acta phys. slov. 41 (1991), No. 2

ACOUSTO-OPTICAL AMPLIFICATION OF ULTRASOUND[1])

BUKOWSKI, R.,[2]) KLESZCZESKI, Z.,[2]) Gliwice

A numerical analysis of the stokessian non-linear acousto-optical interaction is presented in the paper. A detailed consideration of the acoustic wave amplification for the case of stationary, isotropic and a small angle interaction of homogeneous waves was carried out.

I. INTRODUCTION

The phenomenon of light interaction with acoustic wave has been known for over half a century. A particularly intensive development of studies concerning that phenomenon began after the laser had been discovered. In recent years the mutual feedback has been observed — acousto-optical interaction (AOI) has been used among others for an active modulation of the laser resonator's quality factor, for laser modes synchronization, for laser frequency tuning, for steering the laser beam in space, and so on.

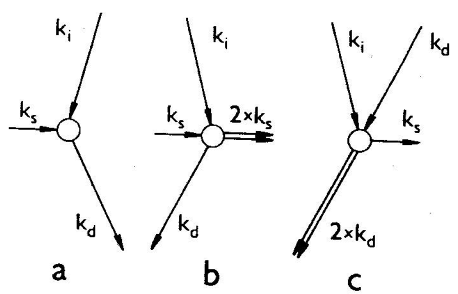


Fig. 1. AOI in quantum image: a) anti-Stokes interaction (A-SI)-photon and phonon annihilation and another phonon creation; b) Stokes interaction (SI) — "stimulated" photon disintegration into phonon and another photon by an acoustic wave; c) acousto-optical generation of ultrasound — "stimulated" photon disintegration into phonon and another photon by light. In all those kinds of interactions the energy conservation law ($\hbar\omega_i = \hbar\omega_d + \hbar\omega_s$) and the principle of conservation of the momentum ($\hbar k_i = \hbar k_d + \hbar k_s$) must be fulfilled ($i$ is the incident light, d is the diffracted light, s is the sound).

[1]) Contribution presented at the 11th Conference of Ultrasonic Method in ŽILINA, August 31-September 2, 1988

[2]) Institute of Physics, Silesian Technical University, ul. B. Krzywoustego 2, 44-100 GLIWICE, Poland

Considering the problem in general, AOI is conditioned by the time-spatial modulation of a medium permittivity tensor through the acoustic wave propagating in the medium (photo-elastic phenomenon). In a quantum image AOI can be presented by a photon-phonon interaction (Fig. 1). It results from this image that in the AOI an acoustic wave can be intensified and generated. On the basis of simple quality considerations (Manley-Rowe's theorem) we can say that all those effects become quite well visible when densities of photon and phonon streams in interacting wave beams are comparable. It means that the light interacting with a typical acoustic wave should have an intensity of the order of tens MW/cm^2. Laser giant pulses can constitute its source.

At such big light intensities different kinds of non-linear effects become of considerable importance. In AOI the phenomenon of electrostriction is of particular significance — stress caused by light "modulates" an acoustic wave which — in turn — changes the conditions of light propagation due to the photo-elastic phenomenon, and so on. Finally, because of the energy flow between interacting waves a complicated spatial distribution of their amplitudes is established. The basic characteristic of that non-linear AOI (NLAOI) is a considerable change of acoustic wave intensity.

Electrostriction influence on the AOI character was analysed in papers [1, 2] (among others). Its results were amplified in the author's papers [3, 4].

