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A SEARCH OF A MECHANISM RESPONSIBLE FOR BREMSSTRAHLUNG ENHANCEMENT IN HADRONIC REACTIONS
I. SHOCK WAVES AND ESCAPING QUARKS

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We study three mechanisms which could effectively lead to the enhancement of bremsstrahlung in hadronic collisions: i) shock waves in the hadron gas formed in the collision, ii) bremsstrahlung of the "electric" type emitted by a quark escaping from the intermediate system formed in such a collision and iii) bremsstrahlung of the "magnetic" type within this approach.

We come to the conclusion that these mechanisms cannot explain the enhancement of very soft photon production observed in hadronic and hadron-nucleus collisions.

I. INTRODUCTION

A few years ago Chliapnikov et al. [1] found an enhanced production of very soft photons in K^+p interactions at 70 GeV/c. They observed prompt photons with $p_T < 40$ MeV/c, concentrated at very low longitudinal photon momenta in the c.m. system: $-0.001 \leqslant x \leqslant 0.008$. The enhancement was consistent with the contribution from hadronic bremsstrahlung, but larger in size by a factor of about four.

The Helios (NA-34) collaboration at CERN have recently found [2] a similar very soft photon enhancement in 450 GeV/c p-Be and p-Al interactions. The excess photons have been observed in the range of 4 MeV/c $\leqslant p_T \leqslant$ 20 MeV/c at $y = y_{c.m} = 0$.

As an important additional information concerning the results of Ref. [1] the Helios collaboration have discovered that very soft photon production depends steeper than linearly (perhaps quadratically) on the associated charged multiplicity at $y \sim 0$. The p_T— spectrum of very soft photons is again similar in shape to the bremsstrahlung from final state hadrons but larger in size by a factor similar as in Ref. [1].

In this paper we shall refer to photons with $p_T \leqslant 20$—40 MeV/c as to very soft photons, in contradistinction to the soft ones corresponding to $100 < p_T < 400$ MeV/c.

The results of Refs. [1, 2], if taken literally, are a sort of a conundrum. The faster than linear (perhaps quadratic) dependence on the associated charged multiplicity indicates that very soft photons might be radiated from the intermediate stage of the collision [3]. On the other hand the Low theorem [4] (see also the recent analysis by Andersson et al. [5]) shows that it is very difficult to produce very soft photons from the intermediate stage.

In particular the uncertainty relation requires that the time for emitting a photon with energy ω is of the order of $t \sim 1/\omega$ which makes[1]) $t \sim 10$ fm/c for $\omega \sim 20$ MeV. In the same way the transverse dimension of the system emitting a photon with $p_T \sim 20$ MeV/c should be of the order of 10 fm.

In a more quantitative way this uncertainty relation is built into the Landau Pomeranchuk [7] mechanism (for a good review see [8]) which might be considered as a classical analogy of the limitations imposed by the Low theorem, applicable when the particles in the intermediate stage can be treated classically.

Perhaps the conflict between the data and the theoretical expectations only indicates that there is something wrong with the data. In fact, the Chliapnikov et al. experiment is challenged by an earlier experiment by Goshaw et al. [15], where π^+p collisions at 10.5 GeV/c were studied by the same technique but no extra very soft photons were found. Moreover, the Chliapnikov et al. result, as well as the better part of the Helios results (the one coming from the p-Be experiment) is only about 3.5σ effect. Consequently, multiplying the theoretical values by a factor of 2 would be enough to comply with these experiments essentially within 2σ.

Although the data on the enhancement of the production of very soft photons are not very convincing, there is still the problem of how such an enhancement could possibly be explained. This question can be dealt with in two different ways: one can try to improve the approximation of the standard bremsstrahlung formula by including correlations between charged hadrons in the final state, or one can go beyond the bremsstrahlung formula and look for

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[1]) A possible formation of a very large intermediate parton system in hadronic collision consisting of very soft partons has been recently proposed by Van Hove [6]. We shall not discuss here this alternative possibility of understanding very soft photon production in hadronic reactions.

new physical mechanisms for the production of extra photons during the collision or after it. We shall explore the former approach in the following paper, and here we confine ourselves to the latter.

The purpose of the present paper is to study three possible candidates for explaining the enhancement of a very soft photon production in hadronic and hadron-nucleus production, namely:

i) phenomena similar to shock waves in the hadron gas formed in hadronic collisions,

ii) bremsstrahlung of the "electric" type emitted by a quark escaping from the intermediate system formed in such a collision and pulled back by the chromoelectric string [9, 10] between the coloured quark and the rest of the partonic system,

iii) bremsstrahlung of the "magnetic" type emitted by the quark trying to escape from the intermediate system in the same scenario.

We are aware that the mechanisms suggested here are not realistic. This is especially true about the scenarios with escaping quarks requiring the existence of strings of the length of about 10 fm since before a string is pulled so far it should break into quarks and antiquarks. The point in investigating these scenarios is to show that even the unrealistic models of the intermediate state do not work.

The paper is organized as follows. In Sect. 2 we review the basic formulae for bremsstrahlung emission. The three candidates for the enhancement of bremsstrahlung are discussed in Sect. 3 and 4. Our (negative) conclusions are summarized in Sect. 5. Some additional material concerning the basic formulae for bremsstrahlung is given for the sake of completeness in the Appendix A.

