acta phys. slov. 41 (1991), No. 3

A SEARCH OF A MECHANISM RESPONSIBLE FOR BREMSSTRAHLUNG ENHANCEMENT IN HADRONIC REACTIONS

II. CORRELATIONS IN THE FINAL STATE

BALEK, V.[1]), PIŠÚTOVÁ, N.[1]), PIŠÚT, J.[1]), Bratislava

Following our preceding paper we study here the possible effects of correlations between the final state hadrons on the emission of bremsstrahlung. In particular we discuss: i) the Bose–Einstein correlations, ii) short range correlations between π^+ and π^-, iii) dependence of $\langle p_T \rangle$ on charged multiplicity and iv) charge transfer dependence on multiplicity. In cases i) and iii) the correlations lead to a faster than linear dependence of the bremsstrahlung emission on multiplicity. Neither of these effects can explain the excess of very soft photons in K^+p interactions at 70 GeV/c, where the correlations have been taken into account. The effects are also too small to explain the enhanced production of very soft photons in 450 GeV/c pBe and pAl interactions.

I. INTRODUCTION

The present paper is a continuation of our study [1] (in the sequel referred to as I) of possible mechanism of very soft photon production in hadronic reactions. Following I we denote as "very soft" photons with c.m.s. momenta of the order of 20 MeV/c in contradistinction of "soft" photons with momenta of the order of 100 MeV/c.

Very soft photon production in hadronic collisions has been studied in Refs. [2, 3, 4]. The results obtained are not yet understood. Goshaw et al. [2] have found that in π^+p interactions at 10.5 GeV/c the production of very soft photons is consistent with the bremsstrahlung emission by final state hadrons. In their calculations of the bremsstrahlung production they have used general formulas (see Appendix A in I) taking into account all correlations between charged particles in the final state. As the input information on the charged particle distributions the authors have taken their own data in the same experiment. Chliapnikov et al. [3] have observed very soft photons in K^+p interactions at 70 GeV/c. Their analysis proceeded in the same way in Ref. [2]. Their bremsstrahlung calculations from final state hadrons are also based on general formulas, taking into account all correlations, and as input the authors have also taken their own data on charge particle distributions obtained in the same experiment. Still, in contradistinction of Ref. [2] Chliapnikov et al. have observed a very soft photon signal, similar in shape to the bremsstrahlung contribution, but with a magnitude about 4 times larger.

If both experimental results [2] and [3] are correct, there should exist some threshold between 10.5 GeV/c and 70 GeV/c above which a new mechanism responsible for very soft photon production is operative.

The most recent information on very soft photon production in hadronic collisions comes from the NA-34 collaboration at CERN SPS [4]. Very soft photons observed in pAl and pBe collisions at 450 GeV/c are again similar in shape to the bremsstrahlung emission and again larger in magnitude by a factor of 4. The information about charged hadrons in the final state in this experiment is rather incomplete and instead of general formulae taking into account all correlations one has to be satisfied with approximative ones (see, e.g., Eq. (4) in I). The data on very soft photons in Ref. [4] are larger by a factor of 4 than the results based on these approximative formulae.

The NA-34 collaboration have discovered [4] that very soft photon production at $y \sim 0$ increases faster than linearly with dN_{ch}/dy at $y \sim 0$. This fact is probably closely correlated with the quadratic dependence of low mass e^+e^- production in pp collisions at $\sqrt{s} = 63$ GeV found by the AFS collaboration at CERN ISR [5].

If the interpretation of the K^+p experiment at 70 GeV/c is correct, it is most likely that the correlations of the final state hadrons play no essential role in the bremsstrahlung enhancement. If, on the other hand the interpretation were wrong, it would be that the correlations in the final state might possibly be responsible for at least a part of the enhancement observed by the NA-34 collaboration.

The purpose of the present paper is to examine the latter option.

The paper is organized as follows. In Sect. II. we analyse the effect of the Bose–Einstein (or the Hanbury–Brown and Twiss) correlations on the bremsstrahlung emission. Section III. deals with the $\pi^+\pi^-$ correlations, like those expected from the ϱ^0 decay on the bremsstrahlung enhancement. The effect of the $\langle p_T \rangle$ increase with dN_{ch}/dy on the bremsstrahlung emission is discussed in Sect. IV. The effect is interesting since it leads to a faster than linear dependence of the bremsstrahlung on dN_{ch}/dy. The dependence of the charge transfer over $y = 0$, $\langle (\Delta Q)^2 \rangle$ on dN_{ch}/dy and its effect on the bremsstrahlung is analysed in Sect. V.