II. GEOMETRY OF AOI

Adequate conditions of matching must be satisfied to make AOI exist effectively. They result from the necessity of fulfilling the energy conservation law and the principle of conservation of the momentum in an elementary act of the

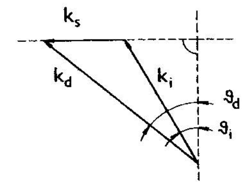


Fig. 2. Spatial matching in AOI.

photon and phonon interaction (Fig. 2). In the case of non-dispersion media it leads to the relation (comp. [5]):

$$\sin\vartheta_i = \frac{\lambda_0 v_s}{2v_s n_i}\left[1 + \left(\frac{v_s}{\lambda_0 v_s}\right)^2 (n_i^2 - n_d^2)\right]$$
$$\sin\vartheta_d = \frac{\lambda_0 v_s}{2v_s n_d}\left[1 - \left(\frac{v_s}{\lambda_0 v_s}\right)^2 (n_i^2 - n_d^2)\right] \tag{1}$$

where ϑ_i, ϑ_d are the angles formed by the wave vectors of the incident and the diffracted light waves with the normal plane to the wave vector of the acoustic wave (Bragg's angles); λ_0 is the light wavelength in vacuo; v_s, ν_s are the acoustic wave velocity and frequency; n_i, n_d are the indices of the refraction of light incident and diffracted waves. The above formulae are correct in both optically isotropis ($n_i = n_d$) and anisotropic media. Using formulae (1) in anisotropic media is not easy as the indices of refraction of light waves are the functions of the directions of their propagation. That problem was analysed in papers [6, 7].

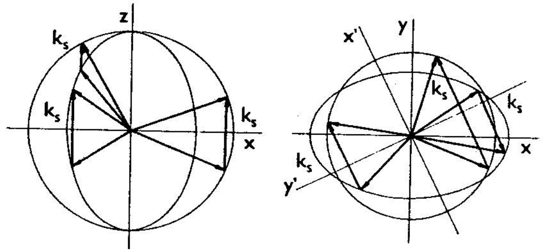


Fig. 3. Exemplary geometries of AOI in anisotropic crystals. Full curves result from cutting the surface of the wave vectors by the AOI plane.

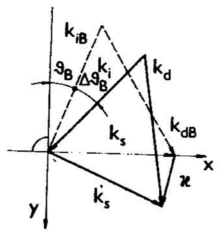


Fig. 4. "Mistuning" of spatial matching: ϑ_B is Bragg's angle, $\Delta\vartheta_B$ is a deviation from Bragg's angle, k_{iB}, k_{dB} are wave vectors of incident and diffracted light beams interacting at Bragg's angle; K_s is a wave vector of the divergent sound beam.

According to the results in those paper the AOI in anisotropic media can be devided into two kinds (Fig. 3). In the first kind the wave vectors of the light waves belong to the same surface of the wave vectors (isotropic interaction), while in the second they belong to different surfaces (anisotropic interaction). In the latter case some particular possibilities of interacting known as Bragg's multiple scattering are worth while to emphasize. Another interesting geometry of the AOI is a collinear interaction of two kinds with co-flow or contra-flow light beams (comp. [8]).

For real wave beams the AOI occurs also at angles somewhat different form Bragg's angles, though in their case is the most effective one. Generally it is caused by the fact that real wave beams are always limited in time and space, which can be characterized by some "distribution" of wave vectors and frequency. To analyse such effects we can assume that there is a certain "mistuning" of matching by introducing the mismatch vector $\varkappa$ (Fig. 4).

III. ANALYTICAL ANALYSIS OF NLAOI

Propagation of light waves and elastic wave which are coupled by the photo-elastic and electrostriction phenomena is described by the following wave equations (comp. [1, 2]):

$$\frac{\partial^2 E_j}{\partial x_k^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\tilde{\varepsilon}_{jk} E_k)$$
$$\varrho \frac{\partial^2 u_j}{\partial t^2} = \frac{\partial \tilde{\sigma}_{jk}}{\partial x_k} + \frac{1}{2}\eta_{jklm}\frac{\partial}{\partial x_k}\frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right) \tag{2}$$

where

$$\tilde{\varepsilon}_{jk} = \varepsilon_{jk} - \frac{1}{2}\varepsilon'_{jl} p_{lmnr}\left(\frac{\partial u_n}{\partial x_r} + \frac{\partial u_r}{\partial x_n}\right)\varepsilon'_{jk}$$
$$\tilde{\sigma}_{jk} = \frac{1}{2}C_{jklm}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right) + \frac{1}{2}\varepsilon_0 \varepsilon'_{nl} p_{lmjk} \varepsilon'_{mr} E_n E_r . \tag{3}$$