II. GENERAL FORMULAE FOR BREMSSTRAHLUNG EMISSION

The standard expression for the number of bremsstrahlung photons radiated by a classical particle moving along the trajectory $\boldsymbol{r}(t)$ reads as follows[2] [11]

$$\mathrm{d}N = \frac{\alpha}{4\pi^2} |\int \mathrm{d}\boldsymbol{r} \times \boldsymbol{n} e^{i\omega(t - \boldsymbol{n}.\boldsymbol{r}(t))}|^2 \frac{\mathrm{d}^3 k}{\omega}, \tag{1}$$

where $\mathrm{d}N$ is the number of radiated photons, $\boldsymbol{k}$, ω are the momentum and energy of the photon, $\boldsymbol{n} = \boldsymbol{k}/\omega$ is the unit vector along the photon momentum, $\alpha = e^2/4\pi = 1/137$. For the case of a single scattering and for $\omega < 1/\Delta t$, where Δt is the time interval during which the scattering occurs, we obtain easily from Eq. (1)

$$\mathrm{d}N = \frac{\alpha}{4\pi^2}\frac{1}{\omega^2}\left|\frac{\boldsymbol{v}' \times \boldsymbol{n}}{1 - \boldsymbol{v}'.\boldsymbol{n}} - \frac{\boldsymbol{v} \times \boldsymbol{n}}{1 - \boldsymbol{v}.\boldsymbol{n}}\right|^2 \frac{\mathrm{d}^3 k}{\omega}. \tag{2}$$

Here $\boldsymbol{v}$ is the velocity of the radiating particle before and $\boldsymbol{v}'$ after the scattering. Eq. (2) is valid also in relativistic quantum theory, where it can be derived from the Feynman diagrams (the argument can be found in any textbook on quantum electrodynamics, see e.g. [14]).

The physics behind Eq. (2) is rather simple. The term $(\boldsymbol{v} \times \boldsymbol{n})/(1 - \boldsymbol{v}.\boldsymbol{n})$ gives the amplitude for the photon radiation corresponding to the immediate stopping of the incident particle, whereas the term $(\boldsymbol{v}' \times \boldsymbol{n})/(1 - \boldsymbol{v}'.\boldsymbol{n})$ determines the amplitude for the immediate acceleration of the final state particle. The formula for bremsstrahlung in the collision

$$a + b \to \sum c_i,$$

where a, b are the initial and c_i the final state particles, can be easily written as

$$\mathrm{d}N = \frac{\alpha}{4\pi^2}\frac{1}{\omega^2}\left|\sum_i Q_i \frac{\boldsymbol{v}_i \times \boldsymbol{n}}{1 - \boldsymbol{v}_i.\boldsymbol{n}} - Q_a \frac{\boldsymbol{v}_a \times \boldsymbol{n}}{1 - \boldsymbol{v}_a.\boldsymbol{n}} - Q_b \frac{\boldsymbol{v}_b \times \boldsymbol{n}}{1 - \boldsymbol{v}_b.\boldsymbol{n}}\right|^2 \frac{\mathrm{d}^3 k}{\omega}, \tag{3}$$

where Q_i, Q_a, Q_b are charges of the corresponding particles in units of the elementary charge e.

In order to obtain the cross-section the right-hand side of Eq. (3) has to be multiplied by the hadronic cross section $\mathrm{d}\sigma/\mathrm{d}^3p_1\mathrm{d}^3p_2 \ldots \mathrm{d}^3p_n$ and integrated over momenta of the final state hadrons.

For the situation as in Ref. [2], when one measures the p_T-dependence of very soft photons at $y = 0$, Eq. (3) can be simplified [12]. We shall give here a simplified derivation of the formula which is perhaps crude but leads to a qualitatively correct result. One divides particle in the final state into two groups. The former contains those with $y > y_0$ and those with $y < -y_0$, the latter contains particles "close" to $y = 0$, that is $-y_0 < y < y_0$. This division is admittedly rather arbitrary and the idea is worth checking by Monte Carlo simulations. Particles with $y > -y_0$ and with $y < -y_0$ are supposed to be moving forward or backward (along the beam axis). Since one measures photons in the direction perpendicular to the beam axis, $\boldsymbol{v}_i\boldsymbol{n} = 0$ for particles in this group. Since the magnitude of velocities in this group $|\boldsymbol{v}_i| \sim 1$, one writes, still for this group,

[2]) Note that in Ref. [11], chapter 14, instead of $\mathrm{d}\boldsymbol{r} \times \boldsymbol{n}$ there is $(\mathrm{d}\boldsymbol{r} \times \boldsymbol{n}) \times \boldsymbol{n}$. For intensity of radiation both formulae are equivalent. The amplitude in the latter formula gives also the correct polarization of the photon.

this group. Since the magnitude of velocities in this group $|\mathbf{v}_i| \sim 1$, one writes, still for this group,

$$\sum_i{}' Q_i \frac{\mathbf{v}_i \times \mathbf{n}}{1 - \mathbf{v}_i \cdot \mathbf{n}} - Q_a \frac{\mathbf{v}_a \times \mathbf{n}}{1 - \mathbf{v}_a \cdot \mathbf{n}} - Q_b \frac{\mathbf{v}_b \times \mathbf{n}}{1 - \mathbf{v}_b \cdot \mathbf{n}} \sim$$
$$\sim -\mathbf{v}_a \times \mathbf{n}\{(Q_a - Q_a') - (Q_b - Q_b')\},$$

where Q_a' (Q_b') is the total charge of particles moving after the collision in the direction of particle a (b). The prime above $\sum$ indicates that the sum proceeds only over particles with $|y| > y_0$. Denoting $\Delta Q(y_0) = Q_a - Q_a'$ and assuming the neutrality of particles within $-y_0 < y < y_0$, leading to $Q_b - Q_b' = -\Delta Q(y_0)$, we obtain

$$\sum_i Q_i \frac{\mathbf{v}_i \times \mathbf{n}}{1 - \mathbf{v}_i \cdot \mathbf{n}} - Q_a \frac{\mathbf{v}_a \times \mathbf{n}}{1 - \mathbf{v}_a \cdot \mathbf{n}} - Q_b \frac{\mathbf{v}_b \times \mathbf{n}}{1 - \mathbf{v}_b \cdot \mathbf{n}} \sim -(\mathbf{v}_a \times \mathbf{n}) 2\Delta Q(y_0).$$

