FINITE SIZE SCALING TEST OF DECONFINEMENT IN SU(2) LATTICE GAUGE THEORY[1])

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The behaviour of the order parameter (Polyakov loop) on a $8^3 \times 4$, $12^3 \times 4$ and $18^3 \times 4$ lattices is investigated by the finite size scaling method. The deconfinement temperature and the critical exponent for the order parameter are calculated. The obtained value of the critical exponent for the order parameter is in very good egreement with those in the three-dimensional Ising model.

I. INTRODUCTION

Lattice calculations by the Monte Carlo (MC) method have so far been the only method for studying temperature phase transitions in gauge theories from the first principles. By their nature, MC calculations are carried out on finite lattices. This is the most serious drawback of a computer simulation approach to the study of critical phenomena, because no finite system with non-singular Lagrangean can exibit a true phase transition. Nevertheless, finite systems remind of phase transitions, and systematic studies of these pseudo-transitions as functions of system size may reveal information about the phase transition in the thermodynamic limit. One way to do this particularly in order to evaluate the critical exponents of the theory, is to use finite size scaling.

The finite size scaling theory has been developed by Ferdinand and Fisher [1] (see also [2—3]) for critical phenomena in spin systems. The validity of this approach has been demonstrated through extensive MC simulations on two- and three-dimensional Ising models (see, for example [4]). In this paper, the SU(2) lattice gauge theory is under consideration. MC data for the order parameter (Polyakov loop) are analysed on the lattices $8^3 \times 4$, $12^3 \times 4$ and $18^3 \times 4$ by the finite size scaling method. The thermodynamic limit values of the critical exponent β for the order parameter and of the deconfinement temperature T_c are presented. The value of β is in excellent agreement with the values of the three-dimensional Ising model.

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II. FINITE SIZE SCALING METHOD

The partition function for the (3 + 1)-dimensional SU(2) gauge theory on an $N_\sigma^3 \times N_\tau$ lattice is defined as

$$Z = \int \prod_{(\mu,\nu)} \mathrm{d}U_{\mu,\nu} \mathrm{e}^{-S(U)}, \tag{1}$$

where $U_{\mu,\nu}$ are the SU(2) matrices on the links (μ, ν), S is the Wilson action

$$S(U) = \frac{4}{g^2} \sum_p \mathrm{Tr}\, U_p; \tag{2}$$

here U_p is the product of link matrices around a plaquette. The temperature T is defined as an inverse size of the lattice in the timelike direction $T = 1/N_\tau a$ (a is a lattice spacing). In the weak coupling limit one can use the renormalization group relation

$$a(g^2) = \frac{1}{\Lambda_L} \exp\left\{ -\frac{12\pi^2}{11g^2} + \frac{51}{121} \ln\left(\frac{24\pi^2}{11g^2}\right)\right\}. \tag{3}$$

The Polyakov loop at the spatial position $\boldsymbol{n}$ is defined as

$$L_n = \frac{1}{2} \mathrm{Tr} \prod_{\tau=1}^{N_\tau} U_{\tau,n,0}, \tag{4}$$

where $U_{\tau,n,0}$ is the SU(2) link matrix at point $(\boldsymbol{n}, \tau)$ in the timelike direction. The deconfining phase transition of the SU(2) lattice gauge theory signifies a spontaneous breakdown of a global Z_2 symmetry of the effective theory, which can be obtained from the partition function (1) by integrations on all links variables except those for the Polyakov loop. The expectation value $\langle L \rangle$ of the Polyakov loop is the corresponding order parameter, which is zero in the confinement phase but is finite in the deconfinement phase. The "temperature" of the effective theory (which is defined in the same spatial volume $V = (N_\sigma a)^3$ as the underlying SU(2) gauge theory) is just the gauge theory coupling $g^2/4$.

