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CLASSICAL AXION FIELD CONFIGURATION INTERACTING WITH AN ELECTRICALLY AND MAGNETICALLY CHARGED STRING

(A MECHANISM TO GENERATE THE DARK MATTER IN GALAXIES)

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The electric and magnetic fields surrounding an axion string left behind after a phase transition in the early universe are calculated. The string attains its electrical charge by a mechanism suggested by Sikivie and its magnetic charge from monopoles created near the G. U. T. phase transition. It is speculated that if two such strings with opposite charges come in contact their fields can relax to yield a matter distribution consistent with that found in the dark matter abounding in galactic arms. The electromagnetic fields annihilating give up their energy to massive particles we know as axions that were postulated to eliminate strong CP violations in Q. C. D. by the P. Q. mechanism.

I. INTRODUCTION

One of the most puzzling and paradoxical facts of observational astrophysics is the existence of dark matter in the galactic arms [1]. Depending on the model used and the restrictions imposed by the spatial curvature various galactic, pancake and superpancake structures are found possible to form. For instance, if one uses neutrinos for the particle content and $\lambda = 1$ (λ = ratio of density to critical density), pancakes can form but galaxies and superpancakes cannot. If one uses baryons with $\lambda = 1$ all three structures can form. In fact, various other possibilities are possible depending on the particle content and the value of λ, [2] (a pancake here refers to a cluster of galaxies and a superpancake refers to a supercluster). The fact that the velocity curves for the galactic arm are not consistent with the predictions for the density based on the luminosity variation with r indicates that there is a large fraction, (perhaps 90%) of dark nonluminous matter in the galactic arms. Whether it is in the form of neutrinos, baryons, cold matter or perhaps axions, it seems at this point to be a completely open question. The scenario adopted in this note is that if an electrically and magnetically charged string forms, [3] the electromagnetic energy given up upon annihilation with an oppositely charged string can give a $\frac{1}{r^2}$ distribution of matter sufficient to form the dark matter of galaxies if the EM energy relaxes into energy of massive particles. Such a mechanism would be in harmony with the observation that galaxies seem to be found along these string-like filamentary structures [4].

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II. THE ELECTROMAGNETIC FIELDS OF A STRING

The appearance of the axion in particle theory was motivated by the recognition on the part of Peccei and Quinn [5, 6], that because of instanton or pseudo particle solutions, Q. C. D. takes on a non-trivial topological vacuum structure with the strong CP violating term induced through instanton effects. Peccei and Quinn suggested that if a chiral U(1) invariance existed in the Lagrangian of the scalar and quark fields, this strong CP violating term could be rotated away through the global chiral U(1) symmetry. However, when the $U(1)_{P.Q.}$ (Peccei Quinn symmetry) is broken, a slightly massive pseudo-scalar appears which we call the axion. M. Dine, et al. [7] have discussed a variant of the axion model with two Higgs doublets and a singlet and has pointed out that the coupling of the axion must be very weak to matter so as not to deplete the thermal energy of red giant stars. They also point out that if the axion decays after recombination, it will generate an observable distortion of the microwave background. Also M. B. Wise at al. [8] have discussed the emergence of the axion from an SU(5) model with the coupling constant to matter and the axion mass being generated by the inverse V. E. V. of the Higgs field that breaks SU(5) down to SU(3) X SU(2) X U(1), their analysis leads to extremely small couplings and mass for the axion and thus it becomes essentially invisible.

The breaking of the Peccei Quinn symmetry, along with the acquistion of a dynamical mass via a Q. C. D. instanton effect after the electroweak phase transition gives the following expression for the low energy Lagrangian for electromagnetism interacting with axions [9, 10]

$$\mathscr{L} = \left[\begin{array}{l} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\partial_\mu a \partial^\mu a}{2} - \frac{m_a^2 V^2}{N^2}\left(1 - \cos\frac{Na}{V}\right) \\ + e^2\left(\Theta_\gamma + \frac{T_\Theta}{2\pi}\right)\frac{Na}{V}\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{64\pi^2 \sin^2\Theta_\omega^0\sqrt{-g}}\right) \end{array}\right]\sqrt{-g} \qquad (1)$$

