acta phys. slov. 42 (1992), No. 5, 267–273

CLASSICAL MASS QUANTIZATION OF A BOUND CONFIGURATION OF ELECTROMAGNETIC DYONS AND BARYONIC DYONS

WOLF, C.[1]), North Adams

By calculating the exterior gravitational field of a configuration of particles with electromagnetic charge and baryonic charge and maintaining that the horizons coincide we arrive at a classical expression for the quantized mass of the system.

I. INTRODUCTION

The existence of magnetic charge was shown by Dirac to lead to a multitude of new phenomena including the existence of Dirac strings and a quantization condition for the electric charge of a particle when the electric charge moves in the field of a magnetic charge [1]. Building upon these ideas, Schwinger demonstrated that for two dyons with charges $e_1\ q_1$; $e_2\ q_2$, the charges obey the condition

$$\frac{e_1q_2 - e_2q_1}{\hbar C} = \frac{n}{2} \qquad (n = 1, 2, \ldots)$$

if the field angular momentum is to be quantized [2]. With regard to non-Abelian gauge theory Prasad and Sommerfield have pointed out that monopole or dyon solutions exist when a group G is broken to a subgroup $H(G \to H \times U(1))$ with a surviving $U(1)$ factor [3]. The quantization of magnetic charge in this case results from the consistency of the solution. Actually Vinciarelli has pointed out that the dual symmetry of Maxwell's equations and hence the symmetry between an electric and a magnetic charge is a direct manifestation of the four-dimensional character of the world [4]. Also, if CP violating terms are present in the lagrangian, Witten has shown that the electric charge degree of freedom need not be quantized for a dyon [5]. With regard to their influence on particle phenomena, Callan has demonstrated that monopoles will catalyze proton decay with strong interaction rates and thus lead to the catalysis of proton decay in the early universe [6]. In the following discussion we extend the idea of a dyon to the case when two Abelian groups are present, the physical motivation comes from the recent discoveries regarding a possible fifth force mediated by a slightly massive Abelian-gauge field coupled to a baryon number [7, 8]. We choose not to discuss the massive nature of the additional gauge field associated with the baryon number but include the

1) Department of Physics, North Adams State College, NORTH ADAMS, MA (01247), U.S.A.

fields mixing with electromagnetism in a representation where the electric-like and magnetic-like charges have known values. Gasperini et al. have pointed out that due to the existence of a fifth force coupled to the baryon number, n, $\overline{n}$ would have different masses in the earth's filed which would affect the n, $\overline{n}$ oscillation strength, also if electromagnetic gauge fields and baryonic gauge fields mix, the existence of a baryon current for a spherically symmetric rotating object would generate a measurable magnetic field even when the object does not possess electric charge [9]. Encouraged by the observational evidence regarding the fifth force as well as the possible implications that dyons would have on dyon fermion dynamics [10–14] and the origin of CP violation in the early universe we study the exterior gravitational field of a composite object composed of an electric and a magnetic charge as well as a baryon electric-like and a baryon magnetic-like charge. Recently Recami and Tonin-Zachin [15] have shown that for a Kerr-Newman de-Sitter solution in the case of strong gravity wherein the mass and the cosmological constant take on values commensurate with hadron dimensions, the coincidence of two of the three horizons leads to stable impermeable black holes with vanishing surface temperature. Inspired by this result we discuss the mass charge relation resulting from a requirement that the two horizons coincide for a Reissner Nordstróm solution carrying both electric-like charges and baryonic-like charges. By insisting that the individual constituents admit the Dirac quantization condition we arrive at a classical quantization rule for the mass of a Reissner Nordstróm black hole admitting both an electric-like and baryonic-like charge with mixing between the two gauge groups. Possible experimental signatures for such mass quantization of charged astrophysical objects would be the existence of highly monochromatic gamma ray bursts that result from the transition from one state to another of the gravitationally bound configuration of an electric-like and a baryonic-like charge.

