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A SEARCH OF A MECHANISM RESPONSIBLE FOR BREMSSTRAHLUNG ENHANCEMENT IN HADRONIC REACTIONS. III. LOW MASS DILEPION PRODUCTION

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Results obtained in our preceding papers for the bremsstrahlung of real photons are generalized to virtual photons experimentally accessible as low mass dileptons. The mass spectrum $d\sigma/dM$ of bremsstrahlung dileptons is similar in shape to but smaller than experimental data obtained by AFS collaboration at the CERN ISR and in other low mass dilepton studies. It is pointed out that data on the k_T — dependence of dilepton production at a fixed dilepton mass could disentangle the question of the origin of low mass dilepton production.

I. INTRODUCTION

Anomalous production of very soft photons has been observed in K^+p interactions at 70 GeV/c by Chliapnikov et al. [1] and in pBe and pAl collisions at 450 GeV/c by the Helios collaboration [2]. An earlier study by Goshaw et al. [3] has shown no anomalous enhancement — a very soft photon yield has been accounted for by the bremsstrahlung off final and initial state hadrons.

Some possibilities to explain the anomalous enhancement [1, 2] have been discussed in the preceding papers of this series [4, 5] (referred to as I and II). Neither of the possibilities analysed gave a satisfactory explanation of the data [1, 2] and the mechanism responsible for very soft photon enhancement remains unknown.

Low mass dilepton production in hadronic collisions has been studied first by the Chicago — Princenton group [6] and subsequently by other collaborations [7]. Most recent data on low mass e^+e^- production in pp collisions at $\sqrt{s}$ = 63 GeV has been obtained by the AFS collaboration [8] working at the CERN ISR.

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Productions of real and virtual photons, materialized as low mass dileptons, are well known to be closely related [9—12]. It is then natural to ask whether the mechanism responsible for a very soft photon production might be also responsible for the low mass dilepton production. A positive answer to this question would impose severe constraints on models proposed for the low mass dilepton production. In fact both the quark gluon plasma model [13] and the soft annihilation model [14] would meet serious difficulties when trying to explain very soft photon enhancement.

The purpose of the present paper is to study the relationship between mechanism responsible for a real very soft photon and low mass dilepton production. The data on single leptons will be analyzed in part IV of this series.

We shall start in Sect. II. with extrapolating basic formulae used in I and II for real photon bremsstrahlung to the case of low mass dilepton production. Technical details are presented in Appendices A and B.

The simplest approximation describing the bremsstrahlung off final and initial state hadrons consists [4, 11] in dividing the total contribution into two parts. The former is a coherent radiation of forward and backward going hadrons and the latter is an incoherent sum of bremsstrahlung emitted by hadrons with roughly the same rapidity as that of the photon. The same decomposition of the amplitude for the dilepton (virtual photon) bremsstrahlung is discussed in Sect. III.

In Sect. III. we also compare results calculated in this way with the data on dilepton mass distribution $d\sigma/dM$. Comments and conclusions are presented in Sect. IV.

II. BREMSSTRAHLUNG OF LEPTON PAIRS IN THE SCATTERING OF A CHARGED PARTICLE

As shown in the Appendix A a generalization of the bremsstrahlung of real photons for a charged particle scattering (on a neutral particle or on a potential) to the case of bremsstrahlung of e^+e^- pairs is given by the expression

$$\frac{d\sigma^{ee}}{dM} = d\sigma^{(0)} 4\pi\alpha(-C_\mu C^\mu) F(M^2) \frac{d^3k}{(2\pi)^3 2k_0}. \tag{1}$$

Here M is the dilepton mass, k the dilepton four-momentum,

$$C_\mu = \frac{p_1'^\mu}{p_1'.k} - \frac{p_1^\mu}{p_1.k}, \tag{2}$$

p_1 and p_1' are four momenta of the charged particle before and after the scattering, and

$$F(M^2) = \frac{2\alpha}{3\pi}\,\frac{M(M^2+2m^2)}{M^4}\sqrt{1-\frac{4m^2}{M^2}}\;\frac{2\alpha}{3\pi}\,\frac{1}{M}, \tag{3}$$

where m is the lepton mass. On the basis of the gauge invariance $C^\mu k_\mu = 0$ we have $C^0 k^0 = \boldsymbol{C}.\boldsymbol{k}$, which implies

$$-C^\mu C_\mu = |\boldsymbol{C}\times\boldsymbol{n}|^2 + \frac{M^2}{k_0^2}|\boldsymbol{C}.\boldsymbol{n}|^2. \tag{4}$$

The first term on the r.h.s. corresponds to the transverse and the second to the longitudinal photons.

The final formula for the dilepton bremsstrahlung

$$\frac{d\sigma^{ee}}{dM} = d\sigma^{(0)}\,\frac{\alpha}{4\pi^2}\,AF(M^2)\,\frac{d^3k}{\omega} \tag{5a}$$

with

$$A = \frac{1}{\omega^2}\left|\frac{\boldsymbol{v}'\times\boldsymbol{n}}{1-\boldsymbol{v}'.\boldsymbol{V}} - \frac{\boldsymbol{v}\times\boldsymbol{n}}{1-\boldsymbol{v}.\boldsymbol{V}}\right|^2 + \frac{M^2}{\omega^2}\,\frac{1}{\omega^2}\left|\frac{\boldsymbol{v}'.\boldsymbol{n}}{1-\boldsymbol{v}'.\boldsymbol{V}} - \frac{\boldsymbol{v}.\boldsymbol{n}}{1-\boldsymbol{v}.\boldsymbol{V}}\right|^2, \tag{5b}$$

where $\boldsymbol{V} = \boldsymbol{k}/k_0 = \boldsymbol{k}/\omega$, recalls very much the formula for the real photon bremstrahlung

$$d\sigma^\gamma = d\sigma^{(0)}\,\frac{\alpha}{4\pi^2}\,\frac{1}{\omega^2}\left|\frac{\boldsymbol{v}'\times\boldsymbol{n}}{1-\boldsymbol{v}'.\boldsymbol{n}} - \frac{\boldsymbol{v}\times\boldsymbol{n}}{1-\boldsymbol{v}.\boldsymbol{n}}\right|^2\frac{d^3k}{\omega}. \tag{6}$$

The only difference is the presence of $F(M^2)$ in Eq. (5) and the unit vector $\boldsymbol{n} = \boldsymbol{k}/k_0$ specifying the direction of the emitted real photon in Eq. (6) is replaced in Eq. (5) by the velocity $\boldsymbol{V} = \boldsymbol{k}/k_0 \equiv \boldsymbol{k}/\omega$ of the virtual photon (the velocity of the dilepton).

The term corresponding to the longitudional polarization of the real photon is, of course, absent in Eq. (6).

