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A CONTRIBUTION TO THE ABR THEORY OF ION COLLECTION BY THE SPHERICAL PROBE[1)]

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An attempt to solve Poisson's equation as part of ABR theory is carried out. In the original paper of Allen, Boyd and Reynolds there is no mention of the solution method. From results of the present paper it is evident that Poisson's equation was in the original paper linearized. Furthermore Poisson's equation was probably solved by the method of two successive integrations, i.e. by a method which is unstable. In the present paper integration was carried out by the integral identities method and without linearizing assumptions. We obtained results, which from the physical point of view are new, as pointed out in the present paper.

I. INTRODUCTION

The ABR theory [1] deals with ion collection by the spherical probe. The ıean free path of the ions is assumed to be large with respect to the probe radius $\lambda_i \gg a$) and the electron temperature is assumed to be much higher than the n temperature ($T_e \gg T_i$). These assumptions are in a low pressure discharge $< 10^3$ Pa) well satisfied.

The ABR theory starts from previous theories, namely that of Bohm, Burop, Massey [2] and Langmuir and Mott-Smith [3].

The ABR theory is based on the fact the probe is so negative that only few ectrons reach the probe surface. Further, the Maxwellian distribution of electron elocities is assumed. Under this condition we can take the following formula for ıe electron density n_e in the neighbourhood of the probe

$$n_e = n_{e0} \exp(-eV/kT_e), \qquad (1)$$

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where n_{e0} is the electron density far from the probe, e is the electron charge, V is the potential and k is Boltzmann's constant. For the ion collection the theory makes the following assumption. If the energy of the ion is large with respect to the initial value of the ion energy, then the ion velocity is $(-2eV/m_i)^{1/2}$ and it has a radial component only. The ion density n_i in the probe neighbourhood is then given as

$$n_i = I_i(e(-2eV/kT_e)^{1/2})^{-1}\frac{1}{r^2}, \tag{2}$$

where I_i is the ion current. The equation (2) holds for such values of r for wich the potential V satisfies the inequality $|V| \gg V_0$, where V_0 is the potential expressing the initial ion energy eV_0.

Under these conditions we can write Poisson's equation for the distribution of the potential in the probe neighbourhood

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}V}{\mathrm{d}r}\right) = -\frac{1}{\epsilon_0}(n_i - n_e)e. \tag{3}$$

The theory further assumes that the ions are charged with one elementary charge only. Introducing the dimensionless potential as

$$\eta = eV/kT_e \tag{4}$$

and the dimensionless distance

$$\xi = r/\lambda, \tag{5}$$

where

$$\lambda = \sqrt{\frac{\epsilon_0 kT_e}{n_{e0}e^2}} \tag{6}$$

is Debye's length, we can with respect to the equations (1), (2) rewrite the equation (3) in the form

$$-\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\eta}{\mathrm{d}\xi}\right) = \frac{I_i}{I_\lambda}\frac{1}{\eta^{1/2}} - \xi^2\exp(-\eta), \tag{7}$$

where

$$I_\lambda = (kT_e)^{3/2}\frac{\epsilon_0}{e}\left(\frac{2}{m_i}\right)^{1/2}. \tag{8}$$

The equation (7) is a nonlinear differential equation of the second order.

II. SOLUTION OF THE EQUATION FOR THE POTENTIAL

Now we are able to solve the equation (7). However, simultaneously difficulties arise. First, it is evident that the problem is a boundary problem. So we must set two boundary conditions. The natural choice of the boundary conditions is

$$\xi = \xi_p \quad \eta = \eta_p, \quad \xi \to \infty \quad \eta = 0, \tag{9}$$

i.e. we put the first boundary condition on the probe surface and the second in infinity. In this manner Laframboise, for exemple, chooses boundary conditions [4]. However, in the ABR theory no comments with respect to boundary conditions are present. Though from consequences it is evident that the first boundary condition is put at ξ=0, not at ξ=ξ_p. This is unnatural. But without such a choice of the first boundary condition the voltampere characteristics of the current cannot be obtained.