E_j are the components of the intensity vector of the electric field in the light wave; u_i are the components of the displacement vector in the acoustic wave; $\varepsilon = \varepsilon' + i\varepsilon''$ is the tensor of complex dielectrics constants; p, C, η are the tensor of the photo-elastic and the elastic constants and viscosity; ε_0, c are the vacuum permittivity and light speed in the vacuum; x_i are the co-ordinates of the Cartesian reference system.

The analysis of the above equations for AOI in crystals of any crystallographic system and in any plane is very complicated even in the case of a linear AOI (i.e. without considering electrostriction). Because of that in most of works an isotropic interaction is considered. The starting equations (2) for this case were analysed in paper [3] in detail and there its general solution in a given amplitude of the pumping field approximation was presented.

Studying that solution it has been stated that in some circumstances it is possible to obtain the stationary state. In the region of interaction, that is under the influence of that state, we can distinguish two subregions — one acoustically close ($x < v_s t_i$, t_i = time of a laser impulse duration) and one acoustically far

$(x > v_s t_i)$. A characteristic feature of the acoustically far subregion is the fact that the distribution of the amplitudes of interacting waves does not depend on the co-ordinate along the propagation of the acoustic wave (disregarding the attenuation). The amplitude distribution of the acoustic wave in the acoustically close subregion — in the case of a pure Stokes interaction of homogeneous waves — is formulated by the following dependence:

$$U_0(x, y) = U_{0p}\,\mathrm{e}^{-\alpha_s x}\left\{1 + \int_0^{\tau} I_1(\zeta)\left[1 + (\mathrm{e}^{2\alpha y} - 1)\zeta^2/\tau^2\right]^{-f} \mathrm{d}\zeta\right\} \tag{4}$$

where

$$\tau = 2\left[\frac{AB}{2\alpha}|E_b^2| x(1 - \mathrm{e}^{-2\alpha y})\right]^{1/2}, \tag{5}$$