The similarity between Eqs. (5) and Eq. (6) permits us to generalize easily all formulas used for real photon production in I and II to the case of the bremsstrahlung of dileptons. For instance, the general formula for dilepton emission in a hadronic collision is obtained from Eqs. (5a, b) in the same way as Eq. (II.2) from Eq. (II.1) (we are now referring to eqs. in paper II).

III. A SIMPLIFIED FORMULA FOR e^+e^- BREMSSTRAHLUNG IN MULTIPARTICLE PRODUCTION

In this Section we shall present a modification of Eq. (I.4) suitable for the production of dileptons with $y_0 \sim 0$ in hadronic collisions. The formula is based on the following approximation: all hadrons in the final (and initial) state are

divided into two groups. The former group consists of hadrons moving approximately along the axis of the collision. The important quantity for this group is the charge transfer $\langle \Delta Q(y_0)^2\rangle$ across $y_0 \sim 0$. Since we are interested in dileptons moving perpendicularly to the axis of the collision, there holds $\boldsymbol{v}.\boldsymbol{n} = 0$ ($\boldsymbol{v}$ is the velocity of the hadron and $\boldsymbol{n} = \boldsymbol{k}/|\boldsymbol{k}|$ is the direction of the dilepton) and as seen from Eq. (5b) longitudinal virtual photons will not contribute. The denominator in the expressions in Eq. (5b) is equal to one, and the absolute value of the numerator is equal to $|\boldsymbol{v}\times\boldsymbol{n}| \sim 1$. The contribution of this term is therefore simply

$$\left(\omega\,\frac{d\sigma^{ee}}{d^3k\,dM}\right)_1 = \sigma^{inel}_{hadr}\,\frac{\alpha}{4\pi^2}\,\frac{1}{\omega^2}\,4\langle \Delta Q(y_0)^2\rangle F(M^2), \tag{7a}$$

the derivation being the same as that leading to Eq. (I.4). Longitudial photons do not contribute in this case because of $\boldsymbol{v}_i.\boldsymbol{n} = 0$. The latter group of hadrons has the rapidities $y \sim y_0 \sim 0$. Assuming that all the hadrons from this group contribute incoherently we obtain for this contribution

$$\left(\omega\,\frac{d\sigma^{ee}}{d^3k\,dM}\right)_2 = \sigma^{inel}_{hadr}\,\frac{\alpha}{4\pi^2}\,\frac{1}{\omega^2}\,\frac{dN_{ch}}{dy}\,F(M^2)(A_T + A_L), \tag{7b}$$

where

$$A_T = \left\langle\left|\frac{\boldsymbol{v}\times\boldsymbol{n}}{1-\boldsymbol{v}.\boldsymbol{k}_T/\sqrt{k_T^2+M^2}}\right|^2\right\rangle \tag{8a}$$

$$A_L = \frac{M^2}{\omega^2}\left\langle\left|\frac{\boldsymbol{v}.\boldsymbol{n}}{1-\boldsymbol{v}.\boldsymbol{k}_T/\sqrt{k_T^2+M^2}}\right|^2\right\rangle \tag{8b}$$

In these expressions $\boldsymbol{k}_T$ is the transverse momentum of the dilepton, M is the dilepton mass, $\omega^2 = k_T^2 + M^2$, $\boldsymbol{v}$ is the velocity of the hadron and $\langle\;\rangle$ indicates that one takes an average over momentum and rapidity distribution of hadrons within $y_0 - 0.5 < y < y_0 + 0.5$.

Using $F(M^2) \sim (2\alpha/3\pi)(1/M)$ and $d^2k_T = 2\pi k_T dk_T$ we can rewrite Eq. (7a) as follows

$$\left(\frac{d\sigma^{ee}}{dy\,dk_T\,dM}\right)_1 = \sigma^{inel}_{hadr}\,\frac{\alpha^2}{3\pi^2}\,\frac{k_T}{M}\,\frac{1}{k_T^2+M^2}\,4\langle \Delta Q(y_0)^2\rangle \tag{9a}$$

and Eq. (7b) as

$$\left(\frac{d\sigma^{ee}}{dy\,dk_T\,dM}\right)_2 = \sigma^{inel}_{hadr}\,\frac{\alpha^2}{3\pi^2}\,\frac{k_T}{M}\,\frac{1}{k_T^2+M^2}\,\frac{dN_{ch}}{dy}\,(A_T + A_L). \tag{9b}$$

To obtain the mass dependence of the dilepton production we have to integrate Eqs. (9a) and (9b) over k_T. Integrating (9a) from 0 to $\varkappa$ we obtain

$$\left(\frac{d\sigma^{ee}}{dy\,dM}\right)_1\bigg|_{y=0} = \sigma_{hadr}^{inel}\,\frac{2\alpha^2}{3\pi^2}\,\frac{\langle \Delta Q(y_0)^2\rangle}{M}\,\ln\left(1+\frac{\varkappa^2}{M^2}\right). \tag{10}$$

The value of $\varkappa$ is a rather uncertain parameter. The classical bremsstrahlung approximation we are using here is appropriate only under the assumption that the energy of the emitted photon (real or virtual) is much smaller than the energy of the particle which emits the photon. Quite frequently the approximation is reasonable for photon energies up to 50% or more of the energy of the radiating particle. The value of $\varkappa$ in p-nucleon collision at $p_L = 450$ GeV/c can thus be about $0.3 * \sqrt{s}/2 \sim 4$ GeV for the leading particles, and somewhat lower for the less energetic ones (in the c.m.s.). In what follows we shall present estimates of this contribution for $\varkappa = 4$ GeV. Note also that the term in Eq. (10) corresponds to a coherent radiation off, forward and backward moving particles and this speaks in favour of a larger value of $\varkappa$.

In calculating $d\sigma/dM$ from Eq. (9b) we have to integrate over k_T of the dilepton and over the directions and the velocities of the radiating hadrons. Calculations are straightforward and similar to those leading to Eq. (I.5). The net result becomes

$$\left(\frac{d\sigma^{ee}}{dy\,dM}\right)_2\bigg|_{y=0} = \sigma_{hadr}^{inel}\,\frac{\alpha^2}{3\pi^2}\,\frac{dN_{ch}}{dy}\,\frac{1}{M}\,(B_T+B_L), \tag{11}$$

where

$$B_T + B_L = \int_{-0.5}^{0.5} dy \int_0^{2\pi}\frac{d\varphi}{2\pi}\int_0^\infty p_T dp_T e^{-Ap_T} A^2 \int_0^{k_{T,\,max}} \frac{k_T dk_T}{k_T^2+M^2} * \\ * \frac{\{(m^2+p_T^2)\,\mathrm{sh}^2 y + p_T^2\sin^2\varphi\} + \left\{\frac{M^2}{k_T^2+M^2}\cdot p_T^2\cos^2\varphi\right\}}{\left[\sqrt{m^2+p_T^2}\,\mathrm{ch}\,y - p_T\cos\varphi\,\frac{k_T}{\sqrt{k_T^2+m^2}}\right]^2}. \tag{12}$$

The first term in curly brackets gives the contribution of B_T and the second one of B_L.