II. THE EXTERIOR GRAVITATIONAL FIELD OF ASTROPHYSICAL OBJECT CARRYING ELECTROMAGNETIC DYON STRUCTURE AND BARYONIC DYON STRUCTURE

We begin by writing the lagrangian describing the interaction of gravity, electromagnetism and baryonic gauge field outside of the configuration of charges.

$$L = \frac{C^4}{16\pi G} R\sqrt{-g} + \left[-\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\pi} B_{\mu\nu}B^{\mu\nu} + \alpha F_{\mu\nu}B^{\mu\nu} \right] \sqrt{-g}. \quad (1)$$

Here

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu}; \quad B_{\mu\nu} = \frac{\partial B_\mu}{\partial x^\nu} - \frac{\partial B_\nu}{\partial x^\mu},$$

α = mixing parameter and A_μ represents the electromagnetic four potential and B_μ the baryonic four potential. For both A_μ and B_μ we have the anti-symmetric

condition when no magnetic-like charges are present

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0 \quad (2)$$

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}) = 0 \quad (3)$$

Varying Eq. (1) with respect to A_μ, B_μ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{1}{4\pi}\sqrt{-g}F^{\mu\nu}\right) - 2\alpha\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}B^{\mu\nu}\right) = 0 \quad (4)$$

$$\frac{\partial}{\partial x^\nu}\left(\frac{1}{4\pi}\sqrt{-g}B^{\mu\nu}\right) - 2\alpha\frac{\partial}{\partial x^\nu}\left(\sqrt{-g}F^{\mu\nu}\right) = 0 \quad (5)$$

For the electric and magnetic-like fields we have

$$F_{14} = E_1(r), \quad F_{23} = r^2 \sin\Theta B_1(r)$$

$$B_{14} = E_2(r), \quad F_{23} = r^2 \sin\Theta B_2(r). \quad (6)$$

Eq. (2) and Eq. (3) give

$$B_1 = \frac{q_1}{r^2} \quad (7)$$

$$B_2 = \frac{q_2}{r^2}. \quad (8)$$

Here q_1 and q_2 are the total magnetic-like charges for the electromagnetic field and the baryonic field. From Eq. (4) and Eq. (5) we have

$$\frac{1}{4\pi}\frac{\partial}{\partial r}\left(r^2 e^{-(\nu+\lambda)/2}E_1\right) - 2\alpha\frac{\partial}{\partial r}\left(r^2 e^{-(\nu+\lambda)/2}E_2\right) = 0 \quad (9)$$

$$\frac{1}{4\pi}\frac{\partial}{\partial r}\left(r^2 e^{-\nu+\lambda)/2}E_2\right) - 2\alpha\frac{\partial}{\partial r}\left(r^2 e^{-(\nu+\lambda)/2}E_1\right) = 0. \quad (10)$$

Here we have used the spherically symmetric metric

$$(\mathrm{d}s)^2 = e^\nu(\mathrm{d}x^4)^2 - e^\lambda(\mathrm{d}r)^2 - r^2(\mathrm{d}\Theta)^2 - r^2\sin^2\Theta(\mathrm{d}\phi)^2, \quad (11)$$

$$x^4 = ct, \quad x^1 = r, \quad x^2 = \Theta, \quad x^3 = \phi.$$

Also since the energy momentum tensor corresponding to equation (1) obeys $T^4_4 = T^1_1$, from the Einstein equations we will find $\nu + \lambda = 0$. Thus Eq. (9) and Eq. (10) give upon integration

$$\frac{1}{4\pi}r^2E_1 - 2\alpha r^2 E_2 = \frac{e_1}{4\pi} \quad (12)$$

$$\frac{1}{4\pi}r^2E_2 - 2\alpha r^2 E_1 = \frac{e_2}{4\pi}. \quad (13)$$

Here e_1, e_2 are electric-like charges for the electromagnetic and the baryonic gauge fields respectively. Solving Eq. (12) and Eq. (13) we have

$$E_1 = \frac{1}{r^2}\left(\frac{e_1 + 8\pi\alpha e_2}{1-64\pi^2\alpha^2}\right) = \frac{K_1}{r^2} \tag{14}$$

$$E_2 = \frac{1}{r^2}\left(\frac{e_2 + 8\pi\alpha e_1}{1-64\pi^2\alpha^2}\right) = \frac{K_2}{r^2} \tag{15}$$

Thus Eq. (7), Eq. (8), Eq. (14) and Eq. (15) represent the complete exterior solution for the electric-like and magnetic-like fields of a charged object with both electromagnetic charges and baryonic charges. We next calculate the energy momentum tensor from Eq. (1)

$$\begin{aligned} T_{\mu\nu} =& \frac{2}{\sqrt{-g}}\frac{\partial L}{\partial g^{\mu\nu}} = \frac{1}{16\pi}g_{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta}) + \frac{1}{16\pi}g_{\mu\nu}(B_{\alpha\beta}B^{\alpha\beta}) \\ &- \frac{1}{4\pi}F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4\pi}B_{\mu\alpha}B^{\alpha}_{\nu} - \alpha g_{\mu\nu}(F_{\alpha\beta}B^{\alpha\beta}) \\ &+ 2\alpha F_{\mu\alpha}B^{\alpha}_{\nu} + 2\alpha B_{\mu\alpha}F^{\alpha}_{\nu} \end{aligned} \tag{16}$$

$$T^1_1 = T^4_4 = \frac{1}{8\pi}[E_1^2 + E_2^2 + B_1^2 + B_2^2] - 2\alpha E_1E_2 - 2\alpha B_1B_2. \tag{17}$$