Note that $|\mathbf{v}_a \times \mathbf{n}| \sim 1$.
The amplitude from particles with $-y_0 < y < y_0$ depends very strongly on their momenta. Assuming further that contributions of these particles are, after averaging over events, mutually incoherent and also incoherent with contributions from particles with $|y| > y_0$, we obtain from Eq. (3) the following approximate expression

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \sigma_{hadr} \left\{ 4 < (\Delta Q(y_0))^2 > + 4R < \frac{dN_{ch}}{dy} > \Delta y \right\}, \tag{4}$$

where $\Delta y = y_0 - (-y_0) = 2y_0$ and

$$4R = \frac{1}{\Delta y} \int_{-y_0}^{y_0} dy \int d^2p_T P(\mathbf{p}_T) \left| \frac{\mathbf{v}(y, \mathbf{p}_T) \times \mathbf{n}}{1 - \mathbf{v}(y, \mathbf{p}_T) \cdot \mathbf{n}} \right|^2 \tag{5}$$

where $\mathbf{v}(y, \mathbf{p}_T)$ denotes the velocity of the final state particle with y and $\mathbf{p}_T$ and $P(\mathbf{p}_T)$ is the properly normalized $\mathbf{p}_T$-distribution, $\int d^2p_T P(p_T) = 1$. Taking now as definite $y_0 = 1/2$ we have

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \sigma_{hadr} \left\{ 4 < (\Delta Q)^2 > + 4R < \frac{dN_{ch}}{dy} > \right\}. \tag{6}$$

In practical applications one often assumes that the charge transfer over $(-y_0, y_0)$ is the same as over $y = 0$. As it follows from the discussion the formula is based on numerous assumptions which should better be tested by numerical studies. In particular it neglects the correlations of particles in the final state. It might be, however, suitable for a qualitative discussion of the influence of various effects and we shall use it below in this sense.

Rückl's constant R [12] is estimated as follows. Take the beam direction as the z axis and the direction of the photon as the x axis. The velocity of the charged particle becomes $\mathbf{v} = (p_T \cos\Phi/E, p_T \sin\Phi/E, p_z/E)$, the energy being $E = (p_T^2 + m^2)^{1/2} \mathrm{ch}y$. Then

$$|\mathbf{v} \times \mathbf{n}|^2 = v_z^2 + v_y^2 = (p_z^2 + p_y^2)/E^2$$
$$\mathbf{v} \cdot \mathbf{n} = p_T \cos\Phi/E,$$

where Φ is the azimutal angle of $\mathbf{v}$ in the (x, y) plane. The expression under the integral in Eq. (5) then becomes

$$\left| \frac{\mathbf{v} \times \mathbf{n}}{1 - \mathbf{v} \cdot \mathbf{n}} \right|^2 = \frac{(m^2 + p_T^2 \mathrm{sh}^2 y) + p_T^2 \sin^2\Phi}{[\sqrt{m^2 + p_T^2}\,\mathrm{ch}y - p_T \cos\Phi]^2}. \tag{7}$$

A realistic $\mathbf{p}_T$ distribution is

$$P(\mathbf{p}_T) = P(p_T, \Phi) = \frac{1}{2\pi} A^2 e^{-Ap_T}, \quad A = 6[\mathrm{GeV/c}]^{-1} \tag{8}$$

where we integrate over $dp_T d\Phi$. Inserting Eqs. (7) and (8) into Eq. (5) we obtain $4R \sim 1.76$, which gives $R \sim 0.44$. The value of R depends on the shape of $P(\mathbf{p}_T)$. The distribution in Eq. (8) corresponds to $\langle p_T \rangle = 2/A = 0.33$ GeV/c. For $\langle p_T \rangle = 0.4$ GeV/c we have $A = 5[\mathrm{GeV/c}]^{-1}$ and we get $R \sim 0.54$. In the original paper by Rückl [12] the quoted value of R has been 0.35, which is somewhat lower than our results. A detailed study of this point requires accurate information about experimental p_T-distributions.

III. BREMSSTRAHLUNG ENHANCEMENT DUE TO THE SHOCK WAVE LIKE MECHANISM

The term "shock-wave-like mechanism" will be understood in this section in a very wide sense as any mechanism in which the transverse velocity of final state particles is rapidly changed during the space-time evolution of the hadronic collision. As a typical situation we shall consider the following one. Suppose that when final state hadrons are created some collective mechanism gives to all of them an additional collective velocity v_c (the velocity of the shock wave) in addition to their individual velocities. The collective mechanism stops at time t_1 and after that time hadrons have the velocity distribution which is observed in the final state. Thus a time dependence of the velocity of a hadron becomes

$$v(t) = v_2 \qquad \text{for } t > t_1$$
$$v(t) = v_1 = \frac{v_2 + v_c}{1 + v_2 v_c} \qquad \text{for } t < t_1. \tag{9}$$

For simplicity we have considered here only a hadron with $y \equiv y_{c.m} = 0$.

