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OBLIQUE INCIDENCE SPECULAR REFLECTION OF UNPOLARIZED AND PARTIALLY POLARIZED IR RADIATION BY UNIAXIAL SEMICONDUCTORS AT PLASMA RESONANCE

FOLTIN, O.[1], Bratislava

The oblique incidence specular reflectivity spectrum of the basal reflection of unpolarized or partially polarized IR radiation is proposed for obtaining the plasma resonance frequency of a uniaxial semiconductor, $\omega_{p\parallel}$, which yields the axial component of the susceptibility effective mass, $m_{\chi\parallel}$.

I. INTRODUCTION

The susceptibility effective mass of carriers, m_χ, can be estimated, within the scope of the Drude—Zener theory (see e.g. [1—6], by means of the measurement of spectral dependence of the normal incidence specular reflectivity at plasma resonance. In the case of optically uniaxial semiconductors separate measurements (using infrared radiation polarized with the electric vector ***E*** parallel and perpendicular, respectively, to the *c*-axis of the crystal) must be performed in order to obtain two components of an anisotropic effective mass. When dealing with the naturally layered crystals, the measurements with radiation polarized parallel to the *c*-axis (yielding the axial component of the susceptibility effective mass, $m_{\chi\parallel}$) are difficult to be performed as there are problems with the production of a reflectance face that is parallel to the *c*-axis (i.e. perpendicular to the easy-cleavage planes). To avoid those problems the method which uses oblique incidence specular reflectivity measurements at the basal (easy-cleavage) plane reflection of *p*-polarized (***E*** parallel to the plane of incidence) radiation was proposed [7]. For such measurements a polarized and a large angle of incidence specular reflectivity unit are required as accessories to the IR spectrophotometer.

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[1] Katedra fyziky, Elektrotechnická fakulta STU, Ilkovičova 3, CS-812 19 BRATISLAVA, CSFR

II. OBLIQUE INCIDENCE REFLECTIVITY

To avoid the use of a polarizer the measurement of oblique incidence specular reflectivity (at basal plane reflection) with unpolarized (natural) or partially polarized IR radiation is proposed. In the case of an unpolarized incident radiation the reflectivity R_u will be

$$R_u = (R_p + R_s)/2, \tag{1}$$

where R_p and R_s are the reflectivities for *p*-polarized and *s*-polarized (***E*** perpendicular to the plane of incidence), respectively, incident radiation. According to [8] the following formulae hold:

$$R_p = \frac{(a_\parallel - \mathrm{c}\cos\varphi)^2 + (b_\parallel - \mathrm{d}\cos\varphi)^2}{(a_\parallel + \mathrm{c}\cos\varphi)^2 + (b_\parallel + \mathrm{d}\cos\varphi)^2}, \tag{2}$$

where

$$2\left\{\begin{matrix} a_\parallel^2 \\ b_\parallel^2 \end{matrix}\right\} = [(n_\parallel^2 - k_\parallel^2 - \sin^2\varphi)^2 + 4n_\parallel^2 k_\parallel^2]^{1/2} \pm (n_\parallel^2 - k_\parallel^2 - \sin^2\varphi), \tag{3}$$

$$c = n_\parallel n_\perp - k_\parallel k_\perp, \tag{4}$$

$$d = n_\perp k_\parallel + n_\parallel k_\perp, \tag{5}$$

and

$$R_s = \frac{(a_\perp - \cos\varphi)^2 + b_\perp^2}{(a_\perp + \cos\varphi)^2 + b_\perp^2}, \tag{6}$$

where

$$2\left\{\begin{matrix} a_\perp^2 \\ b_\perp^2 \end{matrix}\right\} = [(n_\perp^2 - k_\perp^2 - \sin^2\varphi)^2 + 4n_\perp^2 k_\perp^2]^{1/2} \pm (n_\perp^2 - k_\perp^2 - \sin^2\varphi), \tag{7}$$

$n_\parallel(n_\perp)$ and $k_\parallel(k_\perp)$ are the respective real and imaginary components of the complex index of refraction for electromagnetic waves polarized so that the ***E*** vector vibrates parallel (perpendicular) to the *c*-axis. φ is the angle of incidence.