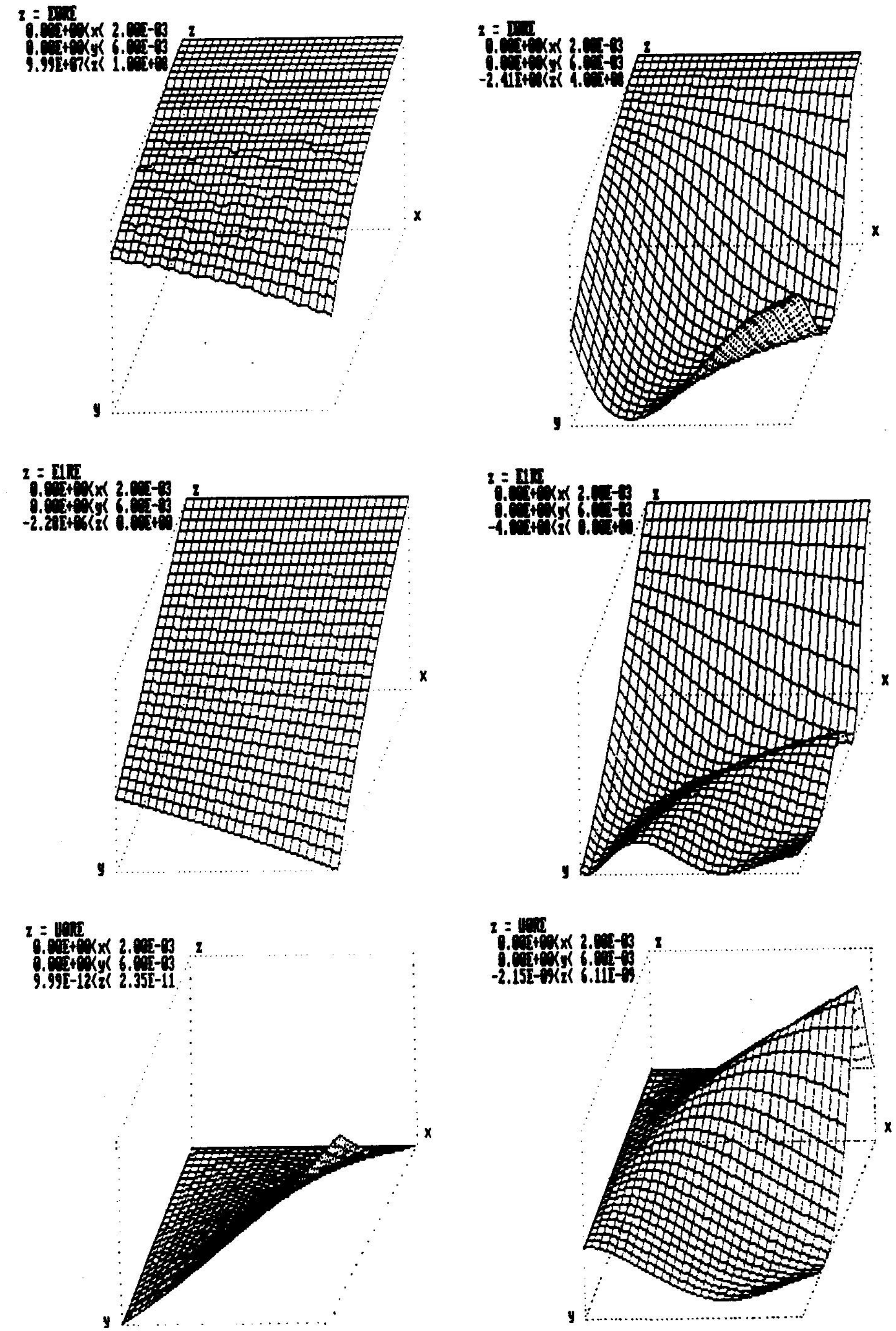
$$f = 1 - \mathrm{i}\,\frac{\varkappa_y}{2\alpha}, \tag{6}$$

where U_{0p} denotes the boundary value of the acoustic wave amplitude. From the property of the modified Bessel function I_1 it results that in a pure Stokes interaction we can obtain the acoustic wave amplification. The maximal amplification is for the interaction under Bragg's angle. When the attenuations are neglected, the amplification coefficient determined as the power ratio at the "output" and "input" of the interaction region is:

$$\gamma = I_0^2(\tau_0) - I_1^2(\tau_0), \quad \tau_0 = 2[AB|E_{0p}|^2 x_0 y_0]^{1/2} \tag{7}$$

(x_0, y_0 is the widt of acoustic and light beams, $x_0 < v_s t_i$).
In the case of deviations from Bragg's angle the amplification is getting smaller and the considered amplitude becomes a complex value. It means that there are phase shifts in relation to the wave interacting at Bragg's angle. They may be interpreted as changes of the phase velocity of the acoustic wave.

Fig. 5. Distributions of interacting wave amplitudes in a pure Stokes interaction of homogeneous waves for their real boundary values and under Bragg's angle. The codes of notation are the following: UO is an acoustic wave amplitude; $E0$, $E1$ are incident and diffracted light waves amplitudes; RE is a real part; Im is an imaginary part. a) $U_{0p} = 10^{-11}$ m, $E_{0p} = 1 \times 10^8$ V/m; b) $U_{0p} = 10^{-9}$ m, $E_{0p} = 4 \times 10^8$ V/m. In this case all imaginary parts of the complex amplitudes are zeros.