Secondly by the authors perform the derivative on the left-hand side of the equation (7) so that this equation is transformed into

$$\eta^{1/2}\xi^2\left(\exp(-\eta) + 2(\frac{\mathrm{d}^2\eta}{\mathrm{d}\xi^2} + \frac{2}{\xi}\frac{\mathrm{d}\eta}{\mathrm{d}\xi})\right) = \frac{I_i}{I_\lambda}. \tag{10}$$

And this equation is then solved numerically. But this manner introduces an instability to the solution due to the presence of the first derivative, which must be approximated by a central difference. For this reason it is better to leave the left-hand side of the equation (7) in the original form and to use an adequate integration method. Such an integration method is, for example, the integral identities method.

Thirdly the greatest problem is in the nonlinearity of the equation. The authors did not comment upon the method used to solve the equation. But all circumstances indicate that the right-hand side was linearized.

Then the first boundary condition was put at zero; the value of the potential was chosen very large, the $\eta(0)$ value cannot be less than 100, otherwise it would be imposible to obtain such values of the potential as discussed in paper [1]). The second boundary condition η=0 was put at a suitable distance from the probe (so that the location of the boundary condition is without effect on the solution).

Thus we tried to repeat this scheme in the first step. Our boundary conditions were chosen as

$$\xi = 0 \quad \eta = 1000, \quad \xi \to \infty \quad \eta = 0. \tag{11}$$

The equation (7) with a linearized right-hand side was integrated by the integral identities method. The results giving a potential distribution are given in Fig. 1.

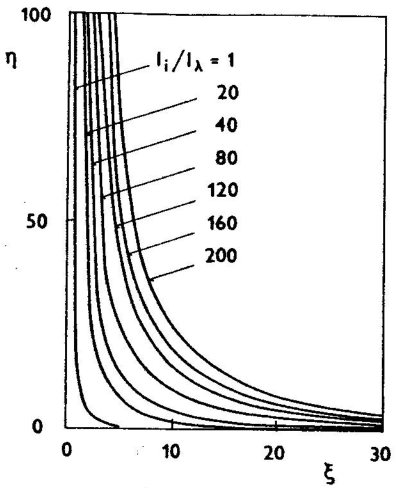

Fig. 1 Distribution of the potential. Linearized version.

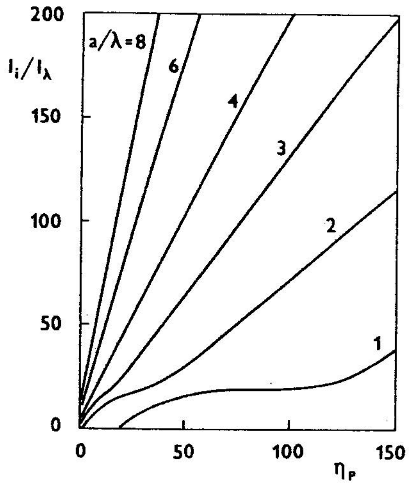

Fig. 2 Voltampere characteristics of the probe. Linearized version.

The evaluation was carried out for the following values of I_i/I_λ: 1, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200. For brevity we present the cases I_i/I_λ: 1, 20, 40, 80, 120, 160, 200 only. If we compare Fig. 1 and Fig. 4 in paper [1], we can state that our results are most probably (the authors of the original paper gave no numerical values) identical. After that we found the voltampere characteristics of the ion current for the following values of a/λ: 1, 2, 4, 6, 8. Our results are presented in Fig. 2. Comparing Fig. 2 and Fig. 5 from paper [1] we can state a practical identity. But, moreover, we state that the form of the characteristics with $a/\lambda < 4$ is different than the form of the remaining characteristics. The characteristics with $a/\lambda < 4$ are not purely concave; at the beginning they are convex. The concavity of the remaining characteristics is identical with the paper [1], but the characteristics are almost linear and have no tendency to saturation.

In the second step we left the right-hand side of the equation (7) in nonlinear form. Integration was carried out by the integral identities method, nonlinearity was iterated by Newton's method. As an initial solution, the solution of the equation

$$-\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\eta}{\mathrm{d}\xi}\right) = 0 \qquad (12)$$

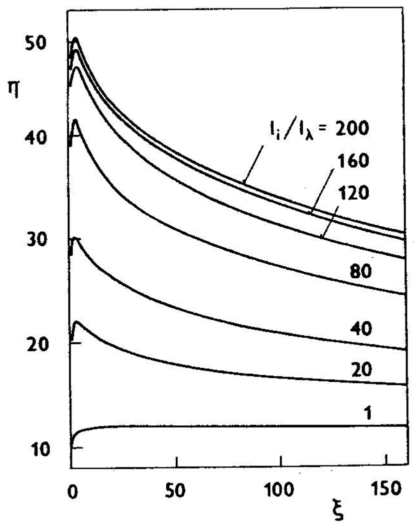

Fig. 3 Distribution of the potential. Nonlinear version, equidistant method.