We have now to find a formula for the bremsstrahlung radiation of a hadron whose velocity rapidly changes during the space-time evolution of a collision. This is easily done by using Eq. (1). Suppose the hadron is created at $t = 0$ and denote

$$A = \int_0^\infty \mathrm{d}\boldsymbol{r} \times \boldsymbol{n}\, e^{i\omega(t - \boldsymbol{n}.\boldsymbol{r}(t))}, \tag{10}$$

under the conditions specified by Eq. (9) we easily find[3])

$$A = A_1 + A_2$$
$$A_1 = \frac{\boldsymbol{v}_1 \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\boldsymbol{v}_1)}\left[e^{i\omega t_1(1 - \boldsymbol{n}.\boldsymbol{v}_1)} - 1\right] \tag{11}$$
$$A_2 = -\frac{\boldsymbol{v}_2 \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\boldsymbol{v}_2)}\, e^{i\omega t_1(1 - \boldsymbol{n}.\boldsymbol{v}_1)}.$$

Even at this stage it is qualitatively obvious that the second term corresponding to the radiation from the final state particle will dominate. Take a very soft photon with $\omega \sim 0.01$ GeV. In that case

$$|A_2| \sim \left|\frac{\boldsymbol{v}_2 \times \boldsymbol{n}}{1 - \boldsymbol{n}.\boldsymbol{v}_2}\right| \frac{\hbar c}{\omega} \sim \frac{0.2\ \text{GeV fm}}{0.01\ \text{GeV}} \sim 20\ \text{fm}, \tag{12}$$

where we have taken the term containing $\boldsymbol{v}_2$ as being roughly 1.

In the A_1 term for $\omega t_1(1 - \boldsymbol{n}.\boldsymbol{v}_1) < 1$ we can expand the exponent to obtain

$$|A_1| \sim t_1. \tag{13}$$

Unless the shock-wave-like mechanism acts for surprisingly large times comparable to 10—20 fm, the term A_1 will be much smaller than A_2 and the additional radiation due to the shock-wave period will be only a negligible correction, but certainly not an enhancement by a factor of four.

Another trouble is caused by different phases of A_1, A_2. A_2 is purely imaginary, whereas A_1 in the approximation made is real

$$A = A_1 + A_2 \sim t_1 + \mathrm{i}\ 20\ \text{fm}, \tag{14}$$

which means that there is no interference term and $|A|^2 = |A_1|^2 + |A_2|^2$ with $|A_2|^2$ giving a typical bremsstrahlung like contribution and $|A_1|^2$ being flat at low ω.

Quantitative estimates are based on the Eq. (5) with the term $|(\boldsymbol{v} \times \boldsymbol{n})/(1 - \boldsymbol{v}.\boldsymbol{n})|^2$ being replaced by the corresponding term of the amplitude given by Eq. (11). The relationship between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ is specified by Eq. (9) and the final state distribution of $\boldsymbol{v}_2$ is given by the function $P(\boldsymbol{p}_T)$ in the Eq. (8). The results depend on values of t_1 and v_c (see Eq. (9)) but for reasonable values of t_1, say $t_1 < 3$—4 fm/c and v_c of the order of velocity of sound in the free relativistic gas, $v_c \sim v_s \sim \sqrt{1/3}$ the bremsstrahlung enhancement is not substantial.

To show that numerically we shall simplify the calculations by considering only bremsstrahlung by a particle with $y = 0$ when the outgoing photons are also registered at $y = 0$. The beam direction is taken as the z-axis and the direction of the outgoing photon as the x-axis. The azimutal angle of the radiating particle in the x, y plane is denoted as Φ.

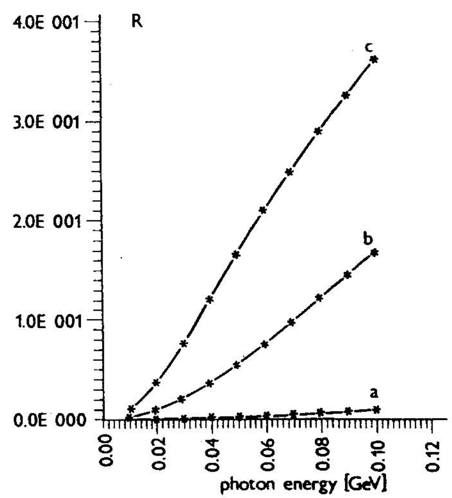

Fig. 1. The ratio R of the additional bremsstrahlung during the shock wave period to the "normal" bremsstrahlung of the final state particle for the shock wave duration a) $t_1 = 1$ fm/c, b) $t_1 = 5$ fm/c, c) $t_1 = 10$ fm/c. The ratio is plotted as a function of the photon energy ω. Note the scale on the vertical axis.

The square of the amplitude A in Eq. (11) can be rewritten as follows

$$|A|^2 = \frac{1}{\omega^2}\frac{(v_2 \sin\Phi)^2}{(1 - v_2\cos\Phi)^2} +$$
$$+ \frac{2v_1\sin\Phi}{\omega^2(1 - v_1\cos\Phi)}\left[\frac{v_1\sin\Phi}{1 - v_1\cos\Phi} - \frac{v_2\sin\Phi}{1 - v_2\cos\Phi}\right][1 - \cos[\omega t_1(1 - v_1\cos\Phi)]], \tag{16}$$

[3]) The generalization of this expression to situations with a larger number of obrupt changes of velocity is given in Appendix A.

where the first term on the right-hand side corresponds to the bremsstrahlung from the final state particle, whereas the second term is an addition due to the shock wave. The contribution of both terms is obtained by multiplying $|A|^2$ by the corresponding v_2 distribution given again by

$$P(p_T, \Phi)\,\mathrm{d}p_T\mathrm{d}\Phi = \frac{1}{2\pi} A^2 \mathrm{e}^{-Ap_T}\mathrm{d}p_T\mathrm{d}\Phi$$

with $A = 6\ (\mathrm{GeV/c})^{-1}$ and $v_2 = p_T/\sqrt{p_T^2 + m_\pi^2}$. The relationship between v_2 and v_1 is given by Eq. (9). Results of calculations are shown in Fig. 1, where we plot the ratio of the second term on the r.h.s. of Eq. (16) to the first term in the r.h.s. of Eq. (16).