IV. NUMERICAL ANALYSIS OF THE STATIONARY NLAOI IN ACOUSTICALLY CLOSE SUBREGIONS

From the above-mentioned works it appears that the isotropic, small angle and the stationary NLAOI in the acoustically close region are described by the following set of equations:

$$\frac{\partial}{\partial y} E_0(x, y) = A\tilde{U}_0(x, y)E_1(x, y) - \alpha E_0(x, y),$$

$$\frac{\partial}{\partial y} E_1(x, y) = -A\tilde{U}_0^*(x, y)E_0(x, y) - \alpha E_1(x, y), \tag{8}$$

$$\frac{\partial}{\partial x} U_0(x, y) = -BE_0(x, y)E_1^*(x, y) - \tilde{\alpha}_s\tilde{U}_0(x, y),$$

where

$$\tilde{U}_0(x, y) = U_0(x, y)\exp(-\mathrm{i}\boldsymbol{\varkappa}\circ\boldsymbol{r}) \tag{9}$$

$$\tilde{\alpha}_s = \alpha_s + \mathrm{i}\varkappa_x \tag{10}$$

$$A = \frac{\varepsilon(\omega)p\omega\omega_s}{4vv_s}, \quad B = \frac{\varepsilon_0\varepsilon^2(\omega)p(1 + \varkappa_y/k_s)}{4\varrho v_s^2}, \tag{11}$$

where $U_0(x, y)$, $E_0(x, y)$, $E_1(x, y)$ are amplitudes of the acoustic wave and the incident and diffracted light waves in point (x, y) within the interaction region; α_s, α are attenuation coefficients of amplitudes of acoustic and light waves; v_s, v are velocities of the acoustic and light waves (in medium); ω_s, ω are circular frequencies of the acoustic and the light waves. Equations (8) must satisfy the next boundary conditions:

$$\begin{aligned} E_0(x, 0) &= E_{0p}(x) \\ E_1(x, 0) &= E_{1p}(x) \\ U_0(0, y) &= U_{0p}(y) \\ x, y &\geqslant 0. \end{aligned} \tag{12}$$

