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THE EFFECT OF A STRONG EXTERNAL ELECTRIC FIELD ON THE PROPERTIES OF SAW[1])

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The dependence of the SAW velocity on the external electric field was computed with the aid of the generalized perturbation theory which includes all important material nonlinearities. The numerical calculations show that this effect is very small in comparison with the temperature effects. The maximum relative velocity change limited by the achievable electric field intensity was 6 ppm on temperature independent ST quartz, which corresponds to the relative change of resonator frequency of about 5 ppm.

I. INTRODUCTION

Due to the piezoelectric effect and the material nonlinearities the strong external electric filed changes both the dimensions of a SAW device and the parameters of a substrate on which a SAW propagates. The accurate computation of SAW device parameters requires to take into account both effects. The results of this computation should be useful, for example, in the design of SAW electric field sensors, for the estimation of the range for the fine tuning of SAW resonators or oscillators or for their frequency modulation.

We suppose, in agreement with experiments, that the amplitude of the strain generated by a SAW is small with respect to the strain created by an external field. The presence of a SAW can be considered therefore as a small perturbation of the state at which the external field acts. This means that the relatively simple perturbation theory [1] can be used instead of the general but complicated nonlinear theory [2]. The perturbation method has been extended and generalized in order to include the piezoelectric effect and other important nonlinearities, namely the electro-elastic effect and the electrostriction [3]. The resulting equations allow to consider any of the important external field.

In this paper we apply these equations to compute the influence of an external electric field on the SAW velocity. According to the perturbation theory the parameters of the substrate under the action of an electric field are computed and

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then used to modify the material parameters of the linear SAW equations. The SAW velocity in the presence of an external electric field is computed by the use of a standard method and the numerical results for important cuts of widely used substrates are given.

II. BASIC EQUATIONS

The use of the perturbation method consists of three steps:
(1) the solution of the nonlinear equations for the initial state,
(2) the modification material parameters,
(3) the solution of the linear SAW equations with modified coefficients.

In this chapter the equations for these steps are given.

The initial state in which only the external field acts is described by these nonlinear equations

$$S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i} + U_{k,i}U_{k,j}) \tag{1a}$$

$$E_i = -\Phi_{,i} \tag{1b}$$

$$T_{ij} = C_{ijkl}S_{kl} + \frac{1}{2}C_{ijklmn}S_{kl}S_{mn} - e_{kijmn}E_kS_{mn} - e_{kij}E_k - \frac{1}{2}H_{klij}E_kE_l + T_{ij}^{(o)} \tag{1c}$$

$$D_i = e_{ikl}S_{kl} + \frac{1}{2}e_{iklmn}S_{kl}S_{mn} + H_{ijkl}E_jS_{kl} + \varepsilon_{ij}E_j + \frac{1}{2}\varepsilon_{ijk}E_jE_k + D_i^{(o)} \tag{1d}$$

$$T_{ij} + T_{jk}U_{i,k} = \rho_o\ddot{U}_i \tag{1e}$$

$$D_{i,i} = 0 \tag{1f}$$

Equation (1a) and (1b) are definitions. The nonlinear medium is described by the constitutive equations (1c) and (1d). The nonlinear equation of motion (1e) and the static electric condition (1f) are needed. Basic medium quantities are the elastic displacement U_i and the electric potential ϕ. Other used quantities are the thermodynamic stress T_{ij}, the elastic strain S_{ij}, the electric field intensity E_i and the electric displacement D_i. $T_{ij}^{(o)}$ and $D_i^{(o)}$ are the stress and the electric displacement due to the external sources. The medium is described by the density ρ_o, the elastic moduli of the second and third rank c_{ijkl} and c_{ijklmn}, the linear and the quadratic piezoelectric stress-tensor components e_{ijk} and e_{iklmn}, the electrostriction H_{ijkl} and the linear and quadratic permittivities ε_{ij}, ε_{ijk}. The Einstein summation rule is used and the space derivatives are given by the indices followed by the comma. The point above the symbol is used for the time derivative.