In Eq. (12) in the same way as in Sect. 2 of I

$$P(p_T, \varphi) = \frac{1}{2\pi}A^2 p_T e^{-Ap_T}, \qquad A \sim 6\ [\mathrm{GeV/c}]^{-1} \tag{13}$$

is the (p_T, φ) distribution of final state hadrons, to be integrated over $dp_T d\varphi$,

φ being the azimutal angle perpendicular to the beam direction, y is the rapidly of the final state hadron, M is the dilepton mass and k_T is the dilepton momentum. The bremsstrahlung approximation we are using might be again reliable only so far as the energy and momentum of the dilepton are smaller than those of the radiating hadron and because of that we are integrating in Eq. (12) only over $0 < k_T < k_{T,\,max}$, where $k_{T,\,max}$ corresponds to the situation when the energy of the virtual photon is equal to the energy of the radiating pion:

$$k_{T,\,max}^2 + M^2 = p_T^2 + m^2,$$

where m, p_T denote pion mass and transverse momentum and M is the dilepton mass.

To compare the results with data [8] we have to divide the results as given by Eqs. (10), (11) and (12) by $d\sigma_{\pi^0}/dy$ and estimate $\langle \Delta Q(y_0)^2\rangle$. We put $d\sigma_{\pi^0}/dy \sim \sim \sigma_{hadr}^{inel}$ and $\langle \Delta Q(y_0)^2\rangle \sim 1$. In this way we obtain

$$X_D(M) = \left(\frac{d\sigma^{ee}}{dy\,dM}\right)_1 \Bigg/ \left(\frac{d\sigma_{\pi^0}}{dy}\right) \sim \frac{2\alpha^2}{3\pi^2}\,\frac{1}{M}\,\ln\left(1+\frac{\varkappa^2}{M^2}\right) \tag{14}$$

for the dipole contribution. In calculating the contribution corresponding to Eqs. (11) and (12) we put

$$\sigma_{hadr}^{inel}\frac{dN_{ch}}{dy} \sim 2\sigma_{hadr}^{inel}\frac{dN_{\pi^0}}{dy} \sim 2\,\frac{d\sigma_{\pi^0}}{dy}$$

obtaining

$$X_T(M) = 2\,\frac{\alpha^2}{3\pi^2}\,\frac{1}{M}\,B_T \tag{15a}$$

$$X_L(M) = 2\,\frac{\alpha^2}{3\pi^2}\,\frac{1}{M}\,B_L \tag{15b}$$

with B_T and B_L given by the two terms in Eq. (12). In Fig. 1 we plot the three contributions $X_D(M)$, $X_T(M)$ and $X_L(M)$ and their sum $X(M)$. The dominating contribution comes from the "dipole" term $X_D(M)$. An accurate estimate of this contribution requires experimental information about the dependence of $\langle \Delta Q(y_0)^2\rangle$ on dN_{π^0}/dy. The total contribution $X(M)$ is by a factor of ~ 5 lower than the experimental data. Note, however, that the data are available mostly in the region above ~ 100 MeV, where also soft (and not only very soft) processes contribute.

Note also that for low dilepton masses $X_L(M)$ is lower than $X_T(M)$, the suppression of longitudinal virtual photons being reflected by the term $M^2/\omega^2 = M^2/(M^2+k_T^2)$ in the second term in the curved bracket in Eq. (12).

Bremsstrahlung radiation has a typical ω^{-2} dependence, which can be seen in Eqs. (7a) and (7b). This typical feature can be studied by measuring the k_T-dependence at fixed a dilepton mass. Proceeding as above we define

$$Y_D(k_T; M) = \left(\frac{d\sigma^{ee}}{dy\, dk_T\, dM}\right)_1 \Big/ \frac{d\sigma_{\pi^0}}{dy}\Big|_{y=0} \tag{16}$$

$$Y_T(k_T; M) = \left(\frac{d\sigma^{ee}}{dy\, dk_T\, dM}\right)_2 \Big/ \frac{d\sigma_{\pi^0}}{dy}\Big|_{y=0} \tag{17}$$

where indices 1 and 2 have the same meaning as in Eqs. (9—12) and T, L refer to contributions from transverse and longitudinal virtual photons. Under the same assumptions on $d\sigma_{\pi^0}/dy$, dN_{ch}/dy as above we obtain the explicit formulas for the k_T-dependence of the dilepton production at a fixed dilepton mass M.

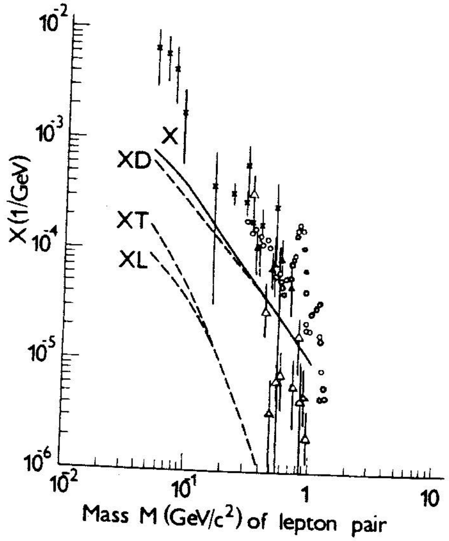

Fig. 1. Comparison of the compilation of experimental data on $(d\sigma/dM)_{y=0}$ with our calculations based on Eqs. (11) and (12). XD — contribution from dilepton radiation related to charge transfer between forward and backward going particles, calculated by Eq. (14), XT, XL — contribution from pions with rapidities $y \sim 0$, calculated by Eq. (15). $X = XD + XT + XL$ — the sum of all contributions.

$$Y_D(k_T; M) = \frac{\alpha^2}{3\pi^2} \frac{4k_T}{M(k_T^2 + M^2)} \tag{18}$$

$$Y_T(k_T; M) + Y_L(k_T; M) = \frac{2\alpha^2}{3\pi^2} \frac{1}{M} \int_{-0.5}^{0.5} dy \int_0^{\pi} \frac{d\varphi}{2\pi} \int_0^{\infty} p_T dp_T e^{-Ap_T} A^2$$

$$\frac{k_T}{k_T^2 + M^2} \frac{\{(m^2 + p_T^2)\, \mathrm{sh}^2 y + p_T^2 \sin^2\varphi\} + \left\{\frac{M^2}{k_T^2 + M^2}\right\} p_T^2 \cos^2\varphi}{\left[\sqrt{m^2 + p_T^2}\, \mathrm{ch} y - p_T \cos\varphi \frac{k_T}{\sqrt{k_T^2 + M^2}}\right]^2}. \tag{19}$$

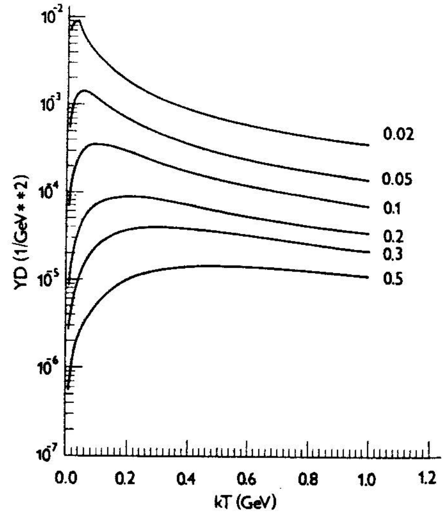

Fig. 2a. "Dipole" contribution to the k_T-dependence of dilepton production at fixed mass. YD denotes $Y_D(k_p; M)$ as defined by Eq. (18).