The results show that the ratio is increasing with ω and it is rather small. The fastest increase naturally apprears for the case $t_1 = 10$ fm/c, which is also expected from the uncertainty relation.

IV. BREMSSTRAHLUNG OF A QUARK TRIYNG TO ESCAPE FROM THE INTERMEDIATE SYSTEM

In this section we shall study a possible enhancement of bremsstrahlung emitted by a quark escaping from the intermediate partonic system formed in hadronic collision and pulled back by the confining force. This mechanism has been proposed in Ref. [9] as a possible signature of the formation of the quark-gluon-plasma (QGP). In Ref. [9] the authors have studied the emission of hard photons, with energies, say above 0.5 GeV. Here we are concerned with another question: whether this mechanism can lead to the enhancement of very soft photons. In the original Ref. [9] the authors have used standard formulae for "magnetic" bremsstrahlung emitted by particles moving in circular accelerators.

These standard formulae are applicable for photon frequencies $\omega > \omega_0$, where ω_0 is the cyclotron frequency, $\omega_0 \sim c/R$, where R is the radius of the charged particle trajectory. Suppose now that the escaping quark moves along a roughly circular path with $R \sim 2$ fm. Then, $\hbar\omega_0 \sim \hbar c/R \sim 0.2$ GeV fm/ /2 fm $\sim$ 100 MeV.

Standard magnetic bremsstrahlung formulae are therefore not applicable to very soft photons and we have to start from the very beginning, namely from Eq. (1).

We shall discuss here first the "electric bremsstrahlung": a quark escapes with the kinetic energy E in the direction perpendicular to the boundary of the

partonic system and is pulled back by the chromoelectric field (see Fig. 2). The motion of the quark in Fig. 2 is controlled by the standard equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{mv}{\sqrt{1-v^2}} = -\sigma, \tag{17}$$

where m is the rest mass of the quark and we have assumed that the quark is pulled back by the constant force given by the string tension $\sigma = 1$ GeV/fm = = 0.2 GeV^2 in units $\hbar = c = 1$.

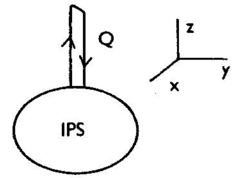

Fig. 2. Quark Q is escaping from the intermediate partonic system (IPS) and is pulled back by the chromoelectric flux tube.

Solving this equation we find that the velocity is roughly constant over the whole trajectory and rapidly changes sign at the turn-over point, see Fig. 3.

In calculating the very soft photon emission we shall thus assume that for $0 < t < T = E/\sigma$ the velocity is constant and equal to v_0, whereas for $T < t < 2T$ it is again constant and equal to $-v_0$. The number of emitted very soft photons is then calculated by using Eq. (1). The result is

$$\left.\frac{\mathrm{d}N}{\mathrm{d}^2k_T\mathrm{d}y}\right|_{y=0} = \frac{\alpha Q^2}{4\pi^2}|\mathbf{v}_0 \times \boldsymbol{n}|^2\{(A_1)^2 + (A_2)^2\}, \tag{18}$$

where Q^2 is the squared quark charge in units of e and

$$A_1 = \frac{1}{\omega(1-\mathbf{v}_0.\boldsymbol{n})}\{\cos[\omega T(1-\mathbf{v}_0.\boldsymbol{n})] - 1\} - $$
$$- \frac{1}{\omega(1+\mathbf{v}_0.\boldsymbol{n})}\{\cos[2\omega T(1+\mathbf{v}_0.\boldsymbol{n})] - \cos[\omega T(1+\mathbf{v}_0.\boldsymbol{n})]\} \tag{19a}$$

$$A_2 = \frac{1}{\omega(1-\mathbf{v}_0.\boldsymbol{n})}\sin[\omega T(1-\mathbf{v}_0.\boldsymbol{n})] - $$
$$- \frac{1}{\omega(1+\mathbf{v}_0.\boldsymbol{n})}\{\sin[2\omega T(1+\mathbf{v}_0.\boldsymbol{n})] - \sin[\omega T(1+\mathbf{v}_0.\boldsymbol{n})]\}. \tag{19b}$$

Note that for $\omega \to 0$ there holds $A_1 \sim \omega$ and $A_2 \sim \omega^2$. This vanishing of both amplitudes at $\omega \to 0$ is easy to see directly from Eq. (1). When performing the integral in Eq. (1) over a closed trajectory at which $\omega(t - \boldsymbol{n} . \boldsymbol{r}(t)) \ll 1$, we obtain

$$A \sim \int \mathrm{d}\boldsymbol{r} \times \boldsymbol{n} e^{i\omega(t - \boldsymbol{n}.\boldsymbol{r}(t))} \sim \int \mathrm{d}\boldsymbol{r} \times \boldsymbol{n} + \int i\omega(t - \boldsymbol{n}.\boldsymbol{r}(t))\,\mathrm{d}\boldsymbol{r} \times \boldsymbol{n} + \ldots \quad (20)$$

The first term gives $\Delta\boldsymbol{r} \times \boldsymbol{n}$, where $\Delta\boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_i$ is the displacement vector between the end point $\boldsymbol{r}_f$ and the initial point $\boldsymbol{r}_i$ of the trajectory. For our simple case this is equal to zero. The second term is also vanishing because the quark goes through every point twice with opposite velocities. Only the term proportional to ω^2 is nonvanishing in our case.