For the $\binom{4}{4}$ component of the Einstein equation we have

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4}r^2T^4_4. \tag{18}$$

Integrating Eq. (18) gives

$$\begin{aligned} e^{-\lambda} = e^{\nu} =& 1 - \frac{2GM}{rc^2} + \frac{GK_1^2}{r^2c^4} + \frac{GK_2^2}{r^2c^4} \\ &+ \frac{Gq_1^2}{r^2c^4} + \frac{Gq_2^2}{r^2c^4} - \frac{16\pi G\alpha K_1K_2}{c^4r^2} - \frac{16\pi G\alpha q_1q_2}{c^4r^2}. \end{aligned} \tag{19}$$

Here $-2GM/C^2$ is the constant of integration representing the total mass of the system of electromagnetic and baryonic charges. Eq. (19) can be written as

$$\mathrm{e}^{-\lambda} = \mathrm{e}^{\nu} = 1 - \frac{2GM}{rc^2} + \frac{\overline{K}}{r^2c^4}, \tag{20}$$

where

$$\overline{K} = GK_1^2 + GK_2^2 + Gq_1^2 + Gq_2^2 - 16\pi G\alpha K_1K_2 - 16\pi G\alpha q_1q_2.$$

The horizons of Eq. (20) are given by

$$\begin{aligned} 0 &= 1 - \frac{2GM}{rc^2} + \frac{\overline{K}}{r^2c^4}, \\ r_{\pm} &= \frac{GM}{c^2} \pm \sqrt{\left(\frac{GM}{c^2}\right)^2 - \frac{\overline{K}}{c^4}}. \end{aligned} \tag{21}$$

For the horizons of Eq. (21) to coalesce we have $r_+ = r_-$,

$$M = \frac{1}{G}\sqrt{\overline{K}}$$

or

$$\begin{aligned} M = \left(\frac{1}{G}\right)^{1/2}\Bigg[&\left(\frac{e_1 + 8\pi\alpha e_2}{1-64\pi^2\alpha^2}\right)^2 + \left(\frac{e_2 + 8\pi\alpha e_1}{1-64\pi^2\alpha^2}\right)^2 + q_1^2 + q_2^2 - \\ &- 16\pi\alpha q_1q_2 - \frac{16\pi\alpha(e_1 + 8\pi\alpha e_2)(e_2 + 8\pi\alpha e_1)}{(1-64\pi^2\alpha^2)^2}\Bigg]^{1/2}. \end{aligned} \tag{22}$$

We now assume that the configuration is composed of an ensemble of electromagnetic dyons and baryonic dyons, thus we have from the Dirac condition

$$\frac{\overline{e}_1\overline{q}_1}{\hbar c} = \frac{n_1}{2}$$

(where $\overline{e}_1$, $\overline{q}_1$ are the individual electric and magnetic charges for the electromagnetic charged particles), and

$$\frac{\overline{e}_2\overline{q}_2}{\hbar c} = \frac{n_2}{2}$$

(where $\overline{e}_2$, $\overline{q}_2$ are the individual baryonic (electric-like) and baryonic (magnetic-like) charges for the baryonic charged particles), these relations give

$$\overline{q}_1 = \frac{\overline{e}_1}{\alpha_1}\left(\frac{n_1}{2}\right), \quad \overline{q}_2 = \frac{\overline{e}_2}{\alpha_2}\left(\frac{n_2}{2}\right) \tag{23}$$

$$\text{where } \alpha_1 = \frac{\overline{e}_1^2}{\hbar c}, \quad \alpha_2 = \frac{\overline{e}_2^2}{\hbar c}, \quad (n_1, n_2 = 1, 2, 3 \ldots).$$