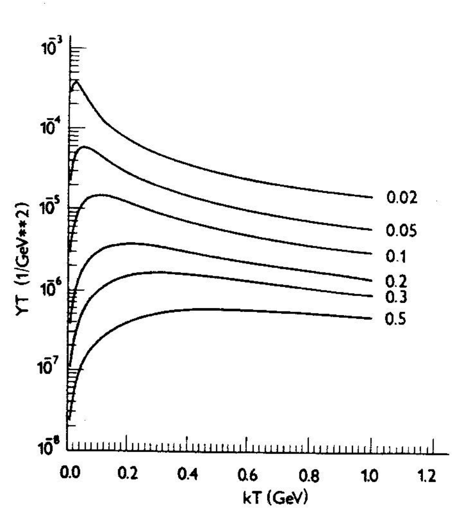

Fig. 2b. Contribution from transverse virtual photons. YT denotes $Y_T(k_T; M)$ as defined by Eq. (19) (only the first term in curly brackets present).

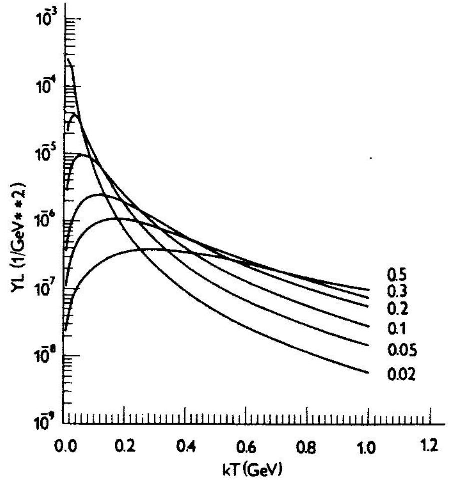

Fig. 2c. Contribution from longitudial virtual photons. $YL = Y_L(k_T; M)$ as defined by Eq. (19) (only the second term in curly brackets present).

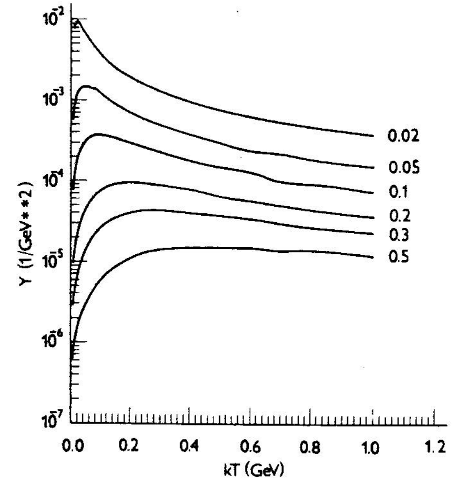

Fig. 2d. The sum of all contributions to the k_T-dependence of dilepton production at fixed dilepton mass M.

The k_T-dependence of the sum of all contribution is given as

$$Y(k_T; M) = \left(\frac{d\sigma^{ee}}{dy\, dk_T dM}\right) \bigg/ \left(\frac{d\sigma_{\pi 0}}{dy}\right)\bigg|_{y=0} = Y_D(k_T; M) + Y_T(k_T; M) + Y_L(k_T; M). \tag{20}$$

Our results on the k_T-dependence of the dilepton production at $y = 0$ and at a selected of dilepton masses are presented in Fig. 2. A dominating contribution is again given by $Y_D(k_T; M)$. This term is proportional to $(k_T/M)/(k_T^2 + M^2)$ and the maximum of $Y_D(k_T; M)$ occurs at $k_T = M$ (Fig. 2a). Contribution from virtual transverse photons (Fig. 2b) is of a similar shape because of the presence of the same factor in Eq. (19), but it is about an order of magnitude smaller. Longitudinal virtual photons (Fig. 2c) contribute by a term which is still somewhat smaller and its shape is different because of the additional term $M^2/(k_T^2 + M^2)$ in Eq. (19). The sum of all contributions $Y(k_T; M)$ (Fig. 2d) is thus very close to $Y_D(k_T; M)$. This permits a very simple quantitative criterium for the identification of the bremsstrahlung contribution to a very soft photon production. The bremsstrahlung contribution is the one whose $Y(k_T; M)$ as defined experimentally in Eq. (16) is proportional to $(k_T/M)/(k_T^2 + M^2)$.

A check of whether a set of very soft photon data and a set of low mass dilepton data are both originated by bremsstrahlung is given by the approximate formula

$$\frac{d\sigma^{ee}}{dy\, dM}\bigg|_{y=0} \sim \frac{2\alpha}{3\pi} \ln\left(1 + \frac{\varkappa^2}{M^2}\right) \frac{1}{2} \frac{d\sigma^{\gamma}}{dk_T}\bigg|_{k_T = M} \tag{20}$$

The formula is derived from Eq. (I.4) neglecting the second term in the r.h.s. and from the "dipole" contribution in Eq. (9a) integrating over k_T. The value of $\varkappa$ is discussed below Eq. (10). A more accurate relationship between a very soft γ and low mass dilepton production can be obtained by including also terms beyond the dipole approximation.

IV. COMMENTS AND CONCLUSIONS

We have derived simple approximate formulas for the bremsstrahlung emission of low mass dileptons. A comparison of our calculations with the experimental data of AFS collaboration [8] shows that the data on $d\sigma^{ee}/dM$ at $y = 0$ are about a factor of five above the bremsstrahlung contribution. The difference is either due to the approximate character of our formulas or to other contributions to a low mass e^+e^- production.

The mechanism responsible for the e^+e^- production could be made clear elucidated by the data on the k_T-dependence of the dilepton production at a fixed dilepton mass M and a fixed rapidity $y \sim 0$. Such data with sufficiently

high statistics could be also studied as function of two variables M, k_T, which could show whether the data posses typical bremsstrahlung features or not.