Applying in equations (8) substitutions

$$E_i(x, y) = e_i(x, y)\exp(-\alpha y), \quad i = 0.1$$

$$\tilde{U}_0(x, y) = \tilde{u}_0(x, y)\exp(-\tilde{\alpha}_s x)$$

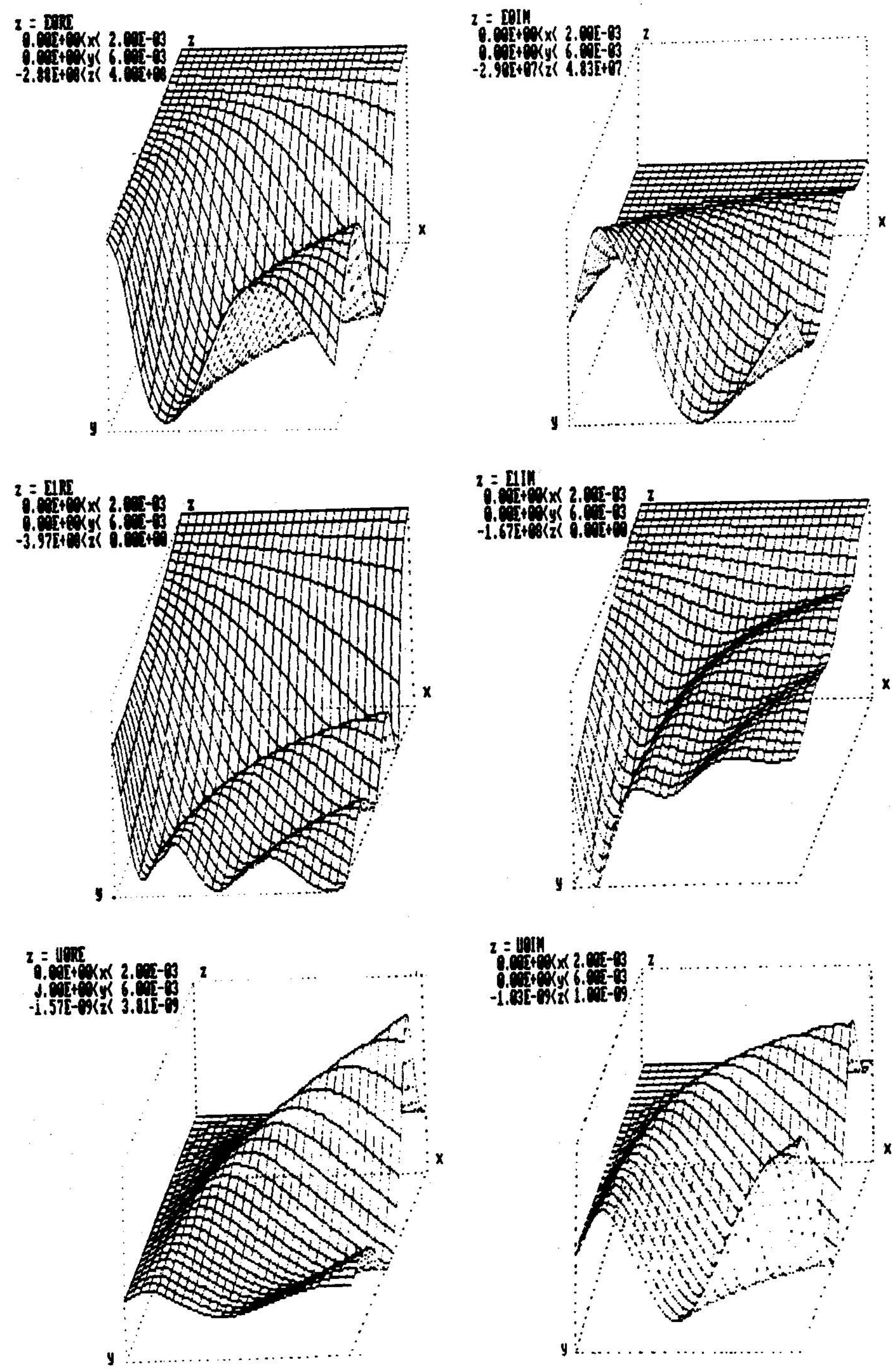
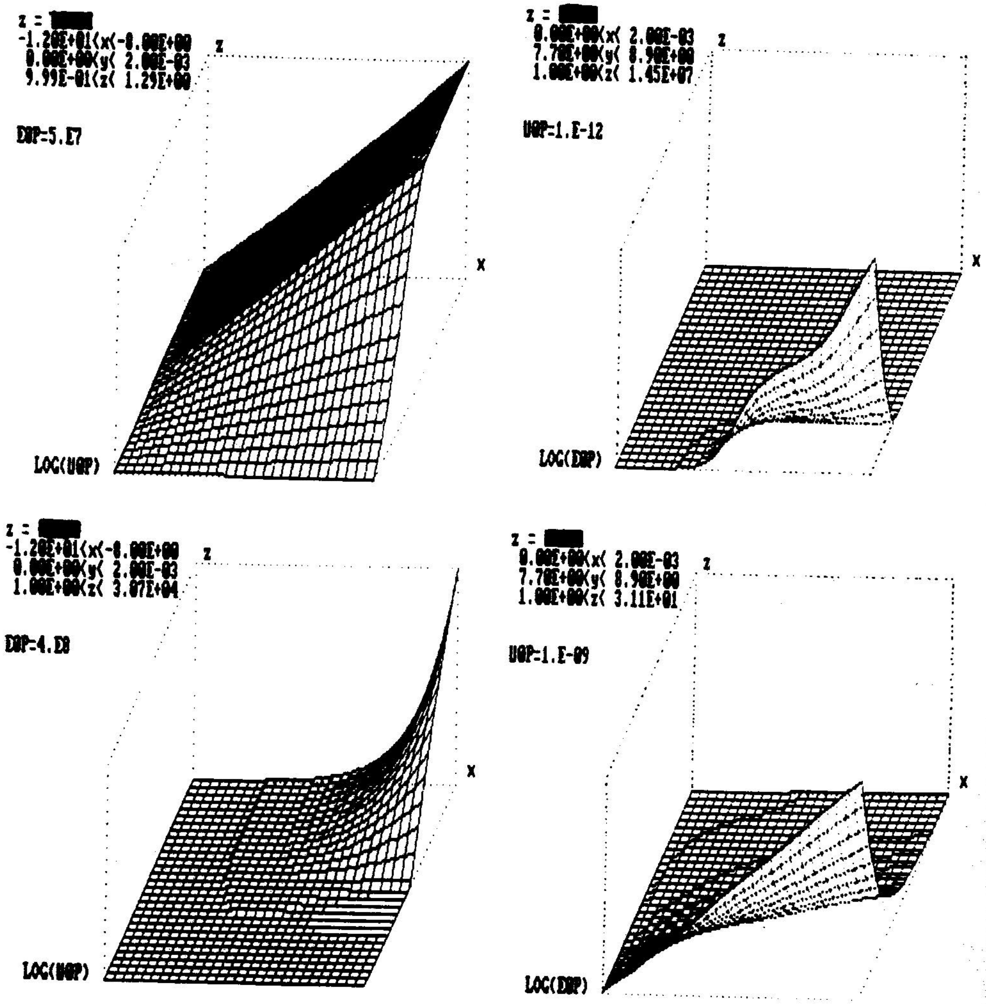


