

## THE POSSIBILITY OF LOCALIZED BAGLIKE EXCITATIONS IN HIGH $T_c$ SUPERCONDUCTORS DUE TO THE LOW DIMENSIONALITY OF THE ELECTRON STATES

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The conditions are determined under which the localized moreparticle baglike excitations in two- and one-dimensional fermion systems can be below the BCS excitations. It is shown that in the twodimensional systems the bags can exist when the characteristic value of the bag potential is comparable with the energy gap, but no more when this potential is determined by the Fermi energy. The bags are always energetically favourable in the one-dimensional fermion system.

It is suggested that the energy changes of the bags by changing the number of the included quasiparticles can be connected with the resonances in the tunnelling characteristics of high  $T_c$  superconductors. In the two-dimensional bags, these values are comparable with the experimental results for the voltage steps in the tunnelling current ( $\approx 40$  mV), whereas in the one-dimensional system these energy changes are always smaller than the energy gap. The Coulomb term due to the quasiparticle localization was also included into the free energy, but the equilibrium quasiparticle number could not be calculated.

It is emphasized that the existence of the localized bags can influence the electro-magnetic, as well as the thermodynamic properties of high  $T_c$  superconductors.

### 1. INTRODUCTION

The discovery of high  $T_c$  superconductors [1, 2] has renewed the interest in many fields of superconductivity research. Basic theoretical mechanism — including some hypothetical and very exotic ones — are studied extensively. The semi-phenomenological Ginzburg-Landau theory of superconductivity and its extensions should be modified due to the very small value of the coherence length, too [3, 4]. The phenomenological theories of current carrying mechanism in high  $T_c$  superconductors (based partially on the Ginzburg-Landau theory including the Josephson effects, etc.) are numerous with very different approach-

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es (see, e.g. references [5] and the literature cited therein). The technological efforts to obtain practically usable conductors with high critical current densities in high magnetic fields are carried out in the expectation of revolutionary changes for many technical branches and the whole social life of the (human) society.

The existence of the energy gap in the quasiparticle excitation spectrum is one of the most important and expressive properties of superconductors. Its experimental verification was one of the most beautiful demonstrations of the validity of the microscopic BCS theory [6]. The existence of the energy gap in high  $T_c$  superconductors is unambiguously proved, although some gapless (but superconducting) region could exist in the immediate vicinity of the critical temperature  $T_c$ .

The energy gap is manifested in various properties of superconductors (interaction with electromagnetic radiation, acoustic waves, etc., thermodynamic properties like specific heat and thermal conductivity, tunnelling currents including the Josephson effects). The tunnelling measurements lead to the most interesting and most precise measurements of the value of the energy gap, including its temperature dependence.

In addition to the usual structures in the tunnelling characteristics (i.e. mainly in the first and second derivatives of the tunnelling current with respect to the applied voltage), the high  $T_c$  superconductors show some nearly periodic resonance-like structures above the gap voltage (see, e.g., [7—11] and the literature cited therein). These resonances can be clearly seen, in spite of the much "richer" structure of the tunnelling characteristics of high  $T_c$  superconductors compared with conventional superconductors. We have shown [10, 11] that these resonances could be connected with the existence of localized electron states in partially normal inclusions near the contacts. In addition, we presented some arguments [8, 10] for the possible existence of localized baglike excitations [12] in the quasiparticle spectrum of high  $T_c$  superconductors, in spite of the fact that for the three-dimensional systems these excitations are lying higher [10] than the one-particle excitations of the BCS theory [6].

The existence of the localized baglike excitations could be important not only for the tunnelling characteristics, but also for other properties of high  $T_c$  superconductors, mainly some thermodynamic ones which are fundamentally determined by the normal quasiparticle excitation spectrum.

In this paper, we calculate the conditions under which the baglike excitations can be favourable in the one-dimensional and two-dimensional system of fermions (Section 2). By changing the number of the involved quasiparticles in the bag, their energy is changed, too (Section 3). These energy changes can lead to the explanation of the experimentally observed resonances in high  $T_c$  superconductors, as given in the Discussion.