Also

$$\begin{aligned} q_1 &= N_1\overline{q}_1 = \frac{N_1\overline{e}_1}{2\alpha_1}(n_1) = \frac{e_1(n_1)}{2\alpha_1} \\ q_2 &= N_2\overline{q}_2 = \frac{N_2\overline{e}_2}{2\alpha_2}(n_2) = \frac{e_2n_2}{2\alpha_2} \end{aligned} \tag{24}$$

where e_1, e_2, q_1, q_2 are the total macroscopic charges of the configuration. Here N_1 particles carry electric-like charges and N_2 carry baryonic-like charges. From Eq. (22) and Eq. (24)

$$\begin{aligned} M = \left(\frac{1}{G}\right)^{1/2}\Bigg[&\left(\frac{e_1 + 8\pi\alpha e_2}{1-64\pi^2\alpha^2}\right)^2 + \left(\frac{e_2 + 8\pi\alpha e_1}{1-64\pi^2\alpha^2}\right)^2 + \frac{1}{4\alpha_1^2}e_1^2n_1^2 + \frac{1}{4\alpha_2^2}e_2^2n_2^2 \\ &- \frac{(16\pi\alpha)(e_1 + 8\pi\alpha e_2)(e_2 + 8\pi\alpha e_1)}{(1-64\pi^2\alpha^2)^2} - \frac{4\pi\alpha}{\alpha_1\alpha_2}(e_1e_2)(n_1n_2)\Bigg]^{1/2}. \end{aligned} \tag{25}$$

Eq. (25) represents a classically quantized mass formula for the black hole configuration that contains both electromagnetic-like charges and baryonic-like charges with mixing between the two gauge fields given by Eg. (1).

We have assumed that all the microscopic dyons making up the macroscopic dyon are in the same quantized state

$$\frac{\bar{e}_1\bar{q}_1}{\hbar c} = \frac{n_1}{2}; \quad \frac{\bar{e}_2\bar{q}_2}{\hbar c} = \frac{n_2}{2}.$$

Transitions between these states would generate gamma ray spectra with a specific frequency that could serve as a specific signature to identify such configurations in the cosmos. It is hoped that a survey of extra-galactic gamma ray spectrum [16] might identify individual anomalous monochromatic signals that would serve to identify such quantized astrophysical objects even if their structure were different from the macroscopic electromagnetic-baryonic dyon discussed above.

ACKNOWLEDGEMENT

I'd like to thank the Physics Departments at Williams College and Harvard University for the use of their facilities.

REFERENCES

[1] Dirac, P. A. M.: Proc. of Roy. Society *A 133* (1931), 60.
[2] Schwinger, J.: Phys. Rev. *144* (1966), 1087.
[3] Prasad, M. K., Sommerfield, C. M.: Phys. Rev. Lett. *35* (1975), 760.
[4] Vinciarelli, D.: *Proc. of the 1976 Int. School of Subnuclear Physics*, Erice. Plenum Press, N.Y. 1978.
[5] Witten, E.: Phys. Lett. *B 86* (1979), 287.
[6] Callan, C.G.,Jr.: Phys. Rev. *D26* (1982), 2058.
[7] Neufeld, D.A.: Phys. Rev. Lett. *56* (1986), 2344.
[8] Thodberg, H.: Phys. Rev. Lett. *56* (1986), 2423.
[9] Bottino, A., De Alfaro, V., Gasperini, M.: Phys. Lett. *B215* (1988), 411.
[10] Olsen, H. A., Ostand, P., Tai Tsum Wu.: Ark. Fys. Semin. Trondheim (Norway) *10* (1989), 197.
[11] Tolkachev, E. A., Tomil' Chik, L. M., Shnir, Ya. M.: Sov. J. Nucl. Phys. (U.S.A.) *50* (1989), 275.
[12] Galtsov, D. V., Ershov, A. A.: Mosc. Univ. Phys. Bull. (U.S.A.) *43* (1988), 40.
[13] Frampton, P. H., Zhang-Jian-Zu, Yong-chang, Qi.: Phys. Rev. *D40* (1989), 3533.
[14] Jian-zu-Zhang.: J. Phys. G. (Nucl. and Part. Phys.) *16* (1990), 195.
[15] Recami, E., Tonin-Zanchin, V.: Report INFN/AE-89/13 1st Naz. Fis. Nucl., Cantania, Italy (8 Nov. 1989).
[16] Ressell, M. T., Turner, M. S.: Fermilab Publ. 89/214A, Oct. (1989).

Received January 4th, 1991
Accepted for publication September 16th, 1991

КЛАССИЧЕСКАЯ КВАНТИЗАЦИЯ МАССЫ СВЯЗАННЫХ КОНФИГУРАЦИЙ ЭЛЕКТРОМАГНИТНЫХ И БАРИОННЫХ ДИОНОВ

При расчете внешних гравитационных полей совокупности частиц с электромагнитным и барионным зарядом при условии совпадения горизонтов найдено классическое выражение квантизации массы системы.