In what concerns phenomenological studies there are some questions deserving a more detailed analysis:

— the accuracy of the approximate formulas used above should be estimated in a realistic situation — one should generate final states of hadronic collisions by a suitable Monte Carlo model and calculate the bremsstrahlung production on the basis of both the exact and the approximate formula.

— data on the mass dependence of the dilepton production should be compared with a model containing both the bremsstrahlung and soft annihilation contributions giving the dilepton production from the intermediate stage of the collision.

Such studies should firmly establish whether there is some truly anomalous enhancement of very soft real and virtual photons and consequently also whether some anomalously large and cold intermediate parton system [15] is formed in hadronic collisions.

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REFERENCES

[1] Chliapnikov, P. V., et al.: Phys. Lett *B 141* (1984), 276; Ajinenko, I. V., et al.: Nuclear Physics *B 162* (1980), 61.

[2] Goerlach, U., et al., (NA-34 Collaboration): *Talk at the XXIV. Int. Conf. on High Energy Physics*, Munich, August 4—10, 1988, to be published in the Proceedings; Goerlach, U.: *Inv. talk at the Hadron Structure '88 Conference*, Nov. 14—18, 1988 Piešťany, Czechoslovakia; Pfeiffer, A.: Heidelberg prepint, MPI-H-1989-V3.

[3] Goshaw, A. T., et al.: Phys. Rev. Lett. *43* (1979), 1065; Phys. Rev. *D24* (1981), 2829.

[4] Balek, V., Pišútová, N., Pišút, J., Zinovjev, G. M.: Bratislava preprint, May, 1988 (referred to as I).

[5] Balek, V., Pišútová, N., Pišút, J.: Bratislava preprint, May, 1988 (referred to as II).

[6] Anderson, K. J., et al.: Phys. Rev. Lett. *37* (1976), 799, Phys. Rev. Lett. *37* (1979), 803.

[7] Corden, M. J., et al.: Phys. Lett. *68B* (1976), 96; Bunnel, K., et al.: Phys. Rev. Lett. *40* (1978), 136; Faessler, M., et al.: Phys. Rev. *D17* (1978), 689; Morse, W. M., et al.: Phys. Rev. *D18* (1978), 3145; Grannan, D. M., et al.: Phys. Rev. *D18* (1978), 3150, Alspector, J., et al.: Phys. Lett *B81* (1979), 397; Haber, B., et al.: Phys. Rev. *D22* (1980), 2107; Dzhelyadin, R. J., et al.: Nucl. Phys. *B179* (1981), 189; Ballmam, J., et al.: Phys. Rev. Lett. *41* (1978), 1207; Stroynowski, R., et al.: Phys. Lett. *B97* (1980), 315; Mikamo, S., et al.: Phys. Lett. *B106* (1981), 428; Stekas, J., et al.: Phys. Rev. Lett. *47* (1981), 1686; Roche, G., et al.: Phys. Rev. Lett *61* (1988), 1069; Stroynowski, R., Phys. Reports *71* (1981), 1.

[8] Hedberg, V., *Thesis*, University of Lund, preprint LUNDFDG/(NFFL-7037)/1987, March 1987.
[9] Craigie, N. S., Physics Reports *47* (1978), 1.
[10] Craigie, N. S., Thompson, H. N.: Nucl. Phys. *B141* (1978), 121.
[11] Rückl, R.: Phys. Lett. *B64* (1976), 39., Cahn, R. N., Phys. Rev. Lett. *29* (1972), 1481; Cahn, R. N.: Phys. Rev. *D7* (1973), 247.
[12] Lichard, P., Pišútová, N., Pišút, J.: preprint CERN-TH-5197/88, to be published in Zeit. f. Phys. C — Particles and fields.
[13] Shuryak, E. V.: Physics Reports *61* (1980), 71.
[14] Černý, V., Lichard, P., Pišút, J.: Zeit. f. Phys. C — Particles and Fields *31* (1986), 163 and references quoted there.
[15] Van Hove, L., preprint CERN-TH. 5236/88, to be published in the "Hermann Feshbach Festschrift", a special issue of Annals of Physics (N. Y.).
[16] Beresteckij, V. B., Lifshitz, E. M., Pitaevskij, L. P., *Relativistic Quantum Theory*, Vol. I., Nauka, Moscow 1968.
[17] Landau, L. D., Lifschitz, E. M., *Teorija Polya* (Theory of Fields), Moscow 1965.

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ПОИСК МЕХАНИЗМА УСИЛЕНИЯ ЭМИССИИ ТОРМОЗНОГО ИЗЛУЧЕНИЯ В РЕАКЦИЯХ АДРОНОВ

III. РОЖДЕНИЕ ЛЕГКИХ ДИЛЕПТОНОВ

Результаты полученные в предыдущих работах о тормозном излучении реальных фотонов обобщены на виртуальные которые в экспериментах знакомы как дилептоны с низкой мяссой. Спектр масс $d\sigma/dM$ торможенных дилептонов соответствует форме но оказывается заниженным в сравнении с экспериментальными данными колаборации АФС-ЦЭРН ИСР как и с другими исследованиями легких дилептонов. Показано, что данные о k_T зависимости рождения дилептонов при фиксированной массе дилептона позволяют ответить на вопрос о природе рождения легкого дилептона.

APPENDIX A

We shall derive here an approximate formula for bremsstrahlung of a lepton pair. The derivation closely follows that for bremsstrahlung of real photons. Consider a scattering of a charged spinor particle on a neutral one. Scattering without radiation is shown in Fig. 3a, scattering with bremsstrahlung of a virtual photon is shown in Figs. 3b, 3c.

The amplitude for scattering without radiation as shown in Fig. 3a is

$$A_0 = \bar{u}(p_1')\,\Gamma(p_1', p_1)\,u(p_1),$$

where we have suppressed momenta of the neutral particle. The amplitude for the massive photon bremsstrahlung in Fig. 3b in conventions of Ref. [16] becomes

$$A_1 = e\sqrt{4\pi}\,\varepsilon_\mu a_1^\mu \tag{A1a}$$

$$a_1^\mu = \bar{u}(p_1')\,\Gamma(p_1', p_1 - k)\,\frac{1}{\hat{p}_1 - \hat{k} - m}\,\gamma^\mu u(p_1).$$

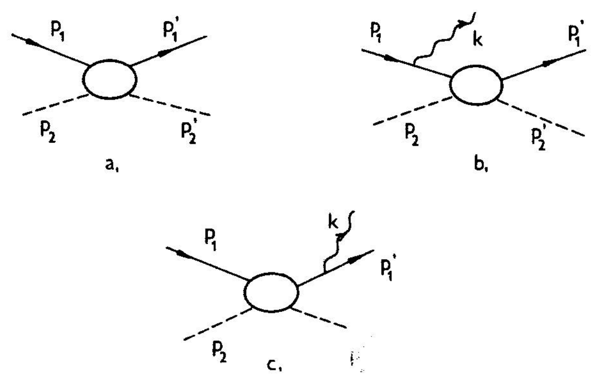

Fig. 3a. Scattering of a charged particle (solid line) on a neutral one without radiation.
Fig. 3b. Bremsstrahlung amplitude off the incoming charged particle.
Fig. 3c. Bremsstrahlung amplitude off the outgoing charged particle.