## II. THE BAGLIKE EXCITATIONS IN THE TWO-DIMENSIONAL AND ONE-DIMENSIONAL FERMION GAS

The importance of the baglike excitations for high  $T_c$  superconductors was introduced by Weinstein [12] who suggested that the bags (localized in the sphere of radius  $R$  in contradiction to the unlocalized BCS excitations) could explain the double or triple peaks in some tunnelling measurements of high  $T_c$  superconductors. His basic result was that the two-particle bag has a lower free energy than the sum of two one-particle bags. Therefore, both excited particles should form a bound state. The calculations were extended to more-particle states of the bags [10]. The results showed that the energy of the baglike excitations is always larger than the energy of the (non-localized plane-wave fermion) BCS excitations (besides a very strong coupling between the fermions) with an equal number of excited quasiparticles. Nevertheless, the energy changes by changing the number of the involved quasiparticles in the bag was approximately the same as the experimental results for the resonance voltage steps, i.e. about 40 meV [7, 8]. Therefore, we suggested that there could be some mechanisms which should favour the existence of the bags [10, 11]: the periodic structure of the bags, the surface contribution to their free energy, the anisotropic properties and the low carrier concentration in the known high  $T_c$  superconductors, and — last but not least — the low-dimensionality of the electronic structure, connected with the Cu-O planes and possibly also chains in high  $T_c$  superconductors.

The general solution of the problem is difficult. Like in the BCS theory, one should treat it selfconsistently, including the spatial variation of the order parameter near the bag. Attempts were made to replace this selfconsistency problem by including surface energy terms (which is from the "transition region" between the superconducting and the normal parts) into the free energy in the three-dimensional problem [12].

As we are interested here mainly in the estimation of bag energies and their changes by changing the number of included quasiparticles, we suppose in the following that the order parameter decreases to zero in the localization region and is unchanged outside this region. This treatment is also quantitatively good for localization radii  $R \gg \xi$ , as the coherence length is determining the distance of spatial change of the order parameter. The generalization of our solution is possible by including a surface contribution to the free energy, too. Such generalization is unambiguously needed for calculations of the most favourable number of quasiparticles in the bag. For making quantitative statements, a selfconsistent solution is required.

Since the quasiparticle excitations are charged particles, their localization contributes also to the Coulomb energy of the bags. This problem is briefly discussed in the Discussion.

We carry out calculations for the quadratic potential (isotropic in the two-dimensional case) like in three dimensions [10, 11]. We have shown [10, 11] that the form of the potential does not influence the results substantially in the three-dimensional case. We assume that the same arguments are true in the two- and the one-dimensional case.

The energy levels in the potential  $U = q/(r/R)^2$  are given by

$$E_{n_2} = \frac{\hbar}{R} \left( \frac{q}{2m} \right)^{1/2} (2n_1 + 1 + 2n_2 + 1) = \frac{\hbar}{R} \left( \frac{q}{2m} \right)^{1/2} 2(n + 1),$$

$$E_{n_1} = \frac{\hbar}{R} \left( \frac{q}{2m} \right)^{1/2} (2n + 1),$$

in the two-dimensional and the one-dimensional cases, respectively. In these expressions,  $n, n_1, n_2 = 0, 1, 2, \dots$

The quantities needed for determining the energy of more — particle states of the bags can be calculated — in analogy to the three-dimensional case — relatively easily [10, 11].

The number of states (including degeneracy) at the level  $N$  is given by  $2(N + 1)$  and 2 for the isotropic two-dimensional and one-dimensional harmonic oscillator, respectively.

The occupation number up to the level  $N$  is then  $M = (N + 2)(N + 1)$  and  $2(N + 1)$  and the sum of the total energy of states filled up to this level (in units of  $(\hbar/R)(q/2m)^{1/2}$ ) is  $L = 2(N + 1)(N + 2)(2N + 3)/3$  and  $2(N + 1)^2$  (see table 1).

