CONDENSATION OF YANG-MILLS FIELD AT HIGH TEMPERATURE IN THE PRESENCE OF FERMIONS

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The possible condensation of the time-component of Yang-Mills field at finite temperature is discussed in the presence of Dirac fermions. We show that the condensation forms regardless of the number of fundamental and adjoint fermion species coupled to the Yang-Mills field. The effect of finite density of fermions is also investigated and it is shown that the magnitude of the condensation is also independent of the densities.

I. INTRODUCTION

In the Euclidean formulation of quantum field theory at finite temperature, the imaginary time variable (τ) is compactified with a period $\beta = T^{-1}$, where T is the temperature of the system [1].

Then a classical background of gauge field $\langle A_0 \rangle \neq 0$ cannot be transformed into $A_0 = 0$ in general, because only gauge transformations which satisfy the periodic boundary condition in the Euclidean time direction are permitted [2].

It has been claimed that a condensation of field A_0 in non-Abelian theory arises at finite temperature [3,4]. Stimulated by this result, in the present paper we examine the condensation of the gauge field at finite temperature in the presence of fermions. If the phenomenon of gauge field condensation has some relevance to the physics of quark-gluon plasma phase [5], the presence of fermions as "quarks" must be crucial for thorough analysis.

As the simplest case, we carry out in this paper the calculations of the effective potential of finite temperature SU(2) gauge theory up to two-loop order with fundamental and adjoint fermions. The generalization to the SU(3) case will be reported in the future.

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II. EFFECTIVE POTENTIAL

We show the effective potential, or, free energy in background space-time $R^d \times S^1$, where S^1 stands for the compact time direction. By global gauge rotation, we can choose a classical background gauge field in a matrix form as

$$A_0 = \frac{T\Phi}{2g}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1}$$

where g is the Yang-Mills coupling constant. The value of A_0 condensation is described by the expectation value of Φ.

The free energy up to two-loop in the pure SU(2) Yang-Mills theory in four dimensions has been obtained by many authors, for example in ref. [4,6]. In general dimensions we find that free energy up to two-loop level is expressed as

$$F_g = -(D-2)\frac{\Gamma(D/2)}{\pi^{D/2}}T^D\sum_{k=1}^{\infty}\frac{2\cos k\Phi + 1}{k^D} +$$
$$+\frac{g^2}{2}\left(\frac{\Gamma(D/2)}{\pi^{D/2}}\right)^2 T^{2D-4}\left(\sum_{k=1}^{\infty}\frac{\cos k\Phi}{k^{D-2}}\right)\left(\sum_{k=1}^{\infty}\frac{\cos k\Phi + 2}{k^{D-2}}\right), \tag{2}$$

where $D = d + 1$ is the dimension of the space-time.

The summations of the series in this expression can be reduced to polynomials provided that D is an even number. Those are known as Bernoulli polynomials $B_s(x)$ [7] and we can write them as

$$\sum_{k=1}^{\infty}\frac{\cos k\Phi}{k^{2n}} = \frac{(-1)^{n-1}}{2(2n)!}(2\pi)^{2n}B_{2n}(\Phi/2\pi), \tag{3}$$

where $0 \leq x \leq 2\pi$ and n is an integer. The Bernoulli polynomials are, for example,

$$B_2(x) = x^2 - x + 1/6,$$
$$B_4(x) = x^4 - 2x^3 + x^2 - 1/30, \tag{4}$$

etc.

By using these polynomials, we can simply expand the effective potential F_g for Φ around $\Phi = 0$. (Note that at one-loop level, or equivalently when g is set to zero, global minimum is at $\Phi = 0$.)

Here we set $D = 4$, our dimensions. We get an expansion

$$F_g = -\left(\frac{\pi^2}{15} + \frac{g^2}{96}\right)T^4 - \frac{g^2}{6\pi}|\Phi|T^4 + \left(\frac{1}{3} + \frac{5g^2}{24\pi^2}\right)\Phi^2 T^4 + O(\Phi^3) \tag{6}$$

From this, one can find that the minimum of the free energy is located at

$$\Phi = \frac{g^2}{4\pi}, \tag{7}$$

in the leading order in g. Then a condensate forms and the value is $\langle A_0 \rangle =$
$$= \frac{gT}{8\pi} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We thus conclude that A_0 condensation is formed at finite temperature in the pure Yang-Mills system with small g. This value of condensation is consistent with $g \ll 1$ and the perturbation is valid.