According to the Drude—Zener theory the real, $\varepsilon_{1\parallel}$ and the imaginary, $\varepsilon_{2\parallel}$, components of the complex dielectric function, $\hat{\varepsilon}_\parallel$, fulfil the relations

$$\varepsilon_{1\parallel} = n_\parallel^2 - k_\parallel^2 = \varepsilon_{\infty\parallel}\left[1 - \frac{1}{(\omega/\omega_{p\parallel})^2 + (1/\omega_{p\parallel}\tau_\parallel)^2}\right] \tag{8}$$

and

$$\varepsilon_{2\parallel} = 2n_{\parallel}k_{\parallel} = \frac{\varepsilon_{\infty\parallel}}{\omega\tau_{\parallel}}\frac{1}{\left(\omega/\omega_{p\parallel}\right)^2 + \left(1/\omega_{p\parallel}\tau_{\parallel}\right)^2} \tag{9}$$

The complex dielectric function $\hat{\varepsilon}_{\perp}$ is given by relations analogous to (8) and (9) with the index $\perp$ instead of the index $\parallel$. τ is the optical relaxation time, ε_{∞} the high-frequency dielectric constant, ω the angular frequency and ω_p the angular frequency of plasma vibrations. There holds

$$\omega_{p\parallel} = \left[\frac{Ne^2}{\varepsilon_0\varepsilon_{\infty\parallel}m_{\chi\parallel}}\right]^{1/2} \tag{10}$$

where N is the concentration of free carriers, ε_0 the permitivity of the vacuum, e the electron charge and $m_{\chi\parallel}$ the axial component of the susceptibility effective mass. A formula analogous to (10) is valid for the radial component of the susceptibility effective mass, $m_{\chi\perp}$, when indices $\parallel$ are replaced by indices $\perp$.

The formula (10), relating the susceptibility effective mass to the plasma frequency, was derived on the assumption that

$$\omega^2\tau^2 \gg 1. \tag{11}$$

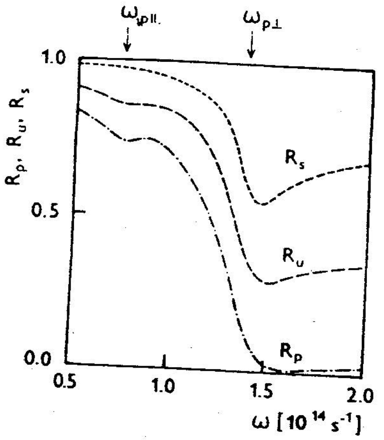

Fig. 1. Frequency dependence of the specular reflectivities R_s, R_u and R_p computed for the basal plane reflection of a uniaxial semiconductor for the angle of incidence $\varphi = 70°$, where $\omega_{p\parallel}$ and $\omega_{p\perp}$ are the angular frequencies of plasma vibrations.

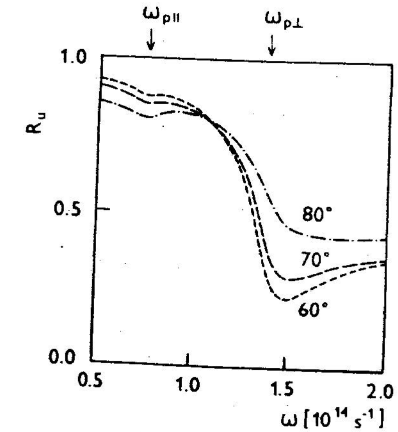

Fig. 2. Frequency dependence of the specular reflectivity R_u computed for an unpolarized radiation incident on the basal plane at the angle equal to 60°, 70° and 80°.