In the vacuum above the medium only the Laplace equation for the electric potential is needed

$$\Phi_{,ii} = 0. \tag{2}$$

We suppose that on the surface of the medium no external surface stress is applied and no free electric charge is on it. The mechanical boundary conditions have therefore the form

$$n_j(T_{ij} + T_{jk}U_{i,k}) = 0, \tag{3a}$$

where n_i are the components of the unit vector normal to the surface. The electric boundary condition on the free surface requires the continuity of the potential and the normal component of the electric displacement

$$\Phi^+ = \Phi^-, \qquad n_iD_i^+ = n_iD_i^-, \tag{3b}$$

where the symbols + and − denote the vacuum and the material side of the surface. The electric boundary condition for the surface covered with a thin perfectly conducting film requires zero potential on it,

$$\Phi = 0. \tag{3c}$$

The elastic displacement and the electric potential must vanish in the infinity.

The modified linear material parameters $c_{ijkl}^{(m)}$, $e_{ikl}^{(m)}$ and $\varepsilon_{ij}^{(m)}$ are given by these equations

$$c_{ijkl}^{(m)} = c'''_{ijkl} + \delta_{li}T_{ij}^{(o)} \tag{4a}$$

$$c'''_{ijkl} = c''_{ijkl} + c''_{ijkn}U_{l,n}$$

$$c''_{ijkl} = c'_{ijkl} + c'_{jnkl}U_{i,n}$$

$$c'_{ijkl} = c_{ijkl} + c_{ijklmn}S_{mn} - e_{nijkl}E_n$$

$$e_{ikl}^{(m)} = e'_{ikl} + e'_{ikn}U_{k,n} \tag{4b}$$

$$e'_{ikl} = e_{ikl} + e_{iklmn}S_{mn} + H_{ijkl}E_j$$

$$\varepsilon_{ij}^{(m)} = \varepsilon_{ij} + \varepsilon_{ijk}E_k + H_{ijkl}S_{kl}, \tag{4c}$$

where ε_o is the permittivity of the free space and δ_{ij} is the Kronecker symbol. The quantities U_l and S_{kl} must be obtained by the solution of the nonlinear equations (1a) through (1f).

The modified material parameters are substituted into well-know linear SAW equations

$$\rho_o \ddot{u}_i = c^{(m)}_{ijkl} u_{l,kj} + e_{kij}\Phi_{,kj} \quad (5a)$$

$$e^{(m)}_{ikl} u_{l,ki} - \varepsilon^{(m)}_{ij}\Phi_{,ij} = 0 \quad (5b)$$

$$\Phi_{,ii} = 0 \quad (5c)$$

$$n_j(c^{(m)}_{ijkl} u_{l,k} + e^{(m)}_{kij}\Phi_{,j}) = 0 \quad (5d)$$

$$\Phi^{+} = \Phi^{-}, n_j \varepsilon_o \Phi^{+}_{,j} = n_j(e^{(m)}_{ikl} u^{-}_{l,k} - \varepsilon^{(m)}_{ij}\Phi^{-}_{,j}) \quad (5e)$$

$$\Phi = 0. \quad (5f)$$

The first and the second equations are the equation of motion and the electric condition for the medium, the equation (5c) is the Laplace equation for vacuum and the remaining equations are the boundary conditions, mechanical (5d) and electrical for free surface (5e) and short-circuited surface (5f). The SAW elastic displacement and potencial are given by the symbols u_i and Φ, respectively.

III. EFFECT OF EXTERNAL ELECTRIC FIELD

From the three steps of solution of this problem outlined at the beginning of the previous chapter the most important and complicated step in the computation of the effect of the external electric field is the solution of nonlinear equations (1a) through (1f) for the initial state. We suppose that only the external electric field acts and there are no free charges on the surface of the medium so that the absolute terms in the constitutive equations (1c) and (1d) are zero. The solution gets considerably simpler if we make two approximations:

(1) the electric field and the mechanical displacement and the strain generated by this field are all homogeneous and static,

(2) the terms with products of strain or of displacement gradients $U_{i,j}$ are negligible.

The first approximation is satisfied satisfactorily because of very small penetration depth and very high frequency of a SAW in SAW devices. The computations show that the second approximation is valid for all the allowable electric field intensities. Even when this approximation cannot be used, the exact solution of nonlinear equations (1) can be performed by an iteration technique.