The total energy of the bag is then [10, 12]

$$E_i = N_i(0) V_i \frac{\Delta^2}{4} + \frac{\hbar}{R} \left( \frac{q}{2m} \right)^{1/2} L_i,$$

where  $V_2 = 4R^2$  and  $V_1 = 2R$  are the "spaces" needed for the existence of the bags (i.e. square and length of size  $2R$ , like the cube or sphere with localization radius  $R$  in the three-dimensional case [10, 12]). The two-dimensional and one-dimensional densities of states,  $N_i$ , are supposed to be nearly constant in the vicinity of the Fermi energy,  $E_F = \hbar^2 k_F^2 / 2m$ , i.e.

$$N_2(0) = \frac{m}{2\pi\hbar^2}, \quad N_1(0) = \frac{m}{2\pi\hbar^2 k_F}.$$

The quantity  $\Delta^2/4$  follows from the microscopic BCS theory and means that about  $N(0) \Delta/4$  particles lower their energy by the factor  $\Delta$  due to the transition into the superconducting state.

Table 1

Some parameters of the isotropic two-dimensional (2d) and one-dimensional (1d) harmonic oscillator.

Level number	Energy at the level $n$		Number of states		Number of occupied states at filling the $n$ -th level	Value of $L$ at filling the $n$ -th level		Minimum value of $L^{2/3}/M L^{1/2}/M$ just at filling the $n$ -th level		
	2d	1d	2d	1d		2d	1d	2d	1d	
0	2	1	2	2	2	2	4	2	1.26	$1/\sqrt{2}$
0	2	1	2	2	2	2	4	2	1.26	$1/\sqrt{2}$
1	4	3	4	4	6	4	20	8	1.23	$1/\sqrt{2}$
2	6	5	6	6	12	6	56	18	1.22	$1/\sqrt{2}$
3	8	7	8	8	20	8	120	32	1.216	$1/\sqrt{2}$
4	10	9	10	10	30	10	220	50	1.215	$1/\sqrt{2}$
5	12	11	12	12	42	12	364	72	1.214	$1/\sqrt{2}$
6	14	13	14	14	56	14	560	98	1.213	$1/\sqrt{2}$
6	14	13	14	14	56	14	560	98	1.2128	$1/\sqrt{2}$
7	16	15	16	16	72	16	816	128	1.2125	$1/\sqrt{2}$
8	18	17	18	18	90	18	1140	162	1.2114	$1/\sqrt{2}$
$\infty$										$1/\sqrt{2}$

The localization radius  $R$  of the bag is obtained in both cases from the equilibrium condition

$$\frac{dE}{dR} = 0.$$

This condition leads to the following relations for the two-dimensional (2d) and the one-dimensional (1d) case:

$$(2d) \quad R_2^3 = \hbar^2 \frac{\pi}{m\Delta^2} \left( \frac{q}{2m} \right)^{1/2} L_2,$$

$$(1d) \quad R_1^2 = 4\pi \frac{\hbar^3 k_F}{m\Delta^2} \left( \frac{q}{2m} \right)^{1/2} L_1.$$

The total energy of the bags is then

$$(2d) \quad E_2 = \frac{3}{2} \left( \frac{1}{2\pi} \frac{q}{\Delta} L_2^2 \right)^{1/3} \Delta,$$

$$(1d) \quad E_1 = \Delta \left( \frac{1}{2\pi} L_1 \right)^{1/2} \left( \frac{q}{E_F} \right)^{1/4}.$$

The choice of the value of  $q$  is of principal importance. Some arguments were given in paper [10] in favour of  $q = \Delta$ , whereas  $q = E_F$  was chosen by Weinstein [12]. As for the three-dimensional case [10, 11], both possibilities are taken into account in our further considerations.

