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Letter to the Editor

THE NEW SIMILARITY LAWS AND THEIR APPLICATION TO PLASMA RESEARCH[1)]

НОВЫЕ ЗАКОНЫ ПОДОБИА И ИХ ПРИМЕНЕНИЕ В ИССЛЕДОВАНИИ ПЛАЗМЫ

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The similarity laws for the plasma characteristics of low-pressure metal vapour (Hg, Cd, Zn)+ RARE gas discharge are derived in this paper. According to new similarity laws the number of independent external parameters which are necessary to describe internal plasma characteristics is less than the known laws give. The application of the similarity laws to the plasma investigation is proposed and illustrated by several examples.

It is known [1-3] that in a wide range of discharge conditions there exist the so-called similarity laws (similarity rules). The gist of these laws is that the description of discharge parameters does not need all of the external parameters, but the less number of their combinations. For example, in the d.c. discharge through a pure gas three external parameters i (the discharge current), R (the radius of the tube) and p (the gas pressure) are converted into two combinations pR and i/R,. This affirmation is often formulated as follows: if the combinations pR and i/R are the same, the internal plasma characteristics like the electron average energy $\bar{\epsilon}$, the degree of ionization n_e/p, the electron velocity distribution function $f(\mathrm{v})/p$, the intensity of the electric field $\mathbf{E}R$ and some others, are also the same or, in other words, ivariant. These discharges are called similar.

The similarity laws are valid in the discharges through the mixture of several gases as well. In this case the similar invariant parameters i/R and p_kR, where p_k are the pressures of gases in the mixture, arise.

There is another very interesting case of the discharge in a gas mixture. It appeares that the roles of the gases in this discharge are often different: one gas (or gases) only

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defines transfer processes of particles and the rest is responsible for the ionization processes and the energy losses of electrons in the plasma. In this work it will be shown that the above-mentioned specific roles of gases in the mixture result in the origin of the new similarity laws. The consideration will be carried out on the Hg+rare gas discharge because of its widest scientific and practical applications. However, the results obtained can evidently be applied to any discharge plasmas where the main assumptions are valid.

The main assumptions are as follows:

- atoms of the noble gas define transfer processes of particles in the plasma and do not take part in the energy losses of electrons;
- mercury atoms are ionized and excited only;
- the electron velocity distribution function is close to the spherically symetrical $f_0(v)$[4];
- the frequency ν with which the discharge conditions are changed is less than ν_a, the frequency of elastic electron collisions with rare gas atoms.

These assummptions appeared to be good enough in a wide range of discharge conditions [5].

In order to derive the similarity laws one has to subject the equations which decribe plasma characteristics [4-7] to the similarity transformation:

$$r \to \Lambda r,\ t \to \Lambda^z t,\ v \to \Lambda^{\alpha_v} v,\ f_0 \to \Lambda^{\alpha_f} f_0,\ n_e \to \Lambda^{\alpha_n} n_e,\ \bar{\epsilon} \to \Lambda^{\alpha_e}\bar{\epsilon},$$
$$\mathbf{E} \to \Lambda^{\alpha_E}\mathbf{E},\ N_m \to \Lambda^{\alpha_N} N_m,\ p \to \Lambda^{\beta_p} p,\ R \to \Lambda R,\ N_0 \to \Lambda^{\beta_N} N_0,\ i \to \Lambda^{\beta_i} i \qquad (1)$$

where Λ is any positive number, z, α_k, β_k are similarity indices. The plasma equations must be invariant under the transformation (1). This requirement defines the similarity indices and therefore invariant combinations of the internal and external plasma characteristics:

$$f_0(v) = N_0 f_0'(v, z_1, z_2, \rho, \tau) \qquad \mathbf{E} = R^{-1}\mathbf{E}'(z_1, z_2, \rho, \tau)$$
$$n_e = N_0 n_e'(z_1, z_2, \rho, \tau) \qquad N_m = N_0 N_m'(z_1, z_2, \rho, \tau); \qquad (2)$$
$$\bar{\epsilon} = \bar{\epsilon}(z_1, z_2, \rho, \tau)$$

$$z_1 = N_0 p R^2,\ z_2 = iRp^2,\ \rho = r/R,\ \tau = N_0 t. \qquad (3)$$

Here N_0 is the mercury atom concentration, p the noble gas pressure, r the running value of a radius, t time. It is seen that the plasma characteristics depend on four combinations of the external parameters z_1, z_2, ρ, τ and this number is less than the known laws [1-3] give.