It is notable that this "phenomenon" is peculiar for $D = 4$. For $s \geq 4$, it is known that $B_s(x)$ does not have the linear term in x. Therefore if gauge field condensation occurs in $D > 4$, the effect of higher-order terms of Φ in the potential must be essential. One can easily find, however, that it is impossible to yield the condensation by perturbative way in the case with $D > 4$.

An approach to non-perturbative effects on Yang-Mills condensation has been studied by the present authors in ref. [8]. We cannot touch on this immense subject in the present paper, and here we concentrate on the perturbative argument.
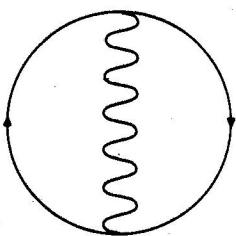

Fig. 1. A two-loop Feynman diagram for a fermion contribution.

III. FERMION DIAGRAM

Next, we compute the fermion diagram. The one-loop result is obtained by the standard technique and is widely known. See for example ref. [9]. The two loop diagram in Fig. 1 contributes to the free energy. We consider Dirac fermions in fundamental and adjoint representations of SU(2). Calculations are straight-forward tasks and after some rearrangements the results of two-loop contributions to free energy of the fundamental and adjoint fermions in general dimensions are represented as follows:

$$F_f(\text{two}-\text{loop}) = -\frac{g^2}{2} 2^{[D/2]} \frac{D-2}{2} T^{2D-4} \times$$
$$\times \left[\frac{1}{2} \left\{ I\left(\pi + \frac{\Phi}{2}\right) + I\left(\pi - \frac{\Phi}{2}\right) \right\} \left\{ I(0) + 2I(\Phi) \right\} - \right.$$

$$\left. -I\left(\pi + \frac{\Phi}{2}\right) I\left(\pi - \frac{\Phi}{2}\right) - \frac{1}{4} \left\{ \left(I\left(\pi + \frac{\Phi}{2}\right) \right)^2 + I\left(\pi - \frac{\Phi}{2}\right)^2 \right\} \right] \quad (8)$$

$$F_a(\text{two}-\text{loop}) = -\frac{g^2}{2} 2^{[D/2]} \frac{D-2}{2} T^{2D-4} \times$$
$$\times \left[2\{ I(\pi + \Phi) + I(\pi - \Phi) \} \{ I(0) + I(\Phi) \} - 4I(\Phi)I(\Phi) - \{ I(\pi + \Phi)^2 + I(\pi - \Phi)^2 \} \right], \quad (9)$$

where [] in the first line of each equation is Gauss' symbol and

$$I(x) = \frac{1}{2\pi^{D/2}} \Gamma\left(\frac{D-2}{2} \right) \sum_{k=1}^{\infty} \frac{\cos kx}{k^{D-2}}. \quad (10)$$

Each I in eqs. (8) and (9) comes from the momentum integration, which corresponds to a line in the diagram (Fig. 1). There are two integrations in each two-loop diagram, because of the momentum conservation. In order to obtain the result of (8) and (9), one needs to sum up the contributions of the diagrams in which SU(2) suffixes are assigned to each line. At the moment, one must notice that each propagator contains the coupling to the background gauge field, i.e. Φ, and it depends on the SU(2) suffix.

Again we can investigate the location of the minimum by expansion with respect to small Φ. For $D = 4$, we get the expansion of the free energy of fermions at one- and two-loop level for each species:

$$F_f \approx -\left(\frac{7\pi^2}{90} + \frac{5g^2}{192} \right) T^4 - \frac{g^2}{24\pi} |\Phi| T^4 + \frac{1}{12} \Phi^2 T^4 + \cdots, \quad (11)$$

$$F_a \approx -\left(\frac{7\pi^2}{60} + \frac{5g^2}{48} \right) T^4 - \frac{g^2}{6\pi} |\Phi| T^4 + \frac{1}{3} \Phi^2 T^4 + \cdots, \quad (12)$$

where we discard the $O(g^2\Phi^2)$ term, which is irrelevant for our purpose. Thus in each case free energy has perturbative minimum; the minimum of F_f and F_a are both located at $\Phi = g^2/4\pi$ when $g \ll 1$. This is a marked result. Total free energy is given by $F_g + N_f F_f + N_a F_a$, where N_f and N_a are the number of fermion species which belong to the fundamental and adjoint representations of SU(2), respectively. Since the total free energy is given as a linear combination of F's, the magnitude of the condensate turns out to be independent of the number of the fermion species.