After some manipulations we get

$$a_1^\mu = \bar{u}(p_1')\,\Gamma(p_1', p_1 - k)\,\frac{2p_1^\mu - k^\mu}{k^2 - 2p_1'k}\,u(p_1)$$

$$-\frac{1}{2}\,\frac{\bar{u}(p_1')\,\Gamma(p_1', p_1 - k)\,[\hat{k}, \gamma^\mu]\,u(p_1)}{k^2 - 2p_1 . k}. \tag{A1b}$$

In the same way the amplitude corresponding to Fig. 3c

$$A_2 = e\sqrt{4\pi}\,\varepsilon_\mu a_2^\mu \tag{A2a}$$

$$a_2^\mu = \bar{u}(p_1')\,\gamma^\mu\,\frac{1}{\hat{p}_1' + \hat{k} - m}\,\Gamma(p_1' + k, p_1)\,u(p_1)$$

can be modified to

$$a_2^\mu = \bar{u}(p_1')\,\frac{2p_1'^\mu + k^\mu}{2p'.k + k^2}\,\Gamma(p_1' + k, p_1)\,u(p_1) + \frac{1}{2}\,\frac{\bar{u}(p_1')[\gamma^\mu, \hat{k}]\,\Gamma(p_1' + k, p_1)\,u(p_1)}{2p_1'.k + k^2}. \tag{A2b}$$

Notation in Eqs. (A1) and (A2) is standard, the polarization vector is assumed to be real. Note that for a massive (virtual) photon $k^2 \neq 0$.

The classical formula for the real photon emission is easily obtained from (A1) and (A2). For a real photon $k^2 = 0$. It is further assumed that the photon momentum is small in comparison with p_1, p_1' and that $\Gamma(p_1', p_1) = \Gamma(p_1' + k, p_1) = \Gamma(p_1', p_1 - k)$. Neglecting k^2 in numerators leads to

$$a_1^\mu + a_2^\mu = \bar{u}(p_1')\,\Gamma(p_1', p_1)\,u(p_1)\left[-\frac{p_1^\mu}{p_1.k} + \frac{p_1'^\mu}{p_1'.k}\right]. \tag{A3}$$

As shown in Appendix to I this leads directly to the classical formula for the bremsstrahlung emission

$$d\sigma^\gamma = d\sigma^{(0)}\,4\pi\alpha\left|\frac{p'.\varepsilon}{p'.k} - \frac{p.\varepsilon}{p.k}\right|^2 \frac{d^3k}{(2\pi)^3 2\omega}, \tag{A4}$$

where $d\sigma^{(0)}$ is the cross-section without radiation. After elementary exercises (A4) leads to

$$d\sigma^\gamma = d\sigma^{(0)}\left|\frac{\alpha}{4\pi^2}\,\frac{\boldsymbol{v}'\times\boldsymbol{n}}{\omega(1 - \boldsymbol{v}'.\boldsymbol{n})} - \frac{\boldsymbol{v}\times\boldsymbol{n}}{\omega(1 - \boldsymbol{v}.\boldsymbol{n})}\right|^2 \frac{d^3k}{\omega}. \tag{A5}$$

Bremsstrahlung of virtual photons with $k^2 \neq 0$ is a more delicate matter. The only safe way is to calculate in detail the corresponding Feynman diagram. Unfortunately in hadronic interactions the dynamics is not known and detailed Feynman diagrams cannot be calculated. In order to obtain an approximate expression for massive (virtual) photon bremsstrahlung we proceed in the same way as above and neglect the k-dependence of $\Gamma(p_1' + k, p_1)$ and $\Gamma(p_1', p_1 - k)$ and second terms in the r.h.s. of (A1b) and (A2b). The resulting amplitude

$$a_1^\mu + a_2^\mu = \bar{u}(p_1')\,\Gamma(p_1', p_1)\,u(p_1)\left[\frac{2p_1^\mu - k^\mu}{k^2 - 2p_1.k} + \frac{2p_1'^\mu + k^\mu}{2p_1'.k + k^2}\right] \tag{A6}$$

is gauge invariant (vanishes when multiplied by k^μ) and for $k^2 \to 0$ gives the amplitude (A3) for the real photon bremsstrahlung. Unfortunately, for $p_1 = p_1'$ (A6) is non-vanishing and therefore unacceptable. In the sence of the bremsstrahlung approximastion we further neglect k in numerators with respect to p_1'

and p_1 and k^2 with respect to $p'.k$ and $p.k$ in denominators. In this way we obtain

$$a_1^\mu + a_2^\mu = \bar{u}(p_1')\,\Gamma(p_1', p_1)\,u(p_1)\left[-\frac{p_1^\mu}{p_1.k} + \frac{p_1'^\mu}{p_1' - k}\right], \tag{A7a}$$

which means that the virtual photon is coupled to external legs in exactly the same way as the real photon.

Note that (A7a) is the lowest term obtained when expanding the amplitude in powers of k.

The choice of Eq. (A3) for real photons is motivated post hoc by agreement with the formulae (A4) and (A5) known from classical electrodynamics. We shall show below in Appendix B that (A7) is obtained in classical electrodynamics of a massive "photon" field. The cross-section for a massive photon corresponding to the amplitude in Eq. (A7a) is

$$d\sigma^\gamma = d\sigma^{(0)}\,\frac{\alpha}{4\pi^2}\,\frac{1}{\omega^2}\left\{\left|\frac{\boldsymbol{v}'\times\boldsymbol{n}}{1 - \boldsymbol{v}'.\boldsymbol{V}} - \frac{\boldsymbol{v}\times\boldsymbol{n}}{1 - \boldsymbol{v}.\boldsymbol{V}}\right|^2 + \frac{M^2}{\omega^2}\left|\frac{\boldsymbol{v}'.\boldsymbol{n}}{1 - \boldsymbol{v}'.\boldsymbol{V}} - \frac{\boldsymbol{v}.\boldsymbol{n}}{1 - \boldsymbol{v}.\boldsymbol{V}}\right|^2\right\}\frac{d^3k}{\omega}, \tag{A7b}$$

where $\boldsymbol{V}$ and M are velocity and mass of the massive photon. Note that in addition to be transversally polarized photons we have also the contribution of longitudinal ones, the latter being suppressed by the factor M^2/ω^2.