The experimental verification of the new similarity laws is carried out with the help of the time similarity of discharges. If the stationary gas discharge are similar ($z_1 = z_1'$, $z_2 = z_2'$), the nonstationary discharges will be similar in "time"$\tau = N_0 t$ as well. Fig. 1 presents the coincidence of some plasma characteristics in time τ, real time scales differing twice.

The existence of the similarity laws means that we *apriori* have some information about the plasma. This can obviously be used for plasma research.

Let as assume that we know the electron average energy $\bar{\epsilon}$ measured on the axis of the Hg + rare gas d.c. discharge as a function of N_0 and i:

$$\bar{\epsilon} = \bar{\epsilon}(N_0, i). \qquad (4)$$

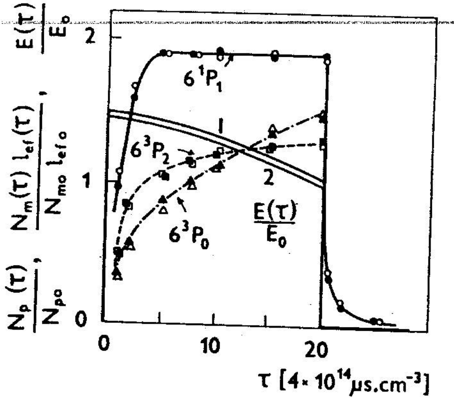

Fig. 1. The intensity of the electric field E and the concentrations of 6^1P_1 and $6^3P_{0,2}$ mercury atoms in the positive column of two similar pulse-periodic dicharges. (1), •, ■, ▲: $N_0 = 4 \times 10^{14} cm^{-3}$, $p = 2$ Torr, $i = 0.6$ A, $T_p = 20\ \mu s$, $T_\alpha = 40\ \mu s$; (2),o, □, △: $N_0 = 2 \times 10^{14} cm^{-3}$, $p = 4$ Torr, $i = 0.15$ A, $T_p = 40\ \mu s$, $T_\alpha = 80\ \mu s$.

In accordance with the similarity laws (2), (3) the electron average energy $\bar{\epsilon}$ must be a function of z_1 and z_2, therefore the parameters N_0 and i can appear only as a part of z_1 and z_2. Because the relations of N_0 and i to z_1 and z_2, respectively are single-valued, one has an opportunity to make a substitution $N_0 \rightarrow z_1$ and $i \rightarrow z_2$:

$$\bar{\epsilon}(N_0, i) = \bar{\epsilon}(z_1, z_2) = \bar{\epsilon}(N_0 p R^2, iRp^2). \qquad (5)$$

Thus the electron everage energy is known as a function of all the external parameters N_0, p, R and i. If the average energy $\bar{\epsilon}$ is measured as a function of other discharge parameters, one has to combine the invariant parameters z_1 and z_2 so that the relations of the new invariant combinations, obtained from z_1 and z_2, to the parameters in question are to be single-valued.

A similar procedure can be applied to any invariant plasma characteristics. In the case of the Hg+noble gas discharge these invariant charactestics are $f(v)/N_0$; n_e/N_0, $\bar{\epsilon}$, RE, N_m/N_0, $I_k/N_0^2R^2$. Here I_k is the intensity of mercury spectral lines emitted by the unit of plasma volume. The efficacy η of the mercury resonance radiation is also invariant in case of a negligible quenching of the resonance levels of mercury atoms.