Comments and conclusions are presented in Sect. VI.

[1]) Department of Physics, Comenius university CS — 842 15 BRATISLAVA, CSFR

II. THE BOSE EINSTEIN CORRELATIONS AND THE BREMSSTRAHLUNG ENHANCEMENT

The emission of very soft photons with wave lengths larger than the dimensions of the radiating system (for $\omega = 20$ MeV the wave length is $\lambda \sim 2\pi \times 10$ fm ~ 63 fm) can be described by classical formulae. The bremsstrahlung of a particle incident with velocity $\boldsymbol{v}$ and scattered to a state with velocity $\boldsymbol{v}'$ is given as

$$\omega \frac{\mathrm{d}N}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2 \omega^2} \left| \frac{\boldsymbol{v}' \times \boldsymbol{n}}{1 - \boldsymbol{v}' . \boldsymbol{n}} - \frac{\boldsymbol{v} \times \boldsymbol{n}}{1 - \boldsymbol{v} . \boldsymbol{n}} \right|^2, \tag{1}$$

where $\mathrm{d}N$ is the number of radiated photons, ω — the photon energy, $\boldsymbol{n}$ the direction of the radiated photon, $\alpha = 1/137$ and $\mathrm{d}^3 k = \omega^2 \mathrm{d}\omega \mathrm{d}\Omega$ is the momentum space volume of the photon.

The bremsstrahlung emission in a hadronic collision is then given as

$$\omega \frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2} \sum_{N^+ N^-} \sigma_h(N^+, N^-) \int \left| \sum_i Q_i \frac{\boldsymbol{v}_i \times \boldsymbol{n}}{\omega(1 - \boldsymbol{v}_i . \boldsymbol{n})} - Q_a \frac{\boldsymbol{v}_a \times \boldsymbol{n}}{\omega(1 - \boldsymbol{v}_a . \boldsymbol{n})} - Q_b \frac{\boldsymbol{v}_b \times \boldsymbol{n}}{\omega(1 - \boldsymbol{v}_b . \boldsymbol{n})} \right|^2 P_{\pi^+}(\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_{N^+}) P_{\pi^-}(\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_{N^-}) * \Pi \mathrm{d}^3 \boldsymbol{p}_i \mathrm{d}^3 \boldsymbol{q}_i . \tag{2}$$

In Eq. (2) the index i refers to the final state pions (we are forgetting now about other particles), $\sigma_h(N^+, N^-)$ is the inelastic hadronic cross-section for production N^+ positive and N^- negative pions and P_{π^+}, P_{π^-} are probability distributions normalized to 1 after integration over the corresponding momenta. Eq. (2) is also written down in a form assuming no correlation between negative and positive pions, since we are dealing here with the Bose–Einstein correlations only.

As discussed in more detail in Eq. (4) in I, the total bremsstrahlung at $y \sim 0$ off the final state particles can be approximately divided into two parts. The former is the dipole radiation of the forward and backward moving particles, proportional to the charge transfer $\langle (\Delta Q)^2 \rangle$ across $y = 0$ and the latter is due to the final state particles with $y \sim 0$. The Bose Einstein correlations between forward (and backward) moving particles are unimportant, so we shall discuss here only the effect of the BE correlations among particles with $y \sim 0$. The width of the relevant rapidity interval can be estimated as follows. According to Eq. (1) the radiation off an outgoing particle is proportional to

$$\left| \frac{\boldsymbol{v} \times \boldsymbol{n}}{1 - \boldsymbol{v} . \boldsymbol{n}} \right|^2 \mathrm{d}\Omega \sim \frac{v^2 \sin^2 \vartheta}{(1 - v \cos \vartheta)^2} \sin \vartheta \mathrm{d}\vartheta,$$