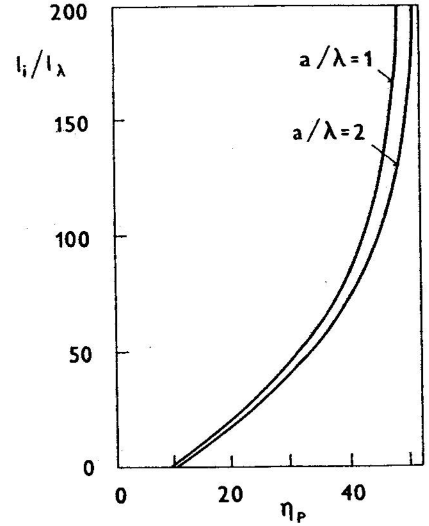

Fig. 4 Typical shape of the voltampere characteristic of the probe. Nonlinear version, equidistant method.

was chosen. Boundary conditions were in the form of (11). The results of the evaluation are strikingly different from the linearized version. The solution for the potential is presented in Fig. 3. The evaluation was carried out for the following values of I_i/I_λ: 1, 5, 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200. Here are presented the results for I_i/I_λ: 1, 20, 40, 80, 100, 120, 140, 160, 180, 200. In the solution the influence of the nonlinearity exp($-\eta$) is evident by a drastic decrease of the potential in the nearest neighbourhood of the probe. The problem is rather complicated and our integration method in the equidistant version is not able to carry out the evaluation of the potential with the accuracy, we should wish. We were limited by the stack memory capacity of the used computer. The nonequidistatnt method up to date has no satisfactory results. It seems, however, that the drop due to the exponential term is at the beginning very steep. Then it is balanced in some region by the second term with a square root. This effect causes a transient increase of the potential, which increases with increasing ratio I_i/I_λ. After that the potential falls to zero. A further surprising effect is that the potential has a greater range than it has in the linearized case. Integration in the nonlinear version cannot be stopped until η=1500.

The accuracy of integration depends on the value of the integration step. The experience with integration so far has been such that the integration step would be very small in the neighbourhood of zero. At greater distances we could choose a disproportionally larger step. Using this process we could decrease the number of integration step and it seems that the accuracy of the solution would be better. It also seems that the number of integration steps is not satisfactory with respect to the influence of the zero point choice on the solution. But with respect to the limit discussed above we cannot increase the number of the integration steps when using the equidistant method. By the reaction of the form of the solution to the stopping point choice and on the value of the integration step, we can explain the form of the voltampere characteristics of the ion current presented in Fig. 4. These characteristic are convex. So it seems to be necessary to increase the accuracy of the solution of the equation (7).

III. CONCLUSION

The results of Allen, Boyd and Reynolds are obtained most probably by the fact of linearization of the right-hand side of the equation (7). So it seems to be necessary to solve the original equation, i.e. the equation without linearization of the right-hand side. This was the attempt of the present paper. The most important results are as follows.

1) The range of the probe potential is much greater than in the original ABR theory. So the plasma will be more affected by the probe presence.

2) The potential is not a purely monotonous (i.e. decreasing) function of the distance. Both nonlinearities are at the beginning contradictory. This fact causes a transient increase of the potential.

3) The values of the potential in the nearest neighbourhood of the probe are problematic and in the present state of investigation cannot be evaluated.

For all these reasons we shall further insist on nonequidistant integration methods which could be capable to evaluate the potential more accurately than the equidistant ones. Nevertheless the most important objection against the ABR theory, i.e. the unnatural choice of the boundary conditions, remains.

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К ТЕОРИИ АБР СБОРА ИОНОВ СФЕРИЧЕСКИМ ЭЛЕКТРОДОМ

В работе приведено решение уравнения Поассона в рамках теории Аллена-Бойда-Рейнолдса, которое в их оригинальной работе не показано. Из результатов предложенной работы следует что уравнение Поассона в оригинальной работе линеаризировано и более вероятно решено с применением метода постепенного интегрирования, что представляет не стабилный метод. В приведенной работе интегрирование проводилось методом интегральных тождеств, который не нуждается в линеаризации. В работе показаны новые физические результаты.