IV. FINITE DENSITY EFFECT

Finally, we examine finite density effect of fermions. Finite density effects of "quarks" may have much importance in the physics of a possible formation of quark-gluon plasma at hadronic collision and very early universe [5].

We introduce chemical potentials for fermion numbers. This can be incorporated in the calculation of diagrams by considering the general phase of the fermionic field ψ which appears in the boundary condition:

$$\psi(\tau + \beta) = -e^{i\delta}\psi(\tau), \quad (13)$$

and setting δ to an imaginary value $-i\mu$, where μ is identified to the chemical potential.

For SU(2) fundamental and adjoint Dirac fermions the contributions up to two-loop to the thermodynamic potential (Ω) including respective chemical potentials as the potential of Φ behave near $\Phi \approx 0$ as

$$\Omega_f \approx -\left(\frac{7\pi^2}{90}T^4 + \frac{\mu^2 T^2}{3} + \frac{\mu^4}{6\pi^2}\right) + g^2\left(\frac{5}{192}T^4 + \frac{3\mu^2 T^2}{32\pi^2} + \frac{3\mu^4}{64\pi^4}\right) - \frac{g^2}{24\pi^2}|\Phi|\left(T^4 + \frac{3\mu^2 T^2}{\pi^2}\right) + \frac{1}{12}\Phi^2\left(T^4 + \frac{3\mu^2 T^2}{\pi^2}\right) + \cdots, \qquad (14)$$

$$\Omega_a \approx -\left(\frac{7\pi^2}{60}T^4 + \frac{\mu^2 T^2}{3} + \frac{\mu^4}{4\pi^2}\right) + g^2\left(\frac{5}{48}T^4 + \frac{3\mu^2 T^2}{8\pi^2} + \frac{3\mu^4}{16\pi^4}\right) - \frac{g^2}{6\pi^2}|\Phi|\left(T^4 + \frac{3\mu^2 T^2}{\pi^2}\right) + \frac{1}{3}\Phi^2\left(T^4 + \frac{3\mu^2 T^2}{\pi^2}\right) + \cdots, \qquad (15)$$

Here we omitted the $O(g^2\Phi^2)$ term again. These expressions are given by the replacement $\delta \to -i\mu$ in analogous calculations to eqs. (8) and (9), and rewriting them using the Bernoulli polynomials. The result of calculations including the "twists" can be found in ref. [8] (note the difference in the definitions of δ). Judging from these, we find the location of the minimum of total thermodynamic potential is unchanged, $\Phi = \frac{g^2}{4\pi}$, in the leading order. It is revealed that the magnitude of the A_0 condensation is independent of the density of fermions in the fundamental and adjoint representations of SU(2) at least in the perturbative regime.

To summarize all the results, we declare that gauge field condensation seems to take place even if fermion fields are present. The finite density of fermions does not affect the magnitude of the condensate. To obtain the next-leading contribution in (Φ), i.e., $O(g^3)$ we have to calculate an infinite sum of diagrams [6]. We leave the higher-order calculation for a future subject.

In ref. [4a], Belyaev claimed that the contribution of ring diagrams of gluons cancels the linear term of Φ in the free energy. Nevertheless, it is yet unknown whether the fermion contribution we have calculated is canceled by higher-order contribution or not. So far, we do not know fermionic diagrams which cancel the linear term. We feel the importance of a further study of the fermionic matter field in the high-temperature YM theory.

The "realistic" case of SU(3) "quarks" will be studied by a similar method with straightforward but tedious calculations. The investigations of perturbative and non-perturbative approaches and analytical and non-analytical methods will progress side by side, and we hope to report the full non-Abelian effects and physical insight in separate publications.

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КОНДЕНСАЦИЯ ПОЛЯ ЯНГА-МИЛЬСА
ПРИ ВЫСОКОЙ ТЕМПЕРАТУРЕ В ПРИСУТСТВИИ ФЕРМИОНОВ

В работе обсуждается возможность конденсации временной компоненты поля Янга-Милльса при конечной температуре в присутствии фермионов Дирака. Показано, что конденсация независит от количества основных фермионов ни от количества окружающих фермионов связанных полем Янга-Милльса. Исследование эффекта плотности фермионов показали, что несуществует зависимость, величины конденсации от плотности.