For a lattice of the infinite spatial size the order parameter $\langle L \rangle$ in the vicinity of the critical point $g_c^2/4$ must behave as follows:

$$\langle L \rangle \sim \left(\frac{g_c^2}{4} - \frac{g^2}{4}\right)^\beta, \tag{5}$$

where β is the critical exponent of the order parameter. According to the iniversality conjecture [5], the SU(2) lattice gauge theory in (3 + 1)-dimensions should have the same critical exponents as the three-dimensional Ising model, where β is evaluated to be 0.3265 ± 0.0025 [6].

In some recent papers [7—12] the critical exponent β of SU(2) order parameter has been calculated. MC simulations were made on lattices of various sizes $N_\sigma = 7 \div 18$, $N_\tau = 3 \div 5$. The value of β varies from $\beta = 0.207$ on a lattice $7^3 \times 3$ [7] to $\beta = 0.409$ on a lattice $18^3 \times 5$ [10]. This discrepancy originates from the fact that a finite lattice cannot, rigorously speaking, undergo a true phase transition. The finite size effects are too large in the very neighbourhood of the pseudo-transition point, and the critical parameters can only aproximately be determined on a finite lattice.

In order to eliminate the finite size effects we use the finite size scaling method, which is extremely useful to guide the extrapolation of MC finite lattice data to the thermodynamic limit. Consider the SU(2) gauge theory on a $N_\sigma^3 \times N_\tau$ lattices for various N_σ and N_τ fix. According to the finite size scaling theory [1], the free energy of a finite gauge system is given by the homogeneous function

$$f\left(\frac{g^2}{4}, N_\sigma\right) = N_\sigma^{[-(2-\alpha)/\nu]}\tilde{F}(tN_\sigma^{1/\nu}). \tag{6}$$

where α and ν are the critical exponents pertaining to the specific heat and to the correlation length, respectively. $\tilde{F}$ is a scaling function of the scale variable $tN_\sigma^{1/\nu}$ only, and $t = \frac{(g^2/4 - g_c^{2*}/4)}{g_c^{2*}/4}$ with $g_c^{2*}/4 = g_c^2(N_\sigma = \infty)/4$. The critical coupling $g_c^2(N_\sigma)/4$ for a lattice of size N_σ differs from $g_c^{2*}/4$ in the following way

$$\frac{g_c^2(N_\sigma)}{4} - \frac{g_c^{2*}}{4} = \text{const}\, N_\sigma^{-1/\nu}. \tag{7}$$

From (6) the scaling properties of the SU(2) order parameter may be derived

$$\langle K \rangle = N_\sigma^{-\beta/\nu}\tilde{M}(tN_\sigma^{1/\nu}). \tag{8}$$

In the limit $N_\sigma \to \infty$, $t \to 0$ (8) has to reduce to the infinite system singular behaviour (5), and therefore, in this limit the order parameter scaling function is given by

$$\tilde{M}(tN_\sigma^{1/\nu}) \sim |tN_\sigma^{1/\nu}|^\beta. \tag{9}$$

It must be emphasized that the critical exponents in (6)—(9) correspond to the thermodynamic limit.

III. NUMERICAL RESULTS

To evalute the critical exponent β we have performed MC simulation on lattices of size $8^3 \times 4$ and $12^3 \times 4$. Our MC data were obtained from SU(2) Metropolis programm after 1000 sweeps for thermalization. In general we performed 10000 sweeps per point. We used also the high precision data on a $18^3 \times 4$ lattice from Ref. [10].

We have performed a fit of MC points with the functional form.

$$\langle L \rangle = A\left|\frac{g^2}{4} - \frac{g_c^2}{4}\right|^\beta\left(1 + B\left|\frac{g^2}{4} - \frac{g_c^2}{4}\right|^{0.5}\right), \tag{10}$$

where A, B and $\frac{g_c^2}{4}$, β are free parameters. A correction of the leading term (the Wegner finite size correction term) is known from the Ising model. This term has to be included in the fit, because the range of validity of the leading term approximation is not known.