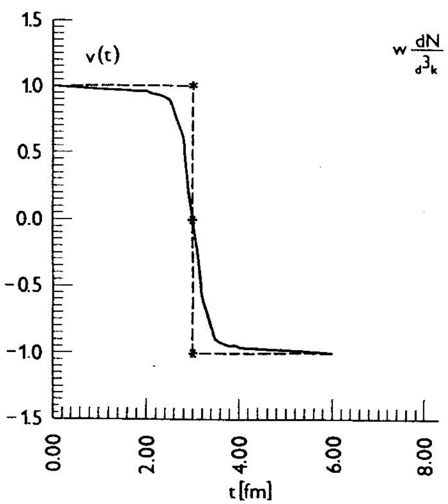

Fig. 3. The dependence of velocity on time for an escaping quark with $E = 3$ GeV. The approximation used in calculating the bremsstrahlung is shown by the dashed line.

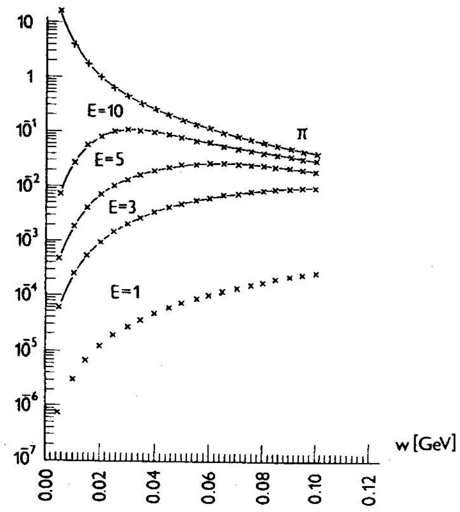

Fig. 4. Comparison of the "electric" bremsstrahlung spectrum of a quark escaping and pulled back with a) $E = 1$ GeV, b) $E = 3$ GeV, c) $E = 5$ GeV and d) $E = 10$ GeV with a typical bremsstrahlung spectrum of a final state pion with energy of 0.5 GeV.

The experimentally observed [1, 2] excess of direct photons is concentrated near $y \equiv y_{c.m} = 0$, hence let us assume for the moment that the intermediate parton system from which the quarks are escaping is formed at $y = 0$ and that the quarks are escaping isotropically. The number of bremsstrahled photons is

obtained from Eq. (18), averaged over all directions of v_0. The result is given by the expression

$$\omega \frac{\mathrm{d}N}{\mathrm{d}^3k} = \frac{\alpha\langle Q^2\rangle}{8\pi^2} \int_0^\pi \sin^3 \upsilon\{(A_1)^2 + (A_2)^2\}\,\mathrm{d}\upsilon, \quad (21a)$$

where A_1, A_2 are given by Eq. (19) with $\boldsymbol{v}_0 . \boldsymbol{n}$ replaced by $v_0 \cos \upsilon$ and $\langle Q^2\rangle$ is a mean square charge of quarks equal to 2/9. Eq. (21a) is easily integrated numerically. The spectrum of radiation emitted by a quark trying to escape and then pulled back is compared in Fig. 4 with the spectrum of the bremsstrahlung radiation of a pion with $E = 0.5$ GeV. In both cases we assume a spherically symmetric source at $y = 0$. The bremsstrahlung by a pion is calculated as

$$\omega \frac{\mathrm{d}N^\pi}{\mathrm{d}^3k} = \frac{\alpha}{8\pi^2\omega^2} \int_0^\pi \sin^3 \upsilon \left(\frac{1}{1 - v_0 \cos \upsilon}\right)^2 \mathrm{d}\vartheta. \quad (21b)$$

The results presented in Fig. 4 show clearly that the radiation emitted by an escaping and pulled back quark has a completely different ω — dependence from the bremsstrahlung of a freely escaping pion. Because of that the radiation of escaping and pulled back quark cannot explain the bremsstrahlung enhancement.

This is true even if we do not restrict the length of a chromoelectric string at all. The contribution of the strings of realistic length to the production of very soft photons is of course negligible.

We shall now briefly discuss the case of the "magnetic" bremsstrahlung of the escaping quark. The term denotes a situation when the escaping quark moves along a more or less rounded trajectory, close to a circular one (see Fig. 5). The terms "electric" and "magnetic" are not strictly exclusive and the difference between both types is for some trajectories rather arbitrary. For our present discussion it is sufficient to refer to the "electric" type trajectories as to those with small $\Delta\boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_i$ and to "magnetic" ones as those with large $\Delta\boldsymbol{r}$.

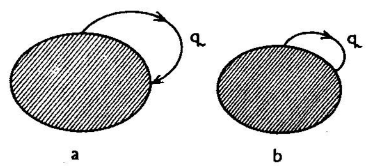

Fig. 5. Trajectories of escaping quarks leading to bremsstrahlung of the "magnetic" type: IPS — intermediate partonic system, a), b): two trajectories as examples.