where ϑ is the angle between $\boldsymbol{v}$ and $\boldsymbol{n}$. To find the maximum of the radiation (the dimension of the cone) we put $x = \cos \vartheta$ and find the maximum of the expression $v^2(1 - x)^2/(1 - vx)^2$. It turns out that $x_{max} = v$, which means $\cos \vartheta_{max} = v$. The bremsstrahlung goes therefore predominantly into the angles $1 - \vartheta^2/2 \geq v$, which implies $\vartheta \leq \sqrt{1 - v^2} = 1/\gamma$. If we detect radiation at $y = 0$, a substantial contribution will come only from particles with $y \leq 1/\gamma$. This is due to $\vartheta \sim \mathrm{tg}\, \vartheta = p/p_T = m_T \mathrm{sh} y / p_T \sim \mathrm{sh} y \sim y$ (note that ϑ is the angle between the direction of the photon and the velocity of the radiating particle). For a pion with $E \sim 0.3$—0.4 GeV we have $\gamma \sim 2$—3. For the sake of definiteness we shall take as the central region (dominantly contributing to the bremsstrahlung at $y \sim 0$) the rapidity interval $-1/2 < y < 1/2$.

The bremsstrahlung of particles from this interval is given as

$$\omega \frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2} \sigma_h(N^+, N^-) \int \left| \sum Q_i a_i \right|^2 P_{\pi^+}(p_1, \ldots p_{N^+}) P_{\pi^-}(q_1, \ldots q_{N^-}) * \Pi \mathrm{d}^3 p_i \mathrm{d}^3 q_i, \tag{3}$$

where

$$a_i = \frac{\boldsymbol{v}_i \times \boldsymbol{n}}{\mathrm{i}\omega(1 - \boldsymbol{v}_i . \boldsymbol{n})} \tag{4}$$

and in contradistinction to Eq. (2) we consider now an event with N_+ positive and N_- negative particles in the rapidity interval $(-1/2, 1/2)$, other particles being irrelevant for the argument.

The square of the amplitude entering Eq. (3) is

$$\left| \sum_i Q_i a_i \right|^2 = \sum_i |a_i|^2 + 2 \sum_{i<j} Q_i Q_j \mathrm{Re}(a_i a_j^*). \tag{5}$$

Suppose now that there are no correlations between particles in the final state. In that situation the probablities P_{π^+} and P_{π^-} in Eq. (3) factorize. After averaging over the events (over P_{π^+} and P_{π^-}) we obtain

$$\left\langle \left| \sum_i Q_i a_i \right|^2 \right\rangle = \sum_i \langle |a_i|^2 \rangle + 2 \sum_{i<j} Q_i Q_j \mathrm{Re}(\langle a_i \rangle \langle a_j^* \rangle). \tag{6}$$

Since $\langle a_i \rangle = 0$, the second term vanishes and we are left with the result

$$\omega \frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2 \omega^2} \sigma_h(N^+, N^-) \Delta y \frac{\mathrm{d}N_{ch}}{\mathrm{d}y} \left\langle \left| \frac{\boldsymbol{v} \times \boldsymbol{n}}{1 - \boldsymbol{v} . \boldsymbol{n}} \right|^2 \right\rangle. \tag{7}$$

Here $\Delta y = 1$ and the last expression in the r.h.s. has been evaluated in I as

$$\left\langle \left| \frac{\boldsymbol{v} \times \boldsymbol{n}}{1 - \boldsymbol{v}.\boldsymbol{n}} \right|^2 \right\rangle = \int d^2 p_T P(p_T) \left| \frac{\boldsymbol{v}(\boldsymbol{p}_T) \times \boldsymbol{n}}{1 - \boldsymbol{v}(\boldsymbol{p}_T).\boldsymbol{n}} \right|^2 = 4R \sim 1.7 \tag{8}$$

As a final result for no correlations we have

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2\omega^2} \sigma_h(N^+, N^-) N_{ch} 1.7, \tag{9}$$

where $N_{ch} = (dN_{ch}/dy)\Delta y$.