Table 1

Parameters of the fit (10) of $\langle L \rangle$ on $N_\sigma^3 \times 4$ lattices ($N_\sigma = 8, 12, 18$)

N_σ	$\frac{4}{g^2}$	$\frac{4}{g_c^2}$	β	A	B	$\frac{\chi^2}{N}$
8	2.29—2.65 2.29—2.70	2.262(05) 2.264(07)	0.499(38) 0.462(75)	1.324(29) 1.141(14)	−0.375(24) −0.313(20)	1.048 1.013
12	2.29—2.90 2.30—2.60	2.281(02) 2.283(05)	0.424(33) 0.490(83)	1.028(15) 1.366(23)	−0.151(06) −0.176(11)	1.920 1.200
18	2.301—2.450 2.305—2.475	2.294(03) 2.264(03)	0.412(57) 0.382(53)	1.331(32) 1.064(09)	−0.139(09) −0.106(06)	0.960 1.140

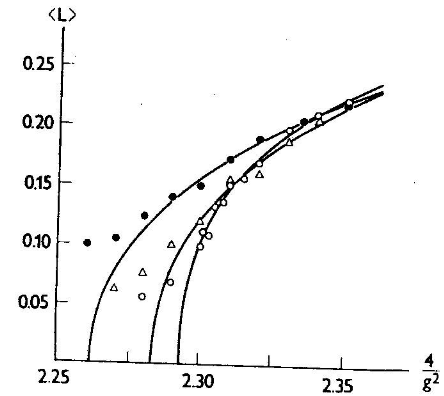

Fig. 1. Order parameter $\langle L \rangle$ on $8^3 \times 4$ (●), $12^3 \times 4$ (Δ) and $18^3 \times 4$ (o) lattices versus inverse "temperature" $\frac{4}{g^2}$. The solid lines are the best fits of $\langle L \rangle$ according to eq. (10).

The average Polyakov loops $\langle L\rangle$ on lattices $8^3 \times 4$. $12^3 \times 4$ and $18^3 \times 4$ are shown in Fig. 1. Results of our fits are given in Table 1.

It follows from Table 1 that the parameters $g_c^2/4$ and β on a single lattice are strongly correlated — a slight change in $g_c^2/4$ leads to a relatively large change in β. The values obtained for β are far from the expected value $\beta = 0.326$, (which is) known from the Ising model. The errors in β from MC data are of the order 10%—15%. The parameters B are of the same order as the leading term amplitudes A. This means that finite size effects are relatively large even on the $18^3 \times \times 4$ lattice. We conclude that MC data for the order parameter $\langle L\rangle$ on a single lattice do not really lead to a conclusive determination of the critical exponent β.

In order to calculate the value of β by the finite size scaling method the scaling function $\tilde{M}(tN_\sigma^{1/\nu}) = N_\sigma^{\beta/\nu}\langle L\rangle$ has been constructed assuming the Ising value of the correlation length exponent, $\nu = 0.63$ [4]. We considered two sets of MC data from $N_\sigma^3 \times 4$ lattices: $N_\sigma = 8,18$ and $N_\sigma = 12,18$. As for the order parameter we have used a two-term fit form:

$$\tilde{M}(X) = A|\bar{X}|^\beta(1 + B|\bar{X}|^{0.5}), \tag{11}$$

where $\bar{X} = tN_\sigma^{1/\nu}$. Results of our fits are given in Tables 2 and 3. The MC data were considered for several initial points in both sets. Contrary to Table 1, the values of the parameters are independent of the initial points. The expected value for the critical exponent $\beta = 0.326$ is very well compatible with MC data for $N_\sigma = 12,18$, but this is not the case for $N_\sigma = 8,18$. It appears from this that only lattices with $N_\sigma \geqslant 12$ are well inside the finite size scaling region described by eq. (6).