a = axion field, $F_{\mu\nu}$ = E.M field tensor, $g_{\mu\nu}$ = cylindrically symmetric metric, also. Where $e^2 = e_s^2 \sin^2 \Theta_w^0$, N = number of vacuum of axion model, a = axion field, m_a is axion mass, V = V.E.V. that breaks the P.Q. symmetry, Θ_w^0 = electroweak angle at G.U.M. scale, e_s = gauge coupling of Q.C.D., $\Theta_\gamma = \Theta_{\text{Q.C.D.}}$ at G.U.M., T_Θ is period of $\Theta = 2\pi$ for Q.C.D. and SU(5) as a G.U.T. group. Suppose we consider Equation (1) as a Lagrangian with given E, B and use it to calculate a. We suppose that a string has been left behind after the P.Q. breaking mechanism and it attains an electric charge by the mechanism of Sikivie in the above reference. If the string also attains a magnetic charge by attracting magnetic monopoles which have condensed after the G.U.T. phase transition we have essentially a string of electric and magnetic charge. Suppose we adopt cylindrical coordinates and write

$$E_r = \frac{\gamma_E}{2\pi r}, \quad B_r = \frac{\gamma_M}{2\pi r}. \tag{2}$$

Now γ_E, γ_M are determined by the electric and magnetic charge density per unit length of string. The equation for a from Equation (1) is

$$-\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left(\sqrt{-g}\, a_{,\beta} g^{\beta\alpha}\right) - \frac{m_a^2 V^2}{N^2}\left(\frac{N}{V}\right)\sin\frac{Na}{V} + $$
$$+ e^2\left(\Theta_\gamma + \frac{T_\Theta}{2\pi}\right)\left(\frac{N}{V}\right)\frac{e^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{64\pi^2\sin^2\Theta_\omega^0\sqrt{-g}} = 0, \tag{3}$$

where

$$\sqrt{-g} = r, \quad \frac{\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}}{\sqrt{-g}} = -8E_r B_r = -\frac{2\gamma_E\gamma_M}{\pi^2 r^2} \tag{4}$$

calling

$$\frac{m_a^2 V^2}{N^2}\left(\frac{N}{V}\right)\left(\frac{N}{V}\right) = A,\ B = e^2\left(\Theta_\gamma + \frac{T_\Theta}{2\pi}\right)\left(\frac{N}{V}\right)\frac{1}{64\pi^2\sin^2\Theta_\omega^0}\left(\frac{2\gamma_E\gamma_M}{\pi^2}\right),$$

$$\sin\frac{Na}{V} \approx \frac{Na}{V}, \text{ for } \frac{Na}{V} < 1.$$

Thus

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}a}{\mathrm{d}r}\right) - Aa - \frac{B}{r^2} = 0 \tag{5}$$

is the equation for the axion field. Equation (5) represent the progression of the axion field away from the string of electric and magnetic charge and we seek to find out what the distribution of the axion field energy density would be in the

presence of the electrically and magnetically charged string. To solve the nonhomogeneous part of Equation (5), that is the axion component generated by the electric and magnetic fields we have

$$r^2 a'' + ra' - A(r^2)a - B = 0 \tag{6}$$

we write

$$a = a_0 + \frac{a_1}{r} + \frac{a_2}{r^2} \cdots = \sum_{i=0}^{\infty} \frac{a_i}{r^i}. \tag{7}$$

Substituting Equation (7) into Equation (6) we have

$$r^2\left(\frac{2a_1}{r^3} + \frac{6a_2}{r^4} + \frac{12a_3}{r^5} + \frac{20a_4}{r^6}\right) + r\left(-\frac{a_1}{r^2} - \frac{2a_2}{r^3} - \frac{3a_3}{r^4} - \frac{4a_4}{r^5}\right) -$$
$$-Ar^2\left(a_0 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \frac{a_4}{r^4}\right) - B = 0,$$

equating the coefficients of various powers to zero we have.

