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HYDROMAGNETIC CONVECTION OF RADIATING RAREFIELD GAS THROUGH A VERTICAL CHANNEL

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The effect of radiation on the combined free and forced convection flow of an electrically conducting rarefied gas through a vertical channel permeated by a uniform transverse magnetic field with slip flow and temperature jump boundary conditions has been considered. The expressions for the velocity, the induced magnetic field, temperature, the flow rate and the heat transfer coefficient due to thermal conduction have been obtained and the influence of radiation on the first three quantities has been shown graphically.

I. INTRODUCTION

The problems of heat transfer in electrically conducting fluids permeated by electromagnetic fields have been studied by many authors. Such studies are of great importance in the design of magnetohydrodynamic generators, cross-field accelerators, shock tubes, pumps, etc., a comprehensive review of which has been given by Romig [1]. The above studies were, however, restricted to the case in which the effect of radiation on heat transfer was absent. However, this effect is of great importance in space applications and higher operating temperatures.

Grief et al [2] obtained an exact solution for the problem of laminar convection of a radiating gas in a vertical channel and Viskanta [3] considered the forced convection flow in a horizontal channel in the presence of a uniform vertivac magnetic field. Gupta and Gupta [4] studied the radiation effect in hydromagnetic convection in a vertical channel.

Now, in the case of rarefield gases, the ordinary continuum approach fails to yield satisfactory results. When the gas in slightly rarefield, results agreeing with the observed phenomena can be analysed by solving the usual Navier-Stokes equations together with the modified boundary conditions allowing for a velocity slip at the boundary surface [5]. For subsonic flows of relatively hot gases,

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the assumption of incompressibility is physically realizable [6]. Such problems have good industrial applications in aircraft response to atmospheric gusts, reentry of a space craft etc.

The purpose of the present paper is to consider the effect of radiation on the combined free and forced convection flow of an electrically conducting rarefied gas through an open-ended vertical channel in presence of a uniform magnetic field perpendicular to the direction of flow. Confining the analysis to the optically thin limit, closed-form solutions are obtained for temperature, velocity, induced magnetic field, flow rate and the heat transfer coefficient by using the first order velocity slip and temperature jump boundary conditions. Variations of temperature, velocity and the magnetic field are shown graphically for different values of the radiation parameter.

II. BASIC EQUATIONS AND THE PROBLEM

The equations for the steady motion of a viscous, incompressible, conducting fluid are (in M. K. S units)

$$(\boldsymbol{v}.\nabla)\boldsymbol{v} = -\frac{1}{\varrho}\nabla p + \frac{1}{\mu\varrho}(\text{rot}\,\boldsymbol{B}) \times \boldsymbol{H} + \nu\nabla^2\boldsymbol{v} + g\beta\Theta\boldsymbol{k}, \tag{1}$$

while the magnetic induction, energy and continuity equations are

$$\boldsymbol{O} = \text{rot}(\boldsymbol{v} \times \boldsymbol{H}) + \frac{1}{\mu\sigma}\nabla^2\boldsymbol{H}, \tag{2}$$

$$(\boldsymbol{v}.\nabla)\Theta = \alpha\nabla^2\Theta - \frac{1}{\varrho C_p}\nabla.\boldsymbol{q}_R + \Phi + \frac{J^2}{\sigma} \tag{3}$$

$$\nabla.\boldsymbol{v} = 0. \tag{4}$$

In the above equations, $\boldsymbol{v}$ is the velocity, $\boldsymbol{B}$ the induced magnetic field, $\boldsymbol{H}$ the total magnetic field, $\boldsymbol{k}$ the unit vector in the vertical direction, ϱ the reference density, p the pressure, μ the magnetic permeability, ν the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion, σ the electrical conductivity, α the thermal diffusivity, C_p the specific heat at constant pressure,

$$\Theta = T - T_{\omega_0}, \tag{5}$$

T being the temperature and T_{ω_0} the reference temperature (taken as a constant), $\boldsymbol{q}_R$ the radiative heat flux, Φ the viscous dissipation function and J^2/σ is the Joule dissipation function, J being the current density. We shall assume that ϱ, μ, ν, g, β, σ, α and C_p are constants.

Let us consider the flow of an electrically conducting rarefied gas through a vertical channel between two non-conducting vertical plates, distant $2L$ apart. The surface temperatures of the plates are assumed to vary along the vertical direction. Let the centre of the channel be taken as the origin, the vertical direction as the z-axis and the x-axis is along the direction normal to the plates. We also assume that a uniform magnetic field B_0 acts in the direction of the x-axis.