To see the conditions for the realization of the bags in the two-dimensional and the one-dimensional case, one has to compare their energy with the energy of the corresponding BCS excitations with the same number of the included quasiparticles  $M$ . For the bags to be energetically favourable, there should be

$$E_i \leq M\Delta.$$

These conditions lead to

$$(2d) \quad \frac{q}{\Delta} = \frac{16\pi M^3}{27 L^2},$$

$$(1d) \quad \frac{q}{E_F} = 4\pi^2 \frac{M^4}{L^2}.$$

The value of  $L$  is changing slightly by adding a state to the already occupied level, but it changes much more at the beginning of the occupation of a new level. Just at filling the  $N$ th level, we have

$$(2d) \quad \frac{M^3}{L^2} = \frac{9}{4} \frac{(N+1)^3(N+2)^3}{(N+1)^2(N+2)^2(2N+3)^2} = \frac{9}{16} \frac{(N+1)(N+2)}{\left(N+\frac{3}{2}\right)^2} =$$

$$= \frac{9}{16} \left[ 1 + \frac{1}{4(N+1)(N+2)} \right]^{-1},$$

$$(1d) \quad \frac{M^2}{L} = \frac{4(N+1)^2}{2(N+1)^2} = 2.$$

Generally, the values of  $M^3/L^2$  and  $M^2/L$ , respectively, are changing between the expressions given above (these are the maximum values for a the given  $N$  value) and the minimum values just after beginning the occupation of a new energy level  $N+1$ :

$$(2d) \quad \frac{M^3}{L^2} = \frac{[N(N+1)(N+2) + 1]^3}{\left[\frac{2}{3}(N+1)(N+2)(2N+3) + N+4\right]^2} = \frac{9}{16} \left[ 1 - \frac{N+1}{(N+2)^2} \right]$$

$$\left[ 1 + \frac{N}{2N^2 + 5N + 6} \right]^2$$

$$(1d) \quad \frac{M^2}{L} = \frac{(2N+3)^2}{2(N+1)^2 + 2N+3} = 2 \left[ 1 + \frac{1}{(2N+3)^2} \right]^{-1}.$$

The differences between the values for different  $N$ , as well as between the maximum and minimum values are not changing very much (see fig. 1), we can therefore use their limiting values

$$\lim_{N \rightarrow \infty} \frac{M^3}{L^2} = \frac{9}{16},$$

$$\lim_{N \rightarrow \infty} \frac{M^4}{L^2} = 4.$$

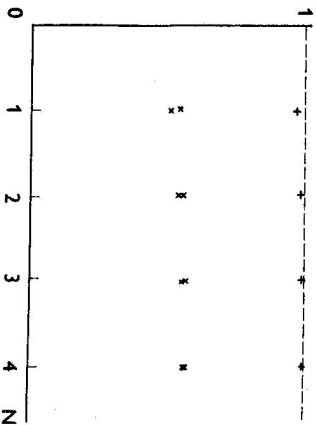


Fig. 1. The results for the maximum (just at filling the  $N$ th level) and minimum (just after beginning the new level  $N+1$ ) values of  $M^3/L^2$  in the two-dimensional bags (crosses  $x$ ) and half of the minimum values of  $M^2/L$  (crosses  $+$ ) for the one-dimensional bags (the maximum just at filling an arbitrary level is equal 2).

Hence, the two conditions for the baglike excitations to be energetically favourable are given by

$$(2d) \quad \frac{q}{\Delta} = \frac{\pi}{3} \approx 1.05,$$

$$(1d) \quad \frac{q}{E_F} = 16\pi^2.$$

Although the last inequality for the one-dimensional bags is valid for both choices of the quantity  $q$  ( $E_F$  and  $\Delta$ , respectively), the condition for the two-dimensional bags is fulfilled only just for  $q = \Delta$ , but no more for  $E_F/\Delta \leq 1.05$  of the second choice ( $q = E_F$ ).

In our opinion [10], the choice of  $q = \Delta$  is physically more relevant since by adding one quasiparticle to the excited bag, we have to take it from the

condensed state already present in the superconductor. This situation is quite different from the case when we had to add this quasiparticle to a system of other quasiparticles filled up to the Fermi energy. However, in the case when in spite of the mentioned argument the value of  $q$  should be larger than 1.054, one would need an additional mechanism for the existence of the two-dimensional bags. Some of them were already suggested [10, 11], e.g. the periodic structure of bags, surface contribution to the free energy, etc.

### III. ENERGY STEPS OF THE BAGS IN CHANGING THE NUMBER OF THE INVOLVED QUASIPARTICLES IN THE BAG

The energy of the excited bags depends on the number of the quasiparticles present in the bag. The equilibrium number can change with the temperature and it can be influenced by structural and other types of fluctuations in the material, too. We obtain thus some quantized energy levels in the bags. These changes can influence the tunnelling characteristics of the superconductor and one can expect some resonance structures at voltages corresponding to these energy differences.