In the situation with correlations the amplitude averaged over events becomes

$$\left\langle \left| \sum_i Q_i a_i \right|^2 \right\rangle = \sum |a_i|^2 + 2 \sum_{i<j} Q_i Q_j \operatorname{Re}\langle a_i a_j^* \rangle, \tag{10}$$

where

$$\operatorname{Re}\langle a_i a_j^* \rangle = \int d^3p_1 d^3p_2 a(\boldsymbol{p}_i) a^*(\boldsymbol{p}_j) P_2(\boldsymbol{p}_1, \boldsymbol{p}_2) \tag{11}$$

and

$$P_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = \int P(\boldsymbol{p}_1, \boldsymbol{p}_2, \dots \boldsymbol{p}_N) d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4 \dots d^3\boldsymbol{p}_N$$

for both P_{π^+} and P_{π^-}.
Eq. (11) shows that the bremsstrahlung is influenced only by two — body correlations and that a realistic estimate can be obtained by taking the experimental information on the Bose – Einstein correlations. The experimental $\pi^+\pi^+$ and $\pi^-\pi^-$ Bose – Einstein correlation functions can be roughly described as

$$P_2(\boldsymbol{p}_1, \boldsymbol{p}_2) = P(\boldsymbol{p}_1) P(\boldsymbol{p}_2) C(\boldsymbol{p}_1, \boldsymbol{p}_2), \tag{12}$$

with

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = c\left[1 + a \exp\left[-\frac{(\boldsymbol{p}_1 - \boldsymbol{p}_2)^2}{2\sigma^2}\right]\right], \tag{13}$$

with $c \sim 1$, $a \sim 1$ and $\sigma \sim 100$ MeV/c.
The first term in the r.h.s. of Eq. (9) is not affected by correlations and leads to the bremsstrahlung production given by Eq. (9). The second term gives the bremsstrahlung enhancement due to the Bose – Einstein correlations

$$\omega \frac{d\sigma^\gamma_{BE}}{d^3k} = \frac{\alpha}{4\pi^2\omega^2} \sigma_h(N_+, N_-) \left(\frac{1}{2} N_+(N_+ - 1) + \frac{1}{2} N_-(N_- - 1)\right) I_{12}, \tag{14}$$

where

$$I_{12} = ca \int P(\boldsymbol{p}_1) P(\boldsymbol{p}_2) \exp\left(-\frac{(\boldsymbol{p}_1 - \boldsymbol{p}_2)^2}{2\sigma^2}\right) \left(\frac{\boldsymbol{v}_1 \times \boldsymbol{n}}{1 - \boldsymbol{v}_1.\boldsymbol{n}}\right) \left(\frac{\boldsymbol{v}_2 \times \boldsymbol{n}}{1 - \boldsymbol{v}_2.\boldsymbol{n}}\right) d^3p_1 d^3p_2. \tag{15}$$

To estimate the r.h.s. in Eq. (14) we first put $N_+ = N_- = N_{ch}/2$, $c \sim a \sim 1$ and evaluate roughly Eq. (15) in a simplified situation when both pions have $y = 0$ and a constant velocity equal to $v = \langle p_T \rangle / \langle E \rangle$. In such a simplified estimate we have

$$I_{12} = \frac{1}{2\pi} \frac{1}{2\pi} \int_0^{2\pi} d\Phi_1 \int_0^{2\pi} d\Phi_2 \frac{v^2 \sin\Phi_1 \sin\Phi_2}{(1 - v\cos\Phi_1)(1 - v\cos\Phi_2)} * \\ *[-2(1 - \cos(\Phi_1 - \Phi_2))/\Delta^2] \tag{16}$$

with $\Delta = \sqrt{2}\,\sigma/p \sim \sqrt{2}.100$ MeV/400 MeV ~ 0.35. A simple numerical estimate gives $I_{12} \sim 0.17$, which leads to

$$\omega \frac{d\sigma^\gamma_{BE}}{d^3k} = \frac{\alpha}{4\pi^2\omega^2} \sigma_h(N_+, N_-) \frac{1}{4} N_{ch}(N_{ch} - 2) 0.17. \tag{17}$$

The comparison with Eq. (9) shows that the ratio

$$\frac{\text{bremsstrahlung enhancement due to BE}}{\text{bremsstrahlung without BE}} \sim 0.1 \frac{N_{ch}}{4}, \tag{18}$$

which shows that the BE correlations cannot explain the factor of 4—5 of the soft photon enhancement [3, 4].

Because of that, the quadratic dependence of the bremsstrahlung enhancement on dN_{ch}/dy cannot be responsible for the experimentally observed steeper than linear dependence of very soft photons on dN_{ch}/dy.