$$a_0 = a_1 = 0, \quad a_2 = -\frac{B}{A}, \quad a_4 = \frac{4a_2}{A} = -\frac{4B}{A^2},$$

$$a_6 = \frac{16a_4}{A} = -\frac{64B}{A^3}, \quad a_3 = a_5 = 0 \ldots$$

$$a(r) = -\left(\frac{B}{A}\right)\left(\frac{1}{r^2}\right) - \frac{4B}{A^2}\left(\frac{1}{r^4}\right) - \frac{64B}{A^3}\left(\frac{1}{r^6}\right). \tag{8}$$

We see that the axion component rapidly decreases outside the string, for the total energy density of the composite system of electromagnetism and axion field we have from the Lagrangian Equation (1)

$$T_{00} = \frac{1}{2}(\boldsymbol{E}\,.\,\boldsymbol{E} + \boldsymbol{B}\,.\,\boldsymbol{B}) + \frac{1}{2}\left(\left(\frac{\partial a}{\partial t}\right)^2 + \left(\frac{\partial a}{\partial r}\right)^2\right), (c = 1)$$
$$+ \frac{m_a^2 V^2}{N^2}\left(1 - \cos\frac{Na}{V}\right) \tag{9}$$

for $\frac{Na}{V} < 1$ this gives

$$T_{00} = \frac{1}{8\pi^2}\frac{\gamma_E^2}{r^2} + \frac{1}{8\pi^2}\frac{\gamma_M^2}{r^2} + \frac{1}{2}\left[\frac{4B^2}{A^2 r^6} - - -\right] + \frac{m_a^2}{2}\left[\frac{B^2}{A^2 r^4} - - -\right], \tag{10}$$

where all terms due to the axion component decrease as $\frac{1}{r^4}$ or faster. It is clear that Equation (10) gives a dominant $\frac{1}{r^2}$ dependence of the total energy density with negligible contribution due to the axion field. Both Zeldovich [1] and Vilenkin [12] have pointed out that strings created near phase transitions in the early universe will produce density fluctuations if the strings have motion either through cosmic expansion or perhaps through local gravitational forces generated by slight inhomogeneities in the matter distribution. If two of the above type strings with opposite electric and magnetic charge became subject to this motion and collide, they would annialate their E M fields and convert the E M energy into massive axions with energy density proportional to $1/r^2$ producing a significant component of the matter distribution surrounding the newly formed galatic structures when the density fluctuations grow. The motion and collision process might occur at appropriate scales to produce the observable $1/r^2$ distribution of dark matter which, in this case would be in the form of axions.

It is understood here that it is possible that not all the matter in the galactic structure is in the form of axions. It is well known that a distribution of massive baryons would yield a $1/r^2$ distribution upon "violent relaxation". However, an additional component here is needed in order to account for the velocity curves and the observed luminosity. The scenario here suggested would have to be investigated from the point of view of stability to see if the field configuration is stable to small perturbations. If it were, it would be a viable candidate for the missing mass, if it is not it would suggest the configuration would just disperse and not provide a model for the missing mass. Actually the interest in the axion as a possible source in the formation of galaxies came from the observation of Vilenkin [13] that the phase of the vacuum expectation value of a complex scalar field should vary over different regions in the universe when the regions are causally disconnected, this variation of the phase, or variation of the Goldstone field would cause density fluctuations sufficient to trigger galaxy formation. The Goldstone field, in our case, could be identified with the axion.

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ВЗАИМОДЕЙСТВИЕ КЛАСИЧЕСКОЙ ФОРМЫ ПОЛЯ ВЗАИМОДЕЙСТВИЕ С ЭЛЕКТРИЧЕСКИ И МАГНЕТИЧЕСКИ ЗАРЯЖЕННОЙ СТРУНОЙ

В работе расчитано электрическое и магнитное поле струны аксиона, которое возникло после фазового перехода на раннем этапе возникновения вселенной. Струны получают электрический заряд механизмом выдвиженным Сикиви и магнитный заряд от монополей рождающихся вблизи GUT фазового перехода. Предпологается, что в случае, когда две струны с обраными зарядами придут в контакт снижается выход расиределения материи, что соответствует темновой материи возникающей в галактических взрывах. Энергия электромагнитного поля передается при аннигиляции тяжелым частицам-аксионам, которые могут устранить сильное CP несохранение в QCD посредством P.Q. механизма.