Fig. 2 presents the data of the experiment (the dots) and the calculations (the broken lines) of n_e, $kT_e = 2/3\bar{\epsilon}$, η in dependence upon the noble gas pressure [8]. The same figure also presents the data of our calculations (the firm lines) made with the help of the similarity laws. The good accordance between our calculations and the experimental data [8] allows us to speak about the validity of the procedure proposed. We have also determined the dependences of the plasma characteristics in question upon the tube radius R which are not presented in [8].

The procedure proposed can apply both our similarity laws and the known laws [1-3], the known laws making it possible to obtain only one additional dependence in comparison with two dependences which are given by our similarity laws. The similarity laws (2),

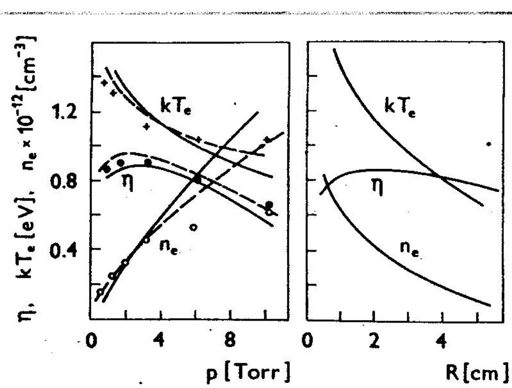

Fig. 2. The electron temperature kT_e, concentration n_e and the luminous efficacy η of the Hg-Ar discharge. Dots - experimental data, broken lines - calculations [8]. Firm lines - results of calculations made with the help of the similarity laws.

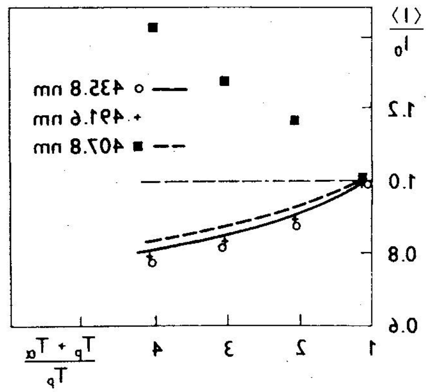

Fig. 3. The time average intensities $< I >$ of some mercury spectral lines as functions of $(T_\alpha + T_p)/T_p$ in pulse-periodic Hg-Ar discharge plasma. The index "0" marks the intensities in the d.c. discharge plasma provided that the electric energy consumed in both regimes (d.c. and pulse-periodic) is the same. Dots - experiment, lines - calculations with the help of the similarity laws.

(3) are especially useful in the case of the discharges in the complex mixtures of gases, e.g. mercury vapour + a few noble gases. The plasma characteristics of these discharges can be derived from the plasma characteristic of the discharges in binary mixtures (mercury + one noble gas). The method described can be applied to the investigation of the non-stationary plasme, giving time-dependent plasma characteristics and their time average values. Moreover, the similarity laws can be useful in studying plasma processes as well. This is illustrated by fig. 3 which presents the time average intensity $< I >$ of some mercury spectral lines as a function of $(T_\alpha + T_p)/T_p$ in pulse-periodic Hg-Ar discharge plasma

(T_p is the duration of the pulses, T_α the time interval between pulses). The behaviour of the time average intensities with $\lambda = 435.8$ nm (7^3S_1) and $\lambda = 491.6$ nm (8^1S_0) agrees with our calculations made with the help of the similarity laws. Note that the intensities of most of the mercury spectral lines behave equally. The behaviour of $< I > /I_0$ with $\lambda = 407.8$ nm (7^1S_0) does not accord with our similar calculation. The analysis of the possible reasons of this disturbance shows that the only cause is the stepwise excitation from the resonance 6^3P_1 state. Similar conclusions were deduced for the 6^3D states as well.

With the help of the data of our measurements we estimated the cross-sections of the stepwise excitation from the resonance 6^3P_1 state in comparison with that of the metastable 6^3P_2 state. For the 7^1S_0 level we get ${}^Q6^3P_1/{}^Q6^3P_2 = 7 \mp 3$; for the 6^3D levels $-{}^Q6^3P_1/{}^Q6^3P_2 = 2.5 \mp 0.8$. It is seen that the cross-section in question are considerably larger than that of the 6^2P_2 state.

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