We conclude that MC data for the order parameter on a lattice with $N_\sigma \geqslant 12$ can be accurately described by a single scaling function $\tilde{M}(\bar{X})$. In Fig. 2 this function is shown together with MC data on $12^3 \times 4$ and a $18^3 \times 4$ lattice. The parameters of $\tilde{M}(\bar{X}))$ are presented in Table 3. It should be noted that the amplitude B of the correction term in (11) is negligibly small as compared to the amplitude of the leading term. This means that the behaviour of the scaling function can be represented by an infinite system form

$$\tilde{M}(\bar{X}) = A|\bar{X}|^\beta. \tag{12}$$

The main results of our calculations is the determination of the deconfinement phase transition parameters $g_c^2/4$ and β. From Table 3 and formula (3) the

Table 2

Scaling function parameters according to the fit (11) for MC data on lattices $8^3 \times 4$ and $18^3 \times 4$.

N_σ	$\frac{4}{g^2}$	$\frac{4}{g_c^2}$	β	A	B	$\frac{X^2}{N}$
8 18	2.320—2.700 2.320—2.475	2.294(03)	0.292(07)	0.668(06)	0.004(03)	1.83
8 18	2.400—2.700 2.330—2.475	2.296(03)	0.288(11)	0.664(06)	0.003(03)	1.58
8 18	2.350—2.700 2.340—2.475	2.296(03)	0.288(11)	0.663(06)	0.004(03)	1.66

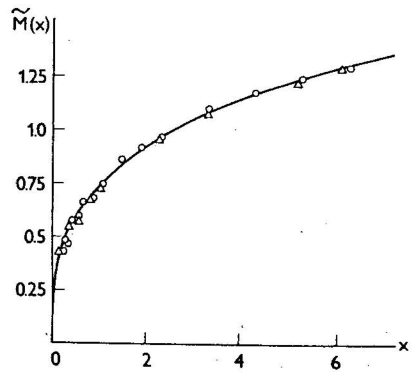

Fig. 2. Scaling function $\tilde{M}(\bar{X})$ for MC data on a $12^3 \times 4$ (Δ) and a $18^3 \times 4$ lattice.

Table 3

Scaling function parameters according to the fit (11) for lattices $12^3 \times 4$ and $18^3 \times 4$.

N_σ	$\frac{4}{g^2}$	$\frac{4}{g_c^2}$	β	A	B	$\frac{X^2}{N}$
12 18	2.320—2.900 2.315—2.475	2.297(02)	0.317(09)	0.749(08)	−0.007(03)	2.70
12	2.330—2.900 2.305—2.475	2.295(01)	0.320(08)	0.751(08)	−0.007(03)	2.17
12 18	2.340—2.900 2.308—2.475	2.295(02)	0.321(09)	0.757(09)	−0.009(03)	2.21

decondinement temperature can be easily found

$$\frac{T_c}{\Lambda_L} = 41.9 \pm 0.2 \tag{13}$$

which in physical units corresponds to $T_c = (209.5 \pm 1)$ MeV ($\Lambda_L = 5$ MeV for the SU(2) group). The value of β presented in Table 3 is in excellent agreement with the critical exponent $\beta = 0.326$ for magnetization in the three-dimensional Ising model, in accordance with the universality hypothesis.

The finite size scaling method thus proved extremely useful in the calculations of the critical exponent for the order parameter in the SU(2) lattice gauge theory. The next problem is to apply this method to the calculations of the critical exponents for the specific heat and susceptibility. Such calculations will finally solve the problem of the validity of the universality hypothesis.

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ИССЛЕДОВАНИЕ ДЕКОНФАЙНМЕНТА В SU(2) КАЛИБРОВОЧНОЙ ТЕОРИИ НА РЕШЕТКЕ МЕТОДОМ КОНЕЧНОРАЗМЕРНОГО СКЕЙЛИНГА

Поведение параметра порядка (петля Полякова) на решетках $8^3 \times 4$, $12^3 \times 4$ и $18^3 \times 4$ исследуется методом конечноразмерного скейлинга. Вычислена температура деконфайнмента и критическая экспонента параметра порядка. Полученное значение критической экспоненты хорошо согласуется с трехмерной моделью Изинга.