The excited bags should, of course, interact with other fields (electromagnetic, acoustic, etc.). The corresponding resonances should appear in these interactions, too.

The energy differences at the transition of the bag from the state  $L_1$  (with number  $M_1 = M + 1$ ) to the neighbouring state  $L_2$  (with particle number  $M_2 = M$ ) are given in both cases by

$$(2d) \quad \delta E_2 = \frac{3}{2} \left( \frac{1}{2\pi} \right)^{1/3} \Delta \left( \frac{q}{\Delta} \right)^{1/3} (L_1^{2/3} - L_2^{2/3}) = 0.81 \Delta \left( \frac{q}{\Delta} \right)^{1/3} Z_2,$$

$$(1d) \quad \delta E_1 = \left( \frac{1}{2\pi} \right)^{1/3} \Delta \left( \frac{q}{E_F} \right)^{1/4} (L_1^{1/2} - L_2^{1/2}) = 0.54 \Delta \left( \frac{q}{E_F} \right)^{1/4} Z_1,$$

Table 2

The difference  $Z_2 = L_1^{2/3} - L_2^{2/3}$  between two neighbouring states with changing the quasiparticle numbers in the bag ( $M_1 = M + 1$ ,  $M_2 = M$ ) for the isotropic two-dimensional quadratic potential well, which is determining the energy differences between two neighbouring states. The values before filling a given energy level, as well as just after beginning a new energy level, are given (the vertical marks between different  $L$  values mean the beginning of a new energy level).

$L$	2	4	8 ... 16	20	26 ... 50	56	64 ... 112	120		
$Z_2$	0.93	1.48		1.02	1.4		1.07	1.36	1.09	1.33
				130 ... 210		220	232 ... 352	364	378 ...	$\infty$
						1.11	1.31	1.127	1.3	... 1.2114

Table 3  
Analogous characteristic numbers to those in table 2 for the one-dimensional quadratic potential well,  $Z_1 = L_1^{1/2} - L_2^{1/2}$ . The vertical marks are again indicating the beginning of new energy levels (they can be occupied by two quasiparticles only).

$L$	1	2	5	8	13	18	25	32	41	50	61			
$Z_1$		1	0.82	0.59	0.77	0.63	0.75	0.657	0.746	0.668	0.74			
						72	85	98	113	128	...			
								0.675	0.734	0.68	0.73	0.683	...	0.707

where

$$Z_2 = L_1^{2/3} - L_2^{2/3}, \quad Z_1 = L_1^{1/2} - L_2^{1/2}.$$

The results for  $Z_2$  and  $Z_1$  for some  $L$  values are given in table 2 and table 3 in the two-dimensional and the one-dimensional case, respectively. One can see immediately that the differences are again not very large by changing the number of the quasiparticles in the bags, therefore one can use in the following considerations their limiting values

$$\lim_{M \rightarrow \infty} Z_2 = \left( \frac{4}{3} \right)^{2/3}, \quad \lim_{M \rightarrow \infty} Z_1 = 2^{-1/2}.$$

In this limiting case we then have

$$(2d) \quad \delta E_2 = \Delta \left( \frac{q}{\Delta} \right)^{1/3},$$

$$(1d) \quad \delta E_1 = 0.38 \Delta \left( \frac{q}{E_F} \right)^{1/4}.$$

For comparing with the experimental results [7-9], we take as characteristic the values for  $E_F = (0.2 - 1) \text{ eV}$ , and suppose approximately  $\Delta = 2k_B T_c \approx 16 \text{ meV}$ . The energy steps are then given by

$$(2d) \quad \delta E_2 \approx \begin{cases} (2.3-4)\Delta \approx 37-64 \text{ meV} & \text{for } q = E_F = 0.2 - 1 \text{ eV,} \\ \Delta \approx 16 \text{ meV} & \text{for } q = \Delta, \end{cases}$$

$$(1d) \quad \delta E_1 \approx \begin{cases} 0.38\Delta \approx 6 \text{ meV} & \text{for } q = E_F \text{ (otherwise independent of } E_F), \\ (0.2 - 0.13)\Delta \approx 3.2-2.1 \text{ meV} & \text{for } q = \Delta. \end{cases}$$

Whereas  $\delta E_2$  is larger or comparable with the value of the energy gap,  $\delta E_1$  is always smaller than  $\Delta$ .

