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COMPUTER MODEL OF THE NON-STATIONARY ELENBAAS-HELLER EQUATION[1)]

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A method for the numerical solution is presented of the non-stationary Elenbaas-Heller equation in the asymptotic region of an arc gas heater with axial gas supply and stabilized by a cold wall. The diagrams of stable stationary solutions are calculated, the arc discharge being regarded as a non-linear element of the electrical circuit with a source of electromotive force and resistance. The transient phenomenon of the formation of the stationary state during the spark ignition phase has also been studied.

The starting equations for the description of an arc plasma is the set of the MHD equations which include the continuity equation, the equation of motion, the energy equation, the Maxwell equations, Ohm's law and the state equation of a perfect gas. These equations are applied in the boundary layer approximation to the asymptotic region of the channel of an arc gas heater (the so-called fully-developed arc, when the physical quantities are the functions only of the radial coordinate r) for such gas flows, when the Mach number $M < 0.3$. Under these conditions the set of MHD equations is reduced to the non-stationary Elenbaas-Heller equation, which can be written in the cylindrical coordinate system (in the non-dimensional variables, see [5]) as follows :

$$\frac{\partial T}{\partial t} = \frac{1}{\rho(T)c_p(T)}\left[\sigma(T)E_z^2 + \frac{1}{Pr}\frac{1}{Re}\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(T)\frac{\partial T}{\partial r}\right) - \psi(T)\right] \qquad (1)$$

with the boundary conditions

$$T(r=1,t) = T_1 \quad \text{(fixed wall temperature)} \qquad (2)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{(the condition of axial symmetry)} \qquad (3)$$

and the initial condition (the initial temperature profile)

$$T(r,t=0) = T_0(r). \qquad (4)$$

Here $Pr = c_{po}\eta_0/\lambda_0$ is the Prandtl number, $Re = \varrho_0 u_0 L_0/\eta_0$ is the Reynolds number, T - temperature, E_z - the electric field strength in the axis direction of the discharge channel z, $\varrho(T)$ - the mass density, $c_p(T)$ - the specific heat under the constant pressure, $\sigma(T)$ - the electric conductivity, $\lambda(T)$ - the thermal conductivity, $\psi(T)$ - the radiative energy emitted per unit volume and unit time.

The equation (1) has been solved numerically by the method of lines. The space discretization has been performed by the method of finite differences with the Lagrange polynomial $L_2(x)$ on both equidistant and non-equidistant mesh of N points over the discharge channel radius. The resulting set of N stiff first-order differential equations has been solved by the Gear-Hindmarsh algorithm [1]. The subprogram GEARB [2] has been used. Numerical computations have been carried out for nitrogen, argon and hydrogen under the atmospheric pressure $p \doteq 10^5$Pa.

Physical results which have been obtained are interesting mainly from the point of view of the discharge stability. Radial stationary temperature profiles, enthalpies, current density profiles and current-voltage characteristics have been calculated for the gases mentioned above and for various discharge channel radii R_0.

Some results of the numerical solution of the equation (1) under the conditions (2) - (4) are shown in Fig. 1 and Fig. 2. These represent the stationary solutions of the equation (1) for argon with discharge channel radius $R_0 = 0.5$ cm. The values of necessary transport and thermodynamic functions σ, λ, ϱ, c_p, ψ were taken from [3]. The arc discharge in the asymptotic region of an arc gas heater was studied as a non-linear element of an electrical circuit with the source of the electromotive force U_N connected in series with the ohmic resistance R, the value of which was chosen $R = 20\ \Omega$. E_z was computed from the equation

$$E_z = \frac{U_N}{L\left(1 + \frac{2\pi R}{L}\int_0^1 \sigma r dr\right)}, \qquad (5)$$

where L is the distance between anode and cathode.

Fig. 1 illustrates the dependence of the electric field strength E_z on the temperature T_{axis} at the axis of the channel in the stationary state. The curve represents the set of all stable stationary solutions of the system (1) - (5). The radial stationary temperature profiles do not exhibit the narrow hot temperature core in the

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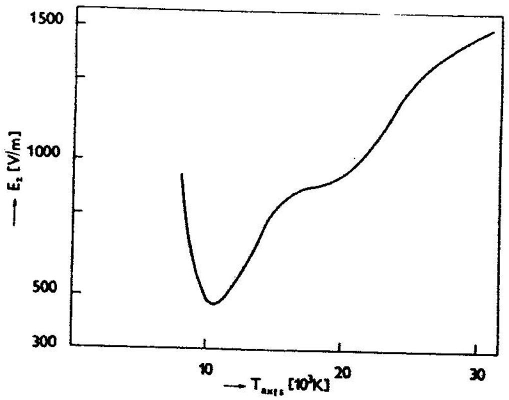

Fig. 1. E_z vs axis temperature T_{axis} in the stationary state. Argon; the radius of the discharge tube $R_0 = 0.5$ cm; the external resistance value $R = 20\ \Omega$.