For magnetic trajectories the expansion in Eq. (20) leads to

$$\boldsymbol{A} \sim \int \mathrm{d}\boldsymbol{r} \times \boldsymbol{n} + i\omega(t - \boldsymbol{n}.\boldsymbol{r}(t))\,\mathrm{d}\boldsymbol{r} \times \boldsymbol{n} =$$
$$= \Delta\boldsymbol{r} \times \boldsymbol{n} + i\omega\boldsymbol{c}_1 + \boldsymbol{c}_2\omega^2 + 0(\omega^3). \quad (22)$$

The square of the amplitude then behaves as

$$|A|^2 = (\Delta \boldsymbol{r} \times \boldsymbol{n} + \boldsymbol{c}_2 \omega^2)^2 + \omega^2 \boldsymbol{c}_1^2 + \dots \tag{23}$$

This behaviour is still qualitatively different from the true bremsstrahlung of free particles in the final state, when the amplitude develops the singularity $\sim 1/\omega$ due to the infinite integration range in Eq. (1). When the amplitude A is inserted into be Eq. (1) the true bremsstrahlung behaves as

$$\left.\frac{dN}{d^2k_T\,dy}\right|_{y=0} \sim \frac{1}{\omega^2},$$

whereas the "magnetic" bremsstrahlung leads to

$$\left.\frac{dN}{d^2k_T\,dy}\right|_{y=0} \sim \alpha + \beta\omega^2.$$

Because of this difference already on the qualitative level, we shall not study the magnetic bremsstrahlung here in more detail.

V. COMMENTS AND CONCLUSIONS

We have studied here three possible candidates for the enhancement of bremsstrahling: i) the enhancement due to "shock-wave like" mechanism, ii) the "electric" type soft photon radiation of the escaping quark and iii) the "magnetic" type radiation of the escaping quark. In cases i) and ii) we have presented also some quantitative results, the item iii) has been discussed only on a qualitative level.

The net result of our study is simple. Neither of the three discussed mechanisms can explain the observed enhancement of the bremsstrahlung [1, 2].

In all cases the factor which excluded a given mechanism has been the Low theorem [4] with the Landau Pomeranchuk type of argument [7] as a quantitative expression of limitations imposed by the Low theorem. There are, of course, also other possible mechanisms which can lead to an enhancement of the bremsstrahlung from final state hadrons, like: the Bose Einstein correlations, other correlations of particles in the final state, multiple scattering of pions in the hadron gas formed during the collision, very soft photon radiation in interactions of partons in the intermediate parton system, radiative decays of heavier resonances, etc. As regards a possible relevance of these possibilities a few comments are in order

— all correlations of hadrons in the final state have been included into the analysis of data in Ref. [1] and therefore no correlations can be responsible for the discrepancy between the data of Ref. [1] and the bremsstrahlung calculation of Ref. [1].
— as regards the discrepancy between the data of Ref. [2] and the calculations based on formulae of the type of Eq. (6) any effect of those mentioned above can contribute
— it is difficult to imagine that a mechanism which would remove the discrepancies between the data and the bremsstrahlung calculations in both Ref. [1] and [2] would not include at some stage a system with a typical space dimension of the order of 10 fm and time intervals of 10 fm/c. Such a scenario has been proposed recently by Van Hove [6] and it seems that his model is a most likely candidate for the sought-for mechanism.
Another, not yet excluded candidate are radiative decays of heavier resonances [13] produced at $y \sim 0$.

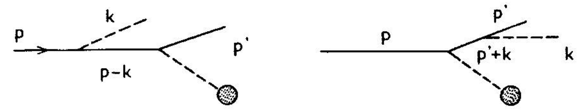

Fig. 6. Feynman diagram for bremsstrahlung emitted during the scattering of a particle on an external potential.

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APPENDIX A

We shall give here the expression for the bremsstrahlung from a charged particle trajectory with the multiple scattering. Eq. (1) can be rewritten as

$$dN = \frac{\alpha}{4\pi^2}\frac{d^3k}{\omega}|A|^2 \tag{A1}$$

with

$$\boldsymbol{A} = \int d\boldsymbol{r} \times \boldsymbol{n}\, e^{i\omega(t - \boldsymbol{n}\cdot \boldsymbol{r}(t))}. \tag{A2}$$

Succesive scatterings and velocities in between these scatterings can be denoted as indicated in Fig. 7. On the basis of Eq. (A2) it is easy to show that $\boldsymbol{A}$ can be decomposed as

$$\boldsymbol{A} = \boldsymbol{A}_1 + \boldsymbol{A}_2 + \dots + \boldsymbol{A}_{n+1}, \tag{A3}$$

where

$$\boldsymbol{A}_1 = e^{i\omega(t_0 - \boldsymbol{n}.\boldsymbol{r}_0)} \frac{\boldsymbol{v}_1 \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\mathrm{v}_1)} [e^{i\omega(t_1 - t_0)(1 - \boldsymbol{n}.\boldsymbol{v}_1)} - 1]$$

$$\boldsymbol{A}_2 = e^{i\omega(t_1 - \boldsymbol{n}.\boldsymbol{r}_1)} \frac{\boldsymbol{v}_2 \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\boldsymbol{v}_2)} [e^{i\omega(t_2 - t_1)(1 - \boldsymbol{n}.\boldsymbol{v}_2)} - 1]$$

$$\vdots$$

$$\boldsymbol{A}_n = e^{i\omega(t_{n-1} - \boldsymbol{n}.\boldsymbol{r}_{n-1})} \frac{\boldsymbol{v}_n \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\boldsymbol{v}_n)} [e^{i\omega(t_n - t_{n-1})(1 - \boldsymbol{n}.\boldsymbol{v}_n)} - 1]$$

$$\boldsymbol{A}_{n+1} = e^{i\omega(t_n - \boldsymbol{n}.\boldsymbol{r}_n)} \frac{\boldsymbol{v}_{n+1} \times \boldsymbol{n}}{i\omega(1 - \boldsymbol{n}.\boldsymbol{v}_{n+1})} [\qquad -1].$$

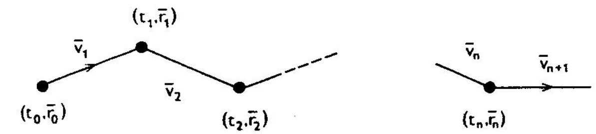

Fig. 7. Multiple scattering of a charged particle.