Due to the small value of the product of potential gradients this basic equation of linear elasticity follows from the equation (1a)

$$S_{ij} = S_{ji} = U_{i,j} = U_{j,i}. \quad (6a)$$

In the equation (1c) the term with a product of strain is not considered. As the electric field is static, the right-hand side of the equation (1e) is zero. After the

substitution from equations (6a) and the reduced equation (1c) into this equation and after neglecting all terms with the product of strain the equation of motion (1e) is reduced to

$$T_{ij} = e_{njk} E_n S_{ik} + \frac{1}{2} H_{nljk} E_n E_l S_{ik}. \quad (6b)$$

Due to the homogeneity of the strain and the electric field the electric condition (1f) is valid identically. After the substitution from equations (6b) into the reduced equation (1c) we obtain a set of linear equations for the strain, which can be solved by standard methods. From the identity (6a) we know the displacement gradients and from equations (1d) we compute the electric displacement.

Since the electric field is given, all quantities in equations (4a) through (4c) are known. The procedure connected with the second step of the previous chapter is very simple and the solution of the third step uses a standard technique.

IV. NUMERICAL RESULTS

The substrates of quartz and lithium niobate were investigated, because for these two materials the full set of nonlinear coefficients is known. In the evaluation of results some strict limitation should be taken into account. The maximum electric field intensity is limited by a vacuum, air or a material electric breakdown. The changes due to the electric field should be compared with the changes produced by other second-order effects, namely temperature effects.

In the case of quartz we did not include the electrostriction because there is a great discrepancy in the published values [4]. We have investigated the well-know cuts of quartz somewhat in detail. On all the investigated cuts the maximum SAW relative velocity change was less than 10 ppm, which corresponds with one exception to the temperature change less than $0.5K$ so that only the temperature independent ST cut quartz should be considered. On this cut the SAW velocity is affected only by the electric field parallel to the SAW beam axis and the sensitivity to the electric field intensity is $8.10^{-12} m/V$. This value corresponds to the value for volume waves [5]. The maximum relative velocity change is therefore about 6 ppm. On rotated ST quartz substrates with respect to thickness the normal electric field acts too and the relative change of SAW velocity by this field for a rotation with an angle of 46 degrees is $5.10^{-12} m/V$. In this case a stronger electric field can be used, unfortunately the phase and group velocity are not collinear.

On cuts of lithium niobate the sensitivity of SAW velocity to the applied external electric field is greater than for quartz, but there are no cuts with a zero temperature coefficient of SAW velocity. For example, on the YZ cut the temperature changes of SAW velocity exceed strongly the field changes.

In SAW resonators and other devices the relative change of frequency is given by both the relative velocity change and the relative dimension change (strain) in the direction of a cavity length. For ST cut quartz the maximum relative frequency change is about 5 ppm.

V. CONCLUSIONS

The generalized perturbation method [3] has been applied to the problem of SAW propagation in the presence of a strong external electric field. It is found that the equations of the initial state, in which the electric field acts only, can be linearized so that their solution can be greatly simplified. The numerical computations provided by a standard technique show that the effect for a practically usable electric field is much less than the effect due to the temperature change. Only the subtrates with a zero temperature coefficient of SAW velocity can be used in applications. For the ST cut of quartz the relative change of SAW velocity is of the same order as for volume waves. The maximal relative change of SAW velocity is limited by the electric breakdown. The maximum velocity change on ST quartz is about 6 ppm.

In SAW resonators and other devices the relative change of substrate dimension must be considered too. For resonators on ST quartz the relative change of resonant frequency due to the maximum available electric field intensity is about 5 ppm. This means that electric field sensors on this cut should have small sensitivity, only a very fine tuning of SAW oscillators can be achieved and frequency modulation is practically impossible. On YZ lithium niobate the sensitivity to the external electric field is greater but the temperature dependence seriously complicates the practical use of this effect in the SAW devices mentioned above.

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ВЛИЯНИЕ СИЛЬНОГО ВНЕШНЕГО ЭЛЕКТРИЧЕСКОГО ПОЛЯ НА СВОЙСТВА ПАВ

С применением общей теории возмущений и учетом нелинейностей материала вычислена зависимость скорости ПАВ на внешнем электрическом поле. Как показали расчеты, в сравнении с температурными эффектами, иследуемый эффект оказывается очень малым. Максимальное изучение относительной скорости ограничено допустимой интенсивностью электрического поля и составляет 6.10^{-6} при независимом от температуры СТ кварце. Это соотвецвует относительному изменению частоты резонатора в 5.10^{-6}.