#### IV. DISCUSSION

From the point of view of the existence of the baglike excitations, it is very important that in the one-dimensional fermion gas these excitations are always below the BCS type excitations (free plane-wave fermions). This is the case for all  $q$  values of the order of all physically meaningful parameters in the superconductors. This means also that no restriction should be made with respect to the interaction strength (weak coupling, strong coupling, very strong coupling) between the quasiparticles leading to the condensed state.

On the other hand, this model cannot explain the experimentally measured voltage steps for the resonances in the tunnelling characteristics (which are of the order of 40 mV), because the theoretical values for the steps, obtained by the energy differences of the bags by changing the number of the involved quasiparticles, are always smaller than the energy gap  $\Delta$ .

The situation is quite converse in the two-dimensional model. The energy of the baglike excitations is nearly of the same order as of the BCS excitations for  $q = \Delta$ , but they are no more energetically favourable by the choice  $q = E_F$ . Therefore, if the choice  $q = E_F$  would be "true" — in spite of the arguments given in paper [10] and in this paper — the existence of the baglike excitations in the two-dimensional fermion system could be questionable. We believe that all effects suggested for the three-dimensional baglike excitations [10, 11] (the strong anisotropy of the energy gap leading to values comparable with the Fermi energy at least in some directions, the periodic structure of the bags, the surface contribution to the free energy, the small carrier concentration and small value of the coherence length in high  $T_c$  superconductors) should facilitate their appearance in the two-dimensional fermion systems, too. The inclusion of all these effects into our model is possible, but could probably be a very difficult task. Some model calculations are in preparation.

On the other hand, the obtained energy steps for changing the number of the quasiparticles involved in the bag are in the range of the experimental results for the steps in the tunnelling characteristics of high  $T_c$  superconductors.

The baglike excitations should not be restricted to the BCS mechanism of the superconductivity (original, as well as extended models with phonons and other quasiparticles), as already stated [10, 11]. They could appear in other superconductivity mechanisms, too, e.g. in the bipolaronic model [13].

These excitations could play a role also in other properties of high  $T_c$  superconductors, mainly in the thermodynamic ones (thermal conductivity, heat capacity).

On the other hand, the resonance-like structures in the tunnelling characteristics of high  $T_c$  superconductors could have another origin, too (geometrical localization, band structure effects, interaction of the tunnelling particles with

phonon and other structures, etc.), see, e.g., the references [8—11, 14] and the literature cited therein.

Finally, we would like to mention the possible role of the Coulomb term in the free energy of the bags. One has to consider such a term as the concentration of the quasiparticles in the bag deviates from the mean concentration. This can be done by including the term  $(M - n)^2 e^2 / 4\pi\epsilon R$ , where  $n$  is the number of (paired and unpaired) quasiparticles with concentration  $n_s$  in the localization volume and  $\epsilon$  the dielectric permittivity.

We suppose that the bags are not occupying a large volume of the superconductor (otherwise, their interaction should be considered, too), therefore the mean concentration does not change considerably in the volume of the superconductor outside the bags.

The localization radius in the two-dimensional case is then given by the following relation:

$$R_2 = h \left( \frac{\pi}{m\Delta^2} \right)^{1/3} \left[ \left( \frac{q}{2m} \right)^{1/2} L_2 + \frac{(M - n)^2 e^2}{4\pi\epsilon h} \right]^{1/3}$$

The energy of the bag is  $E_2 \sim R_2^2$ , from which one can obtain a condition for the most favourable number of quasiparticles in the bag. In the limiting case  $M \gg 1$  (i.e.  $L_2 = 4M^{3/2}/3$ ), one has

$$R_2 \sim [4MM^{3/2} + B(M - n)^2]^{1/3},$$

where  $A$  and  $B$  are constants. From  $dE_2/dM = 0$  we obtain

$$fM^{1/2} + (M - n) = 0, \quad (1)$$

where  $f = 3A/4B = 0.04\epsilon$  and  $0.012e$  for  $q = E_F$  and  $\Delta$ , respectively.