III. $\pi^+\pi^-$ CORRELATIONS

The Bose – Einstein correlations concern only $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs. Because of that Eq. (3) contained no $\pi^+\pi^-$ correlations. But such correlations are present in the data and they are rather strong. Their presence is not surprising either one expects for instance that $\pi^+\pi^-$ coming from the ϱ^0 decay will have strong back-to-back correlations.

The purpose of the present section is to show that such back-to-back correlations of $\pi^+\pi^-$ lead to the enhancement of the bremsstrahlung relative to the case of the uncorelated $\pi^+\pi^-$. The effect is easy to understand. In the case of uncorrelated pions, π^+ and π^- can have sometimes similar momenta $\boldsymbol{p}_+, \boldsymbol{p}_-$ and because of opposite charges, their bremsstrahlung is suppressed. For a back-to--back correlation this destructive interference never happens.

In order to obtain a rough idea of the magnitude of the effect we shall consider a very simplified situation. Suppose that we are observing a bremsstrahlung photon at $y = 0$ in a direction perpendicular to the beam. The cross-

-section for the bremsstrahlung of π^+ and π^- both at $y = 0$ and produced with the same $v = |\mathbf{v}_{\pi^-}| = \langle p_T \rangle / (\langle p_T \rangle^2 + m_\pi^2)^{1/2}$ is

$$\omega \frac{d\sigma^\gamma}{d^3k} = \sigma^0 \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \left| \frac{Q_+ \mathbf{v}_{\pi^+} \times \mathbf{n}}{1 - \mathbf{v}_{\pi^+} . \mathbf{n}} + \frac{Q_- - \mathbf{v}_{\pi^-} \times \mathbf{n}}{1 - \mathbf{v}_{\pi^-} . \mathbf{n}} \right|^2 . \tag{19}$$

If $\pi^+ \pi^-$ are uncorrelated, the cross-section should be averaged over the directions of pions in the plane perpendicular to the beam axis. In this way we obtain

$$\omega \frac{d\sigma^\gamma}{d^3k} = \sigma^0 \frac{\alpha}{4\pi^2} \frac{v^2}{\omega^2} I_{no\text{-}corr}$$

$$I_{no\text{-}corr} = \frac{1}{(2\pi)^2} \iint d\Phi d\Phi' \left| \frac{\sin \Phi}{1 - v \cos \Phi} - \frac{\sin \Phi}{1 - v \cos \Phi'} \right|^2 d\Phi d\Phi' . \tag{20}$$

A simple calculation gives

$$I_{no\text{-}corr} = \frac{2}{2\pi} \int_0^{2\pi} d\Phi \frac{\sin^2 \Phi}{(1 - v \cos \Phi)^2} = \frac{2}{v^2} (\gamma - 1).$$

For the case of a complete back-to-back correlation the cross-section is given again by Eq. (20) with $I_{no\text{-}corr}$ replaced by

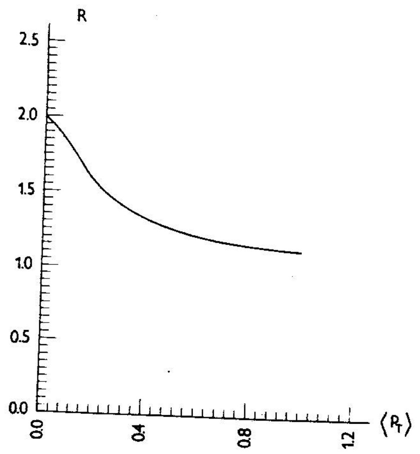

Fig. 1. The dependence of the ratio $R = I_{corr} / I_{no\text{-}corr}$ on the mean transverse momentum $\langle p_T \rangle$.

$$I_{corr} = \frac{1}{2\pi} \int_0^{2\pi} d\Phi \left[\frac{\sin \Phi}{1 - v \cos \Phi} - \frac{\sin (\Phi + \pi)}{1 - v \cos (\Phi + \pi)} \right]^2 \tag{21}$$

and a simple calculation gives

$$I_{corr} = \frac{4}{2\pi} \int_0^{2\pi} \frac{\sin^2 \Phi}{(1 - v^2 \cos^2 \Phi)^2} d\Phi = 2\gamma.$$

The ratio $R = I_{corr} / I_{no\text{-}corr} = 1 + \gamma^{-1}$ depends on a single parameter $v = \langle p_T \rangle / (\langle p_T \rangle^2 + m_\pi^2)^{1/2}$. This dependence as a function of $\langle p_T \rangle$ is shown in Fig. 1. For realistic values of $\langle p_T \rangle \sim 400$ MeV the ratio is about 1.3. In general the ratio is decreasing with increasing $\langle p_T \rangle$. Note that our simplified model probably overestimates the effect.