Having specified the coupling of a virtual photon to charged particle we can proceed to the calculation of the bremsstrahlung of dilepton pairs. The corresponding Feynman diagrams are obtained simply by attanching e^+e^- legs to virtual photon lines in Figs. 3b and 3c.

The sum of amplitudes for Figs. 3b and 3c modified in this way is

$$M_{fi} = M_{fi}^0(-i)\,\frac{4\pi e^2}{k^2}\,c_\mu\,\bar{u}(k_1, \lambda)\,\gamma^\mu v(k_2, \lambda), \tag{A8}$$

where

$$c_\mu = \frac{p_1'^\mu}{p_1'.k} - \frac{p_1^\mu}{p_1.k} \tag{A9}$$

k_1, k_2 are dilepton momenta and λ, λ' denote dilepton polarizations. $M_{fi}^{(0)}$ is the amplitude for the scatterring without bremsstrahlung of the lepton pair.

Summing over spins in the final state we get for the square of the amplitude

$$|M_{fi}|^2 = |M_{fi}^{(0)}|^2 \left(\frac{4\pi\alpha}{k^2}\right) Sp\{(\not{k}_1 + m)\,\gamma^\mu (\not{k}_2 - m)\,\gamma^\nu\}\, c_\mu c_\nu. \tag{A10}$$

The total cross-section can be written as

$$\sigma = \int d^4k\, T_{\mu\nu}(k) f^{\mu\nu}(k), \tag{A11}$$

where

$$T_{\mu\nu} = \frac{(2\pi)^4}{4qW}\int |M_{fi}^{(0)}|^2 c_\mu c_\nu\, \delta(p_1 + p_2 - k - p_1' - p_2')\, \frac{d^3p_1}{2E_1'(2\pi)^3}\, \frac{d^3p_2}{2E_2'(2\pi)^3}$$

$$f^{\mu\nu} = \int \delta^4(k - k_1 - k_2)\left(\frac{4\pi\alpha}{k^2}\right)^2 Sp\{(\not{k}_1 + m)\,\gamma^\mu(\not{k}_2 - m)\,\gamma^\nu\}$$

$$\frac{d^3k_1}{(2\pi)^3 2k_1^0}\, \frac{d^3k_1}{(2\pi)^3 2k_2^0}, \tag{A12}$$

where $W = \sqrt{s}$ is the total c.m.s. energy of the collision and $q = |\boldsymbol{q}|$ is the magnitude of momenta of colliding particles in the c.m.s. This decomposition follows Ref. [16, 17] and facilitates somewhat the calculations by permitting us to use the method of invariant integration. The tensor $f^{\mu\nu}$ depends only on k^μ and both tensors $T^{\mu\nu}$ and $f^{\mu\nu}$ are gauge invariant in the sense that $T^{\mu\nu}k_\nu = k_\mu T^{\mu\nu} = 0$ and $f^{\mu\nu}k_\nu = k_\mu f^{\mu\nu} = 0$. Because of that

$$f^{\mu\nu} = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{1}{3} f_\lambda^\lambda. \tag{A13}$$

A straightforward calculation gives

$$\frac{1}{3} f_\lambda^\lambda = \frac{1}{3}\left(\frac{4\pi\alpha}{k^2}\right)^2 4(k^2 + 2m^2)\,\frac{1}{(2\pi)^3}\,\frac{\pi}{2}\sqrt{\frac{k^2 - 4m^2}{k^2}}, \tag{A14}$$

where m is the lepton mass. The term $k_\mu k_\nu$ in Eq. (A13) gives zero when contracted with a gauge invariant tensor $T^{\mu\nu}$. In the sense of the bremsstrahlung approximation we neglect k in $\delta(p_1 + p_2 - k - p_1' - p_2')$ in Eq. (A12) and insert this Eq.

$$1 = \int dM^2\, \delta(k^2 - M^2)\, \Theta(k^0).$$

As a net result we find for the bremsstrahlung of dilepton with mass M and momentum k

$$\frac{d\sigma}{dM} = d\sigma^{(0)} 4\pi\alpha(-c_\mu c^\mu)\, F(M^2)\, \frac{d^3k}{(2\pi)^3 2k^0}, \tag{A15}$$

where

$$F(M^2) = \frac{2\alpha}{3\pi}\,\frac{M(M^2 + 2m^2)}{M^2}\sqrt{1 - \frac{4m^2}{M^2}} \sim \frac{2\alpha}{3\pi}\,\frac{1}{M}. \tag{A16}$$

APPENDIX B

We shall show here now one can arrive at the formula for the bremsstrahlung of massive photons in the framework of the classical field theory. The field equation for a massive vector field is (in the conventions of Ref. [17] with $c = \hbar = 1$)

$$(-\partial_t^2 + \Delta - M^2)\, A^\mu = -4\pi j^\mu, \tag{B1}$$

where M is the mass of the field, A^μ is the 4-potential and j^μ is the R-current. We are interested in the Fourier components of the 4-potential rather than in the 4-potential itself. The equation for these Fourier components is effectively the same as in the case of the massless field

$$(\Delta + k^2)\, A_\omega^\mu = -4\pi j_\omega^\mu, \tag{B2}$$

the only difference being that the quantity k (the magnitude of the wave vector) is a more complicated function of the frequency ω, $k = (\omega^2 - M^2)^{1/2}$. The solution to Eq. (B2) has the familiar form

$$A_\omega^\mu = \int \frac{1}{R}\, e^{ikR} j_\omega^\mu(\boldsymbol{r}')\, dV', \tag{B3}$$

where $R = |\boldsymbol{R}_0 - \boldsymbol{r}'|$, $\boldsymbol{R}_0$ is the radius vector of the place of observation, and far from the source this reduces to

$$A_\omega^\mu = \frac{1}{R_0}\, e^{ikR_0} \int e^{-i\boldsymbol{k}.\boldsymbol{r}'}\, j_\omega^\mu(\boldsymbol{r}')\, dV', \tag{B4}$$

where $\boldsymbol{k} = k\,\boldsymbol{n}$, $\boldsymbol{n}$ is the unit vector in the direction of $\boldsymbol{R}_0$. For the field generated by a pointlike particle we obtain

$$A_\omega^\mu = \frac{1}{R_0}\, e^{ikR_0} q \int_{-\infty}^{\infty} e^{i\omega t - i\boldsymbol{k}.\boldsymbol{r}}\, v^\mu\, dt, \tag{B5}$$

where q is the charge of the particle, $\boldsymbol{r}$ is the radius vector of the particle and

$v^\mu = (1, \boldsymbol{v})$, $\boldsymbol{v}$ is the velocity of the particle. Note that $\boldsymbol{k} = \omega \boldsymbol{V}$, where $\boldsymbol{V}$ is the group velocity of the propagation of the radiation, therefore

$$A^\mu_\omega = \frac{1}{R_0} e^{ikR_0} q \int_{-\infty}^{\infty} e^{i\omega(t - \boldsymbol{V}.\boldsymbol{r})} v^\mu \, dt. \tag{B6}$$