III. RESULTS

Fig. 1 shows the calculated spectral dependence of R_u, R_s and R_p for the angle of incidence $\varphi = 70°$. Fig. 2 shows the spectral dependence of R_u for the angles of incidence φ equal to 60°, 70° and 80°. The formulae (1) to (9) were used for the calculations of the reflectivities. The values of physical parameters have been chosen so as to resemble the n-type Bi_2Se_3 single crystal with the concentration of free carries of about 3×10^{19} cm^{-3} [9], namely $\omega_{p\parallel} = 0.74 \times 10^{14}s^{-1}$, $\omega_{p\perp} = 1.38 \times 10^{14}s^{-1}$, $\varepsilon_{\infty\parallel} = \varepsilon_{\infty\perp} = 29$ (see [9, 10]). The isotropic [6, 9] optical relaxation time was considered to be $\tau = 5 \times 10^{-14}$s. A singularity of reflectivity R_u (as well as of R_p) can be seen near $\omega_{p\parallel}$. The slight shift of the singularity from the considered value of $\omega_{p\parallel}$ represents only a few per cent of the value of $\omega_{p\parallel}$. The larger the angle of incidence, the more pronounced is the singularity. There may exist combinations of values of physical parameters which will not show a singularity of reflectivity.

When using an arbitrarily polarized incident radiation with the polarization factor Q defined by

$$Q = (I_p - I_s)/(I_p + I_s) \tag{12}$$

where I_p and I_s are the relevant irradiances, then the reflectivity R will be [8]

$$R = [(1 + Q)R_p + (1 - Q)R_s]/2. \tag{13}$$

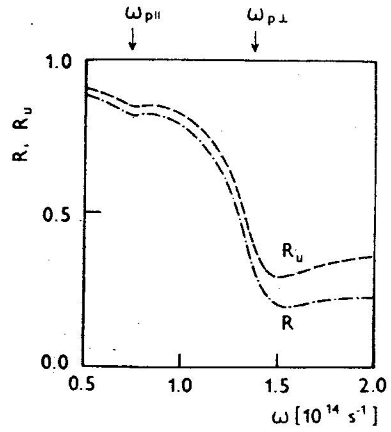

Fig. 3. Frequency dependence of the specular reflectivity computed for an unpolarized (R_u) and for a partially p-polarized (R) radiation, incident on the basal plane at the angle $\varphi = 70°$.

Further computations have shown, if there is a singularity of R_p at $\omega_{p\parallel}$, so there is a singularity of R at $\omega_{p\parallel}$, provided neither the s-polarization prevails in the incident radiation, nor is there a sudden change of the polarization factor in the considered spectral range ($\omega_{p\parallel}$ is assumed to differ substantially from $\omega_{p\perp}$). This is demonstrated in Fig. 3 which shows the spectral dependence of reflectivity for an unpolarized or a partially polarized IR radiation. The curve for a partially polarized incident radiation corresponds to the case when the polarization factor grows linearly with the angular frequency, namely $Q = 0.2 + 0.1 \times 10^{-4}\omega$.

It results that the spectral measurements of a specular reflectivity at oblique incidence of an unpolarized or a partially p-polarized IR radiation on layer-type uniaxial semiconductors (at basal plane reflection) can be used for obtaining the value of $\omega_{p\parallel}$, which yields the axial component of the susceptibility effective mass, $m_{\chi\parallel}$.

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ЗЕРКАЛЬНОЕ ОТРАЖЕНИЕ НЕПОЛЯРИЗОВАННОГО И ЧАСТИЧНО ПОЛЯРИЗОВАННОГО ИНФРАКРАСНОГО ИЗЛУЧЕНИЯ ПРИ НАКЛОННОМ ПАДЕНИИ НА ОДНООСНЫЕ ПОЛУПРОВОДНИКИ ПРИ ПЛАЗМЕННОМ РЕЗОНАНСЕ

Предложено использовать частотную зависимость зеркального отражения неполяризованного и частично поляризованного инфракрасного излучения при его наклонном падении на плоскость скола одноосных полупроводников для определения плазменной частоты, $\omega_{p\parallel}$, дающей аксиальную компоненту эффективной массы для восприимчивости, $m_{\chi\parallel}$.