In realistic situation one could thus expect about 20% of the bremsstrahlung enhancement due to the back-to-back correlations of $\pi^+ \pi^-$ (and of other charged particles) caused by the resonance decays. A more detailed study of this mechanism could be made by using the Monte Carlo models of multiparticle production with the possibility of having pions produced either directly or via decays of neutral resonances.

IV. THE INCREASE OF $\langle p_T \rangle$ WITH dN_{ch}/dy

The amount of the bremsstrahlung by a particle accelerated abruptly to velocity v (created with velocity v) increases with v. Because of this an increase of $\langle p_T \rangle$ of secondary particles in hadronic collisions leads also to an increase in the bremsstrahlung emission. The available experimental data [7] give evidence of the increase of $\langle p_T \rangle$ with a charged multiplicity.

In this section we shall give some rough qualitative estimates of this effect. For the sake of simplicity we shall consider only the radiation emitted at $y = 0$ by final state particles with $y = 0$. The bremsstrahlung emission in this case is given as

$$\omega \frac{dN}{d^3k} = \frac{\alpha}{4\pi^2 \omega^2} \left\langle \left| \frac{\mathbf{v} \times \mathbf{n}}{1 - \mathbf{v} . \mathbf{n}} \right|^2 \right\rangle \tag{22}$$

where we average over all directions and magnitudes of $\mathbf{v}$ in the (x, y) plane, with the z-axis parallel to the beam direction.

The averaging can be done by using the phenomenological p_T-distribution function

$$P(p_T) p_T dp_T = A^2 e^{-Ap_T} p_T dp_T \tag{23}$$

with $A = 2/\langle p_T \rangle$ and $v = p_T / (p_T^2 + m_\pi^2)^{1/2}$.
The bremsstrahlung emission for one final state pion then becomes

$$\omega \frac{dN^\gamma}{d^3k} = \frac{\alpha}{4\pi^2 \omega^2} I(\langle p_T \rangle)$$

$$I(\langle p_T \rangle) = \int_0^\infty A^2 p_T e^{-Ap_T} dp_T \int_0^{2\pi} \frac{d\Phi}{2\pi} \frac{v^2(p_T) \sin^2 \Phi}{(1 - v(p_T) \cos \Phi)^2}, \tag{24}$$

where

$$\int_0^{2\pi} \frac{\mathrm{d}\Phi}{2\pi} \frac{v^2 \sin^2 \Phi}{(1 - v\cos\Phi)^2} = \gamma - 1 = \frac{(m_\pi^2 + p_T^2)^{1/2}}{m_\pi} - 1 \doteq \frac{p_T}{m_\pi}.$$

In Fig. 2 we present the dependence of $I(\langle p_T \rangle)$ on $\langle p_T \rangle$. An interesting point is that the dependence of $I(\langle p_T \rangle)$ on $\langle p_T \rangle$ is practically linear.

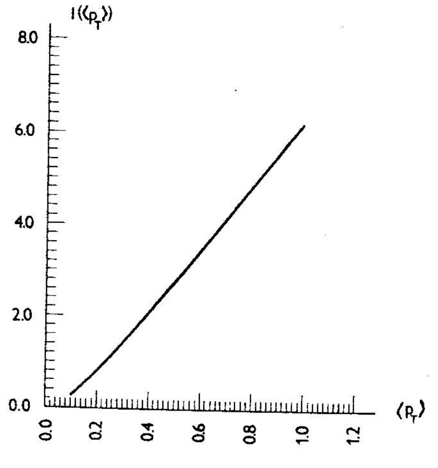

Fig. 2. The dependence of bremsstrahlung emission on the average transverse momentum of charged pions at $y = 0$. $I(\langle p_T \rangle)$ is defined in Eq. (24).