Fig. 6. It is analogous to Fig. 5 but we have a 3′ deviation from Bragg's anagle ($U_{0p} = 10^{-10}$ m, $E_{0p} = 4 \times 10^8$ V/m). ▶

and using the integral form we get:

$$e_0(x, y) = E_{0p}(x) + A(x)\int_0^{y} \tilde{U}_0((x, y')e_1(x, y')\,dy',$$

$$e_1(x, y) = E_{1p}(x) - A^*(x)\int_0^{y} \tilde{U}_0^*((x, y')e_0(x, y')\,dy', \qquad (14)$$



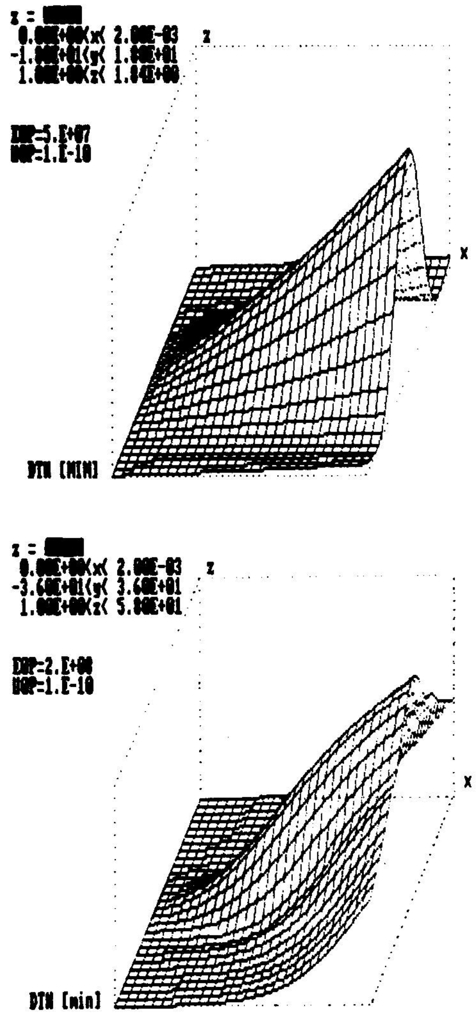
7a

7b

$$\tilde{u}_0(x, y) = \tilde{U}_{0p}(y) - \int_0^{x} B(x', y)e_0(x', y)e_1^*(x', y)\,dx',$$

where

$$A(x) = A\exp(-\tilde{\alpha}_s x), \quad B(x, y) = B\exp(-2\alpha y + \tilde{\alpha}_s x). \qquad (15)$$



