji}^* \overline{\Gamma}_j\right]. \tag{5.5}$$

Hence the deviation of the group velocity from the direction of the phase velocity of the surface wave is

$$\tan^{-1}\frac{P_{T,3}}{P_{T,1}} = \tan^{-1}\left[\frac{\mathrm{Re}\{\sqrt{-1}\xi_1^2 v_s |A_1^{(1)}|^2 l_{j1}^* \overline{\Gamma}_j\}}{\mathrm{Re}\{\sqrt{-1}\xi_1^2 v_s |A_1^{(1)}|^2 l_{ji}^* \Gamma_j\}}\right].$$

The repeated suffix j indicates summation over it from 1 to 5, is the complex conjugate of l_{j1} and $l_{11} = l_{11}^* = 1$.

$$\Gamma_i = \sum_{j=1}^{5} \frac{m_j + n_j P_i^* + \Theta_j Q_i^*}{\xi_1\alpha_j + \xi_1^*\alpha_i^*}. \tag{5.6}$$

$$\overline{\Gamma}_i = \sum_{j=1}^{5} \frac{\overline{m}_j + \overline{n}_j P_i^* + \overline{\Theta}_j Q_i^*}{\xi_i\alpha_j + \xi_i^*\alpha_i^*}. \tag{5.7}$$

where

$$\begin{aligned} m_j &= l_{j1}\{ic_{11} - c_{12}P_j\alpha_j - c_{14}Q_j\alpha_j + ie_{11}R_j - \frac{\lambda_{11}}{\xi_1}T_j\}, \\ n_j &= l_{j1}\{ic_{66}P_j - c_{66}\alpha_j + ic_{56}Q_j - e_{26}R_j\alpha_j\}, \\ \Theta_j &= l_{j1}\{ic_{56}P_j - c_{56}\alpha_j + ic_{55}Q_j - e_{25}R_j\alpha_j\}, \\ \overline{m}_j &= l_{j1}\{ic_{15} - c_{56}\alpha_j + ic_{56}P_j + ic_{55}Q_j + ie_{25}R_j\alpha_j\}, \\ \overline{n}_j &= l_{j1}\{ic_{14} - c_{24}P_j\alpha_j - c_{44}\alpha_j Q_j + ie_{14}R_j - \frac{\lambda_{23}}{\xi_1}T_j\}, \\ \overline{\Theta}_j &= l_{ji}\{ic_{13} - c_{34}Q_j\alpha_j + ic_{13}R_j - \frac{\lambda_{33}}{\xi_1}T_j\}. \end{aligned}$$

In the expressions for m_j, n_j, Θ_j, $\overline{m}_j$, $\overline{n}_j$, $\overline{\Theta}_j$ the repeated suffix does not indicate summation.

VI. DISCUSSION

To summarize the above analysis we recall that the characteristic determinantal equation (3.1) relates the decay constants $\alpha(=\frac{\xi_2}{\xi_1})$, the phase velocity $v_s(=\frac{\omega}{\xi_1})$ and the wave number ξ_1. These parameters also satisfy the other determinantal equation (4.10) obtained from the boundary conditions of the problem. In such wave propagation problems, numerical techniques are used to evaluate the values for the decay constants α phase velocity v_s and the wave number ξ_1; see, Nandy [14], White and Tseng [3] etc. These techniques consist in assigning values of the wave number ξ_1 and phase velocity v_s (or angular frequency ω) in the characteristic determinantal equation (3.1) and then solving for the decay constant. These decay constants together with the preassigned values for the wave numbers and the phase velocity are then substituted in the determinantal equation obtained from the boundary conditions to see whether the equation is satisfied. If not, new values for the wave number ξ_1 and phase velocity are chosen and by substituting then in the characteristic equation (3.1) new values for the decay constants α are derived. This process is continued until a set of values for the decay constants, phase velocity and wave number are obtained for which both determinantal equations are satisfied.

The values for the decay constants, phase velocity and wave number thus determined are substituted in the equation (4.11) to obtain the final expressions for the displacement components, piezoelectric potential and temperature decaying with depth into the medium. Putting these values in the equations (5.4), (5.5), (5.6) and (5.7) the power flow components and the angle of deviation can be determined.

Substituting h, the distance of the shorting plane above the free surface of the half space equal to zero and infinity in the determinantal equation (4.10) derived from the boundary conditions, proceeding in the usual manner, we get the corresponding results for the surface wave propagation when the free surface is covered with a thin conducting film or is electrically free and open to vacuum.

Further, substituting the prestress components σ_{ik} and the thermal relaxation parameter τ_0 equal to zero the results obtained in the present paper reduce to those already derived for such wave propagation in a monoclinic piezoelectric halfspace with classical thermal coupling by Pal [6].

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РАСПРОСТРАНЕНИЕ ПРОДОЛЬНОЙ ПОВЕРХНОСТНОЙ ВОЛНЫ В ОБОБЩЕННОМ ТЕРМО ПЬЕЗОЭЛЕКТРИЧЕСКОМ ПОЛУПРОСТРАНСТВЕ С МОНОКЛИННОЙ СИММЕТРИЕЙ, НАХОДЯЩИМСЯ ПОД ДАВЛЕНИЕМ

Распространение продольной поверхностной волны на поверхности пьезоэлектрического полупространства с моноклинной симметрией, которое находится под давлением, изучено с применением обобщеной тепловой связи в рамках модели Лорд–Шилмана. Изменения пьезоэлектрического потенциала, размещения частицы, температуры и т.д. с толщиной материала, пьезоэлектрического вектора Пойтинга относительно компонент потока мощности и угол между фазовой и групповой скоростьями определены при предположении существования закорочивающей плоскости с конечным расстоянием над сбододной поверхностью полупространства.