The available data [6, 7] on the dependence of $\langle p_T \rangle$ on $\mathrm{d}N_{ch}/\mathrm{d}y$ show an increase of $\langle p_T \rangle$ with $\mathrm{d}N_{ch}/\mathrm{d}y$. The effect is observed at the highest ISR [7] and at the SppS collider [6] energies.

As a net result we obtain a *faster than linear* dependence of the bremsstrahlung emission on $\mathrm{d}N_{ch}/\mathrm{d}y$. This is seen from Eq. (24) multiplied by the number of the radiating particles

$\Delta y\, \mathrm{d}N_{ch}/\mathrm{d}y$ (compare with Eq. (7))

$$\omega \frac{\mathrm{d}N^\gamma}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2 \omega^2} \Delta y \frac{\mathrm{d}N_{ch}}{\mathrm{d}y} I(\langle p_T \rangle). \tag{25}$$

The linear term comes from the proportionality of the bremsstrahlung emission to $\mathrm{d}N_{ch}/\mathrm{d}y$ and an additional power dependence comes from the increase of $\langle p_T \rangle$ with $\mathrm{d}N_{ch}/\mathrm{d}y$ and thereby from the increase of $I(\langle p_T \rangle)$ with $\mathrm{d}N_{ch}/\mathrm{d}y$.

The effect deserves a careful study, but it seems that the increase of $\langle p_T \rangle$ with $\mathrm{d}N_{ch}/\mathrm{d}y$ is too small to give anything similar to the quadratic dependence as observed by the AFS group [5] for virtual γ's and by the Helios collaboration for soft real γ's.

A comment is in order. In the paper by Černý et al. [8] the authors claimed that a quadratic dependence of soft (virtual or real) photons on $\mathrm{d}N_{ch}/\mathrm{d}y$

is a clear sign of the origin of photons from the intermediate stage of the collision. The argument contains a tacit assumption that if there is no intermediate stage the radiation of soft photons by a particular final state hadron is not influenced by the presence of other hadrons in the final state. The correlation between $\langle p_T \rangle$ and $\mathrm{d}N_{ch}/\mathrm{d}y$ is a clear example of such a correlation, since a larger $\langle p_T \rangle$ means more bremsstrahlung. This correlation is still some kind of a collective effect, which leads to a faster than linear dependence of the bremsstrahlung production on $\mathrm{d}N_{ch}/\mathrm{d}y$.

V. CHARGE TRANSFER DEPENDENCE ON $\mathrm{d}N_{ch}/\mathrm{d}y$

An approximate formula for the bremsstrahlung emission in multiparticle production has been derived by Rückl [9] (see also Eq. (6) in I). The formula expresses the overall bremsstrahlung emission as a sum of the radiation of particles with a rapidity close to that of the photon and the radiation due to the charge transfer between forward and backward moving particles. The formula reads as follows

$$\omega \frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}^3 k} = \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \left\{ 4\langle \Delta Q^2 \rangle + 4R \frac{\mathrm{d}N_{ch}}{\mathrm{d}y} \right\}, \tag{26}$$

where the value of Rückl's constant R is about 0.4 (see I). In fact Eq. (26) is written in a somewhat oversimplified form. In a more detailed way we should sum over semiinclusive hadronic production with a specified value of $\mathrm{d}N_{ch}/\mathrm{d}y$ and make $\langle \Delta Q^2 \rangle$ and R dependent on $\mathrm{d}N_{ch}/\mathrm{d}y$. As discussed in detail in the preceding section R depends on $\mathrm{d}N_{ch}/\mathrm{d}y$ because of the dependence of $\langle p_T \rangle$ on $\mathrm{d}N_{ch}/\mathrm{d}y$.

In this section we shall discuss the dependence of $\langle \Delta Q^2 \rangle$ on $\mathrm{d}N_{ch}/\mathrm{d}y$.

The charge transfer across $y_{c.m.} = 0$ has been studied in detail in bubble chamber experiments [10—15] and analysed in cluster [16—19] and parton models [20, 21]. The data show that [13] $\langle \Delta Q^2 \rangle$ is increasing roughly linearly with $\ln(s)$ and that at a fixed s $\langle \Delta Q^2 \rangle$ increases linearly [12] with the total charge miltiplicity N_{ch}. The data [12, 23, 24, 25] seem to suggest a linear increase of $\langle \Delta Q^2 \rangle$ with $(\mathrm{d}N_{ch}/\mathrm{d}y)_{y=0}$ and the same conclusion seems to follow also from neutral cluster [17, 18, 19] and parton models [20, 21]. The conjecture should be however checked in more detail in particular for high values of $(\mathrm{d}N_{ch}/\mathrm{d}y)_{y=0}$.