Let us now consider a particle scattering off some external centre of force. The full spectrum of the radiation of such a particle depends of course on details of the scattering, but one can always compute the low frequency part of the spectrum which does not. The resulting expression is what one calls the bremsstrahlung formula. The relevant frequency interval is $\omega \ll 1/\Delta t$, where Δt is the time interval during which the scattering takes plase. (In the case of the massive field one has to impose this condition simultaneously with $\omega \geq M$ since for $\omega < M$ the field is exponentially damped at large distances from the source). If only small frequencies are considered, the trajectory of the particle can be replaced by a broken line, and from Eq. (B6) one finds

$$A^\mu_\omega = \frac{1}{R_0} e^{ikR_0} q \left[-\frac{v^{\mu'}}{i\omega(1 - \boldsymbol{v}'.\boldsymbol{V})} + \frac{v^\mu}{i\omega(1 - \boldsymbol{v}.\boldsymbol{V})} \right], \tag{B7}$$

where $\boldsymbol{v}$ and $\boldsymbol{v}'$ are the celocities of the particle before and after the scattering, respectively. (The primitive functions at $t = \pm\infty$ do not contribute since they are zero in the sense of the theory of distributions).

The quantity we wish to compute is the full energy radiated into the solid angle $d\Omega$ in the frequency interval $d\omega$. This energy is given by

$$d\varepsilon = \boldsymbol{S}_\omega . \boldsymbol{n} R_0^2 d\Omega \frac{d\omega}{2\pi} \tag{B8}$$

where $\boldsymbol{S}$ is the energy flow of the radiation at the distance R_0 from the place where the scattering occurred. The energy flow is given by the nondiagonal components of the energy-momentum tensor which can be computed in a standard way from the Lagrangian of the field. For the massless field the energy flow is just the Poynting vector, and from the mass term in the Lagrangian $\Delta L = M^2 A^2/8\pi$ we obtain the additional term to the energy flow $\Delta \boldsymbol{S} = M^2 A^0 \boldsymbol{A}/4\pi$. The Fourier component of this term is $\Delta \boldsymbol{S}_\omega = M^2 A^0_\omega \boldsymbol{A}_\omega/2\pi$ and using the Lorentz condition $\omega A^0_\omega - \boldsymbol{k}.\boldsymbol{A}_\omega = 0$, which can be easily proved to be valid for the 4-potential given by (B6) as well as (B7), we find

$$\Delta \boldsymbol{S}_\omega . \boldsymbol{n} = M^2 \frac{1}{2\pi} \frac{k}{\omega} |\boldsymbol{A}_\omega . \boldsymbol{n}|^2. \tag{B9}$$

For the Poynting vector we have

$$\boldsymbol{S}_{Poynt,\, \omega} . \boldsymbol{n} = \frac{1}{2\pi} k\omega |\boldsymbol{A}_\omega \times \boldsymbol{n}|^2 \tag{B10}$$

and putting these two expressions together, inserting them into Eq. (B8) and using Eq. (B7) we get

$$d\varepsilon = \frac{q^2}{4\pi^2} \left[\left| \frac{\boldsymbol{v}' \times \boldsymbol{n}}{\omega(1 - \boldsymbol{v}'.\boldsymbol{V})} - \frac{\boldsymbol{v} \times \boldsymbol{n}}{\omega(1 - \boldsymbol{v}.\boldsymbol{V})} \right|^2 + \frac{M^2}{\omega^2} \left| \frac{\boldsymbol{v}'.\boldsymbol{n}}{\omega(1 - \boldsymbol{v}'.\boldsymbol{V})} - \frac{\boldsymbol{v}.\boldsymbol{n}}{\omega(1 - \boldsymbol{v}.\boldsymbol{V})} \right|^2 \right] d^3k. \tag{B11}$$

The cross section for the production of massive photon is equal to this quantity divided by ω (the energy of one photon in the units in which $\hbar = 1$) and multiplied by $d\sigma^{(0)}$ (the cross section of the scattering of the radiating particle). If we put $q = e$ and use $e^2 = \alpha$, we obtain the bremsstrahlung formula (A7b).

APPENDIX C

We shall give here an interpretation of the formula for the bremsstrahlung of dileptons relating this process to the creation of dileptons via the decay of real massive photons. The differential cross section of the bremsstrahlung of dileptons is

$$d\sigma = d^4k \, dT_{\mu\nu} df^{\mu\nu} \tag{C1}$$

or, equivalently

$$d\sigma = \frac{M}{\omega} d^3k \, dM \, dT_{\mu\nu} df^{\mu\nu}, \tag{C2}$$

where $dT_{\mu\nu}$, $df^{\mu\nu}$ are given by the formulae (A12) in which only the integrations necessary to get rid of the δ-functions are performed. Neclecting k^μ in the argument of the δ-function in $dT^{\mu\nu}$ we get $dT^{\mu\nu} = d\sigma^{(0)} c_\mu c_\nu$, and using the expression of the differential cross section of the bremsstrahlung of real massive photons given in Eq. (A4) as well as the definition of c^μ given in Eq. (A9) we find

$$d^3k \, dT^{\mu\nu} = d\sigma^\gamma \frac{(2\pi) 2\omega c_\mu c_\nu}{4\pi\alpha(-c^2)}. \tag{C3}$$

The quantity $df^{\mu\nu}$ is related to the differential probability of the decay of a massive photon with the given polarization ε^μ into the dilepton in 1 sec. If we denote this probability $\mathrm{d}w(\varepsilon^\mu)$, we have

$$\varepsilon_\mu \varepsilon_\nu \mathrm{d}f^{\mu\nu} = \frac{1}{(2\pi)^4} \frac{4\pi\alpha}{k^4} 2\omega \mathrm{d}w(\varepsilon^\mu). \tag{C4}$$

Inserting Eq. (C3) into Eq. (C2) and making use of Eq. (C4) as well as of the relation $k^2 = M^2$ we arrive at

$$\mathrm{d}\sigma = \frac{2\omega}{\pi M^3} \mathrm{d}M \, \mathrm{d}\sigma^\gamma \mathrm{d}w(c^\mu/(-c^2)^{1/2}). \tag{C5}$$

In such a way the production of dileptons from virtual photons can be viewed as the production of real massive photons with the lifetime ω/M^2 (which can be understood in terms of the uncertainty principle) and the mass distribution function $2/\pi M$, followed by the decay of the massive photons into the dileptons. This interpretation, however, cannot be pushed too far, as it is seen from the fact that the photons should be produced as nonpolarized and at the same time decay as polarized with the polarization vector $\varepsilon^\mu = c^\mu/(-c^2)^{1/2}$.