The condition of minimum energy (1) can be fulfilled only for  $M < n$ , as expected ( $A$  and  $B$  are positive). The quadratic equation (1) can be solved in the form  $M(n)$  or  $n(M)$ :

$$M = n + \frac{f^2}{2} - \left( nf^2 + \frac{f^2}{4} \right)^{1/2},$$

$$n = M + f^2 M^{1/2} = M(1 + f^2 n M^{1/2}).$$

The latter form is more simple and more suitable for our further considerations.

One obtains in this way

$$E_2 \sim A^{2/3} M \left[ 1 + \frac{3f}{4M^{1/2}} \right]^{2/3}.$$

In spite of the determining equation (1) for the deviation of the quasiparticle density in the bag from the mean electron concentration,  $n - M = fM^{1/2} \sim fn^{1/2}$

(due to  $f \ll 1$ ), we cannot give the exact number of quasiparticles in the bag (and therefore the energy of the bag), because  $R_2$  is a function of  $M$  and thus of  $n$ , too. Only the inclusion of the "surface" term (or more precisely, the self-consistent calculation of  $M$ ) should solve this problem.

Nevertheless, for our purposes, i.e. the possible explanation of the resonances in the tunnelling current, this is not very important, as the required quantities  $Z_2$  and  $Z_1$  are not changing very much with changing  $M$ .

The full treatment of the problem would require a microscopic self-consistent description [12, 16, 17], as the quasiparticle excitations "deform" the gap in their immediate vicinity. The situation resembles the self-trapped polaron (see, e.g. [18]). Numerical calculations lead to cigar- and star-shaped one- and two-particle bags within the gap. One should therefore expect that the more particle bags will also be possible, because in the more-particle bags the gap deformation takes place only in the transition region bag-condensate.

Attempts were made to replace the self-consistency problem by including a surface term into the free energy [12] from the transition region between the bag and the condensate. The results did not change substantially. Nevertheless, we believe that the inclusion of such a term (or, more precisely, the self-consistent calculation of the problem) is needed for the determination of the equilibrium number of quasiparticles in the bag. This should be very important for calculating the thermodynamic properties of superconductors in the presence of localized baglike excitations.

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#### О ВОЗМОЖНОСТИ ОПРЕДЕЛЕНИЯ МЕШКОБРАЗНЫХ ВОЗБУЖДЕНИЙ В ВЫСОКОТЕМПЕРАТУРНЫХ СВЕРПРОВОДНИКАХ СУЩЕСТВУЮЩИХ ЗА СЧЕТ НИЗКОДИМЕНЗИОНАЛЬНОСТИ ЭЛЕКТРОННЫХ СОСТОЯНИЙ

Определены условия при которых локальные многочастичные мешкообразные возбуждения в двух и одномерных системах фермионов лежат ниже БКШ возбуждениям. Показано, что в двумерных системах мешки существуют, если характеристическая величина — потенциал мешка сравнима с энергетической щелью, но не могут существовать, если такой потенциал определяет энергии Ферми. Также надо отметить, что мешки оказываются энергетически выгодными в одномерной системе фермионов.

Отмечается, что изменения энергии в мешках с изменением числа включенных квазичастиц связаны с резонансами тунсипирующих характеристик высокотемпературных сверхпроводников. В случае двумерных мешков эти величины сравнимы с экспериментальными результатами скачка напряжения туннельного тока ( $\sim 40$  мВ), когда в одномерной системе эти изменения оказываются меньше энергетической щели. Кулоновский член при определении квазичастиц тоже учитывается в свободной энергии, но в расчетах не удалось определить число равновесных квазичастиц. Надо подчеркнуть, что существование отдельных мешков влияет на электромагнитные и термодинамические свойства высокотемпературных сверхпроводников.