Moreover, as discussed in detail in I, the relevant variable entering Eq. (26) and denoted as $\langle \Delta Q^2 \rangle$ is probably not the charge transfer across $y = 0$ but across a central rapidity interval, roughly of the size $-0.5 < y < 0.5$

It seems therefore that the term with $\langle \Delta Q^2 \rangle$ in Eq. (26) leads to a linear dependence of the bremsstrahlung emission on $(\mathrm{d}N_{ch}/\mathrm{d}y)_{y=0}$, but the issue deserves a more detailed experimental and theoretical study.

VI. COMMENTS AND CONCLUSIONS

We have studied here the effects of correlations between final state particles on the bremsstrahlung emission. The correlations studied included:
i) the Bose–Einstein correlations between identical pions,
ii) short-range correlations between π^+ and π^- corresponding to correlations caused by $\pi^+\pi^-$ decays of neutral resonances,
iii) dependence of $\langle p_T \rangle$ on $(dN_{ch}/dy)_{y=0}$, and
iv) dependence of the charge transfer $\langle \Delta Q^2 \rangle$ across $y = 0$ on $(dN_{ch}/dy)_{y=0}$.
Neither of these correlations can explain the discrepancy between the data on very soft photon production in K^+p interactions at 70 GeV/c [3] and the calculated emission of the bremsstrahlung by final state hadrons, since all the correlations are already included there.

Neither of these effects can explain the full discrepancy of factor 4 between a very soft photon production and the bremsstrahlung calculations in pBe and pAl collisions at 450 GeV/c [4]. The effects can however enhance the theoretical results by some 30% or so. If this could be increased by some additional effects to 100%, the data would be satisfied within 2σ (see the discussion of the data in the introduction of I).

Another interesting aspect of data is a faster than linear dependence of a very soft photon production on $(dN_{ch}/dy)_{y=0}$ observed by the Helios collaboration [4]. Out of the effects discussed, i) and iii) do lead to a faster than linear dependence. For ii) a faster than linear dependence would be obtained only if the ratio of neutral resonances to directly produced pions increased with $(dN_{ch}/dy)_{y=0}$.

The BE correlations lead to only a small (10%) enhancement of an uncorrelated production. A larger effect can be expected from item iii), which deserves a more detailed experimental and theoretical study. A both experimental and theoretical understanding of item iv) is also desirable, although the mechanism is not expected to bring anything very surprising.

ACKNOWLEDGEMENTS

One of the authors (J. P.) would like to thank the CERN Theory Division for hospitality during the time when a part of the present work was carried out. We are indebted to V. Černý, C. Fabjan, J. Ftáčnik, D. Jackson, P. Lichard, L. Van Hove, W. Willis and G. M. Zinovjev for comments and discussions.

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Received June 8th, 1989
Accepted for publication November 18th, 1989

ПОИСК МЕХАНИЗМА УСИЛЕНИЯ ЭМИССИИ ТОРМОЗНОГО ИЗЛУЧЕНИЯ В РЕАКЦИЯХ АДРОНОВ
II. КОРРЕЛЯЦИИ В ФИНАЛЬНОМ СОСТОЯНИИ

В продолжении предыдущей статьи изучаются возможные эффекты влияния корреляций финальных состояний адронов на эмиссию тормозного излучения. Детально изучаются: 1) корреляции Бозе–Эйнштеина, 2) корреляции между π^+ и π^- на коротком расстоянии, 3) зависимость $\langle p_T \rangle$ на множественности заряда, 4) зависимость переноса заряда на множественности.

В случаях 1) и 3) корреляции приводят к усиленной, в сравнении с линейной зависимостью эмиссии тормозного излучения на множественности. Ни один из эффектов не позволяет пояснить завышение очень мягких фотонов во взаимодействии K^+p при 70 ГэВ/с, где корреляции учтены. Эффекты оказываются также малыми для пояснения эмиссии очень мягких фотонов при взаимодействиях pBe и pA при энергии 450 ГэВ/с.