7c

Fig. 7. The amplification coefficient of an acoustic beam in NLAOI (z) versus the width of the light beam and: a) the boundary amplitude of the acoustic wave; b) the boundary amplitude of the incident light wave; c) the deviation from the Bragg angle.

Applying in equations (14) a numerical integration instead of the analytical we obtain an adequate differential scheme. In this case we apply the trapezian method. The attainment set of the differential equations was solved by the iteration method. As starting values we utilized an amplitude distributions before the interaction. The above algorithm was realized in FORTRAN for a microcomputer of the IBM PC/AT type. The demanded iteration numbers were between a few and tens (relative to the boundary conditions).

The results of the calculations are presented in Figs. 5—7. All figures are presented by the stokessian NLAOI in the interaction region $(x, y) \in 2 \times 6$ mm^2 for homogeneous boundary conditions (i.e. $E_{0p}(x)$, $U_{0p}(y) = \text{const}$). In all cases the ruby laser light interacts with the acoustic wave propagated in quartz. All the numerical values are given in fundamental SI units.

V. CONCLUSIONS

From the presented numerical solutions of equations (14) there arise interesting inferences. Firstly the possibility of the acoustic wave amplification was confirmed. From Fig. 7 we can see that the value of the amplification coefficient can be very big. The maximum value of this coefficient is attained at the end of the acoustically close region. For the typical materials ($v_s \cong 5 \times 10^3$ m/s) and intensive laser impulses ($t_i \cong 10$ ns) we have $x_{max} \cong 0.05$ mm. In our calculations this value is equal 2 mm. Fig. 7a is very interesting. It shows a loss of the acoustic beam amplification ability with an increase of its wave input intensity. Another interesting information is evident from fig. 7c. We can see that we have an increase of the angular with of NLAOI with an increase of the incident light beam intensity. Moreover the amplification coefficient is asymmetrical relative to the deviation from the Bragg angle.

VI. ACKNOWLEDGEMENTS

This work has been done within the Central Program of Fundamental Researches (CPBP point 02.02, subject 3.7), which is co-ordinated by the Institute of Fundamental Technical Researches of the Polish Academy of Sciences.

REFERENCES

[1] Kroll, N. M.: J. Appl. Phys. 36 (1965), 34.
[2] Shkerdin, G. N., Proklov, V. V., Gulayev, Yu. V.: Fiz. Tv. Tela *19* (1977), 424.
[3] Bukowski, R., Kleszczewski, Z.: phys. st. sol. (b) *143* (1987), 281.
[4] Bukowski, R., Kleszczewski, Z.: phys. st. sol. (b) *147* (1988), 409.
[5] Dixon, R. W.: IEEE J. Quan. Electr. QE-*3* (1967), 88.

[6] Lemanov, V. V., Shakin, O. V.: Fiz. Tv. Tela *14* (1972), 229.
[7] Pisarevskii, Yu. V., Silvestrova, I. M.: *18* (1973), 1003.
[8] Balakshii, V. I., Parygin, V. N., Chirkov, L. E.: *Physical Principles of Acoustooptics*. Radio i Sviaz, Moscov 1985.

Received December 21st, 1988
Accepted for publication October 12th, 1989

АКУСТООПТИЧЕСКОЕ УСИЛЕНИЕ УЛЬТРАЗВУКА

В работе нумерически анализируется нелинейное стоксовское акустооптическое взаимодействие. Детально рассмотрено усиление однородых акустических волн при стационарном, изотропном и малоугловом взаимодействиях.