Note that the formula has a necessary property that when in some of the scatterings the velocity is not changed, the scattering can be omitted from Fig. 7 and also from the sum in Eq. (A3). The formula simplifies in a situation when all the exponents in square brackets are much smaller than 1. In such a situation we have

$$\boldsymbol{A}_1 = e^{i\omega(t_0 - \boldsymbol{n}.\boldsymbol{r}_0)} (\boldsymbol{v}_1 \times \boldsymbol{n})(t_1 - t_0)$$

$$\boldsymbol{A}_2 = e^{i\omega(t_1 - \boldsymbol{n}.\boldsymbol{r}_1)} (\boldsymbol{v}_2 \times \boldsymbol{n})(t_2 - t_1)$$

$$\vdots$$

$$\boldsymbol{A}_n = e^{i\omega(t_{n-1} - \boldsymbol{n}.\boldsymbol{r}_{n-1})} (\boldsymbol{v}_n \times \boldsymbol{n})(t_n - t_{n-1})$$

with $\boldsymbol{A}_{n+1}$ remaining unchanged.

REFERENCES

[1] Chliapnikov, P. V. et al.: Phys. Lett. *B141* (1984), 276, Ajinenko, I. V., et al.: Nuclear Physics *B162* (1980), 61.

[2] Goerlach, U., et al.: (Na-34 collaboration): *Talk at the XXIV. th Int. Conf. on High Energy Physics*, Munich, August 4—10, 1988, to be published in the Proceedings; Goerlach, U.: *Inv. talk at the Hadron Structure '88 conference*, Nov. 14—18, 1988 Piešťany, Czechoslovakia, Pfeiffer, A.: *Heidelberg preprint*, MPI H — 1989 — V3.

[3] Černý, V., Lichard, P., Pišút, J.: Zeit. f. Physik *C31* (1986), 163.

[4] Low, F. E.: Phys. Rev. *110* (1958), 974.

[5] Andersson, B., Dahlquist, P., Gustafson, G.: Lund University preprint, *LUTP 88-1* (1988).

[6] Van Hove, L.: CERN-TH. 5236/88, to be published in the "Herman Feshback Festschrift", a special issue of Annals of Physics (N. Y.).

[7] Landau, L. D., Pomeranchuk, I. Ya.: Doklady Akad. Nauk SSSR *92* (1953), 535; 735; (Papers No. 75 and 76 in the English edition of L. D. Landau's collected works).

[8] Nikolaev, N. N.: Usp. fiz. Nauk *134* (1981), 369. (Engl. translation Soviet Physics, Uspekhi *24* (1981), 531).

[9] Golovizin, V. V., Snigirev, A. M., Zinojev, G. M.: Yadernaja Fizika *46* (1988), 897; Golovizin, V. V., Snigirev, A. M., Zinoviev, G. M.: Bielefeld preprint BI-TP 87/12, Sept. 1987.

[10] Bannerjee, B., Glendenning, N., Matsui, T.: Phys. Lett. *127 B* (1983); Casher, A., Neuberger, H., Nussinov, S.: Phys. Rev. *D 20* (1979), 179.

[11] Jackson, J. D.: *Classical Electrodynamics*, Wiley, New York, 1975, ch. 14.

[12] Rückl, R.: Phys. Lett. *64B* (1976), 39; Cahn, R. N.: Phys. Rev. Rev. *D7* (1973), 247.

[13] Černý, V.: private communication, Ryskin, M. G.: *Lectures at the 24th Leningrad Winter School*, Leningrad Institute of Nuclear Physics, preprint 1989, to be published in the Proceedings.

[14] Beresteckij, V. B., Lifshitz, E. M., Pitaevskij, L. P.: *Relativistic Quantum Theory*, Vol. I, Nauka, Moskva 1968.

[15] Goshaw, A. T., et al.: Phys. Rev. Lett. *43* (1979), 1065; Phys. Rev. *D24* (1981), 2829.

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ПОИСК МЕХАНИЗМА ОТВЕТСТВЕННОГО ЗА УСИЛЕНИЕ ТОРМОЗНОГО ИЗЛУЧЕНИЯ В АДРОННЫХ РЕАКЦИЯХ.

1. УДАРНЫЕ ВОЛНЫ И ВЫЛЕТАЮЩИЕ КВАРКИ

Исследуются три механизма, которые могли бы эффективно вызвать усиление тормозного излучения в столкновениях адронов: во-первых, ударные волны в адронном газе соаданном при столкновении, во вторых, тормозное излучение «электрического типа» испускаемое кварком вылетающим из промежуточной системы, которая образуется при таком столкновении, и в третьих, тормозное излучение «магнитного типа» в рамках этого подхода. Показывается, что эти механизми не способны объяснить усиление продукции очень мягких фотонов наблюдаемое в адронных и адронно-ядерных реакциях.