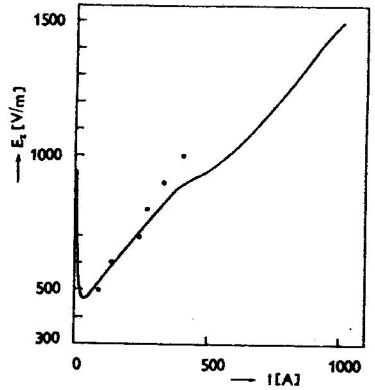

Fig. 2. Current-E_z characteristic. Argon; $R_0 = 0.5$ cm; $R = 20\ \Omega$. The points represent the experimental values published in [6].

axial region which is in a qualitative agreement with the numerical solutions of other authors (see, e.g. [4]).

In Fig. 2 the dependence E_z vs the total current I is computed (static current-electric field strength characteristic). For the stable stationary states which are presented, the known Kaufman criterion of stability $R + R_{pl} > 0$ is valid, where R_{pl} is the differential resistance of plasma. The current-E_z characteristic does not contain any current jumps in contrast to the same characteristic of nitrogen [5]. The points mark the experimentally measured values published in [6].

The time development of the stationary temperature state from the initial narrow conductive channel in the axis region (spark ignition model) has also been studied. Numerical calculations show that a critical temperature exists for a given initial value of E_z (computed from (5)) and a given initial temperature profile of a narrow conductive channel from which the stationary state can develop.

Fig. 3 shows the formation of the temperature profile for nitrogen plasma in time for the channel radius $R_0 = 1$ cm. The transport and thermodynamic functions for nitrogen were obtained from [7] (under the atmospheric pressure). The initial condition of temperature was chosen in the form of a Gaussian profile

$$T(r, t = 0) = (T_{axis} - T_1)\exp\left(1 - \frac{\epsilon^2}{\epsilon^2 - r^2}\right) + T_1, \qquad (6)$$

with the initial axis temperature $T_{axis} = 8500$ K and the temperature of the cold wall $T_1 = 500$ K. The initial temperature profile width was $\epsilon = 0.12$ cm. During

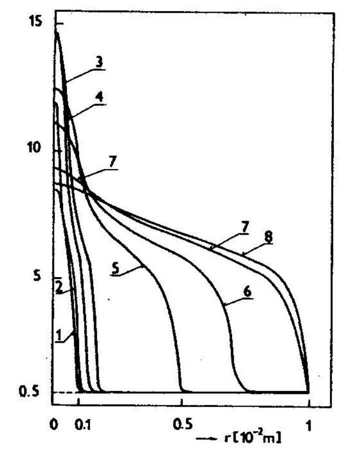

Fig. 3. Time formation of the stationary temperature state from a narrow conductive channel. Nitrogen, electromotive force $U_N = 1000$ V; external resistance value $R = 20\Omega$. Curve 1 - initial condition ($t = 0$ s); 2 - $t = 0.01$ ms; 3 - $t = 0.05$ ms; 4 - $t = 0.1$ ms; 5 - $t = 1$ ms; 6 - $t = 3$ ms; 7 - $t = 6$ ms; 8 - $t = 30$ ms.

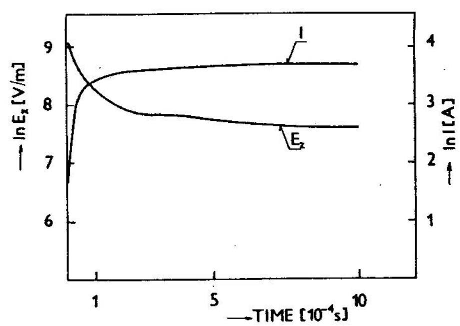

Fig. 4. Time development of E_z and I during the formation of the stationary state. Nitrogen.

the first 0.1 ms the temperature in the axis region highly increases to the value of 14 700 K. The Joule heat produced in this region is transported towards the cold wall and passed over to the neighbouring gas. The temperature in the region near the wall increases during several ms and the stationary temperature profile is established in about 30 ms.

The time development of E_z and I is apparent in Fig. 4. An abrupt change of both these quantities takes place during the initial 0.1 ms of the formation of a hot conductive axial region. The change of E_z and I is later significantly lower and develops approximately exponentially.

CONCLUSIONS

We hope that we have demostrated the usefulness of the non-stationary model for the description of the interaction of an arc discharge with the flowing gas in the discharge channel of an arc gas heater. We have proved the realizability of the non-stationary model, the applicability of the Gear-Hindmarsh algorithm for the time integration of the resulting set of differential equations and the sufficient accuracy of the generation of the material and thermodynamic functions by means of simple cubic spline interpolation.

It was shown that stable stationary states for a given working gas can efficiently be computed with the assistance of the model of the arc discharge considered as a non-linear element of the electric circuit with electromotive force and resistance. Within this framework the transient phenomena occurring during the formation of the stationary state (the ignition process) can also be studied.

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КОМПЮТЕРНАЯ МОДЕЛЬ НЕСТАЦИОНАРНОГО УРАВНЕНИЯ ЭЛЕНБААС-ХЕЛЛЕРА

Разработан метод нумерического решения нестационарного уравнения Эленбаас-Хеллера для асимптотической области нагрева дуги газа с его аксиальной подачей и стабилизацией холходной стеной. Вычислены диаграммы устойчивых стационарных выражений. Дуговой разряд представлен нелинейным электрическим элементом с источником электродвижущей силы и сопротивлением. Также изучается переходное явление возникновения стационарного состояния в процессе зажигания искры.