Now for a fully developed laminar flow in a uniform magnetic field, the velocity and the induced magnetic have only a component in the vertical direction and all the physical variables expect temperature and pressure are functions of x. Assuming that the temperature T inside the fluid has the vertical gradient N (a constant), we can take [4]

$$T = T^*(x) + N.z. \tag{6}$$

The momentum equations in the x and z-direction are

$$\frac{\partial p}{\partial x} + \frac{B}{\mu}\frac{dB}{dx} = 0, \tag{7}$$

$$\nu\frac{d^2 v}{dx^2} + \frac{B_0}{\mu\varrho}\frac{dB}{dx} + g\beta(\Theta^* + N.z) - \frac{1}{\varrho}\frac{\partial p}{\partial z} = 0 \tag{8}$$

where

$$\Theta^* = T^*(x) - T_{\omega_0}, \tag{9}$$

v is the velocity and B is the induced magnetic field in the z-direction.

The equations of continuity is satisfied identically. The energy and the magnetic induction equations are

$$Nv = \alpha\frac{d^2\Theta^*}{dx^2} - \frac{1}{\varrho C_p}\frac{\partial q_R}{\partial x}, \tag{10}$$

$$\frac{d^2 B}{dx^2} + \sigma\mu B_0\frac{dv}{dx} = 0. \tag{11}$$

In equation (10), we have neglected viscous and ohmic dissipation. The fluid does not absorb its own emitted radiation in the case of an optically thin limit, that is, there is no self absorption, but the fluid does absorb radiation emitted by the boundaries. Using the relation [7]

$$\frac{\partial q_R}{\partial x} = 4(T - T_\omega)\int_0^\infty K_{\lambda\omega}\left(\frac{de_{b\lambda}}{dT}\right)_\omega d\lambda \tag{12}$$

for an optically thin limit and for non-grey gas near equilibrium, it is easy to see that the equation (10) gives

$$Nv = \alpha \frac{d^2\Theta^*}{dx^2} - C\Theta^*, \tag{13}$$

where

$$C = \frac{4}{\varrho C_P} \int_0^\infty K_{\lambda_0} \left(\frac{de_{b\lambda}}{dT}\right)_0 d\lambda$$

and K_λ is the absorption coefficient, $e_{b\lambda}$ the Planck function and the subscript 0 (zero) indicates that the quantities have been evaluated at the reference temperature T_{ω_0}. We shall limit our study to small variations of temperature.

Integrating (7) with respect to x, we get

$$p = -\frac{B^2}{2\mu} + f_1(z).$$

Substituting this into (8) we have

$$v\frac{d^2v}{dx^2} + \frac{B_0}{\mu\varrho}\frac{dB}{dx} + g\beta\Theta^* = \frac{1}{\varrho}\frac{df_1}{dz} - g\beta Nz. \tag{14}$$

The l.h.s. is a function of x, only and the r.h.s. is a function of z only. Then each side must be equal to the same constant C_1 (say). Thus

$$v\frac{d^2v}{dx^2} + \frac{B_0}{\mu\varrho}\frac{dB}{dx} + g\beta\Theta^* = C_1. \tag{15}$$

This constant C_1 depends on the physics of the problem. It may be determined either from the end conditions of pressure to which the channel is subjected or from the mass flow through the channel. If we introduce the following non-dimensional quantities

$$\eta = x/L, \quad u = Lv/\alpha, \quad t = -\Theta^*/NL, \quad b = B/B_0 \tag{16}$$

M = the Hartmann number = $B_0 L(\sigma/v\varrho)^{\frac{1}{2}}$
R_a = the Rayleigh number = $g\beta NL^4/v\alpha$
P_m = the Magnetic Prandtl number = $\alpha\sigma\mu$,

equations (15), (11) and (13) reduce to

$$\frac{d^2u}{d\eta^2} + \frac{M^2}{P_m}\frac{db}{d\eta} - R_a . t = C_2, \tag{17}$$

$$\frac{1}{P_m}\frac{d^2b}{d\eta^2} + \frac{du}{d\eta} = 0,$$

$$\frac{d^2t}{d\eta^2} - F.t = -u, \tag{18}$$

where

$$F = \frac{L^2C}{\alpha}, \quad C_2 = \frac{C_1L^3}{\alpha}. \tag{19}$$

The first order velocity slip and the temperature jump boundary conditions, neglecting the thermal creep term are [8]

$$u = \mp h\frac{du}{d\eta} \quad \text{at} = \pm 1, \tag{20}$$

$$t = \mp f\frac{dt}{d\eta} \quad \text{at} = \pm 1, \tag{21}$$

where

$$h = \frac{2 - f_1}{f_1} L_1,$$

f_1 being the reflection coefficient and

$$f = \frac{2 - f_2}{f_2} \cdot \frac{2v}{v+1} \cdot \frac{L_1}{P},$$

f_2 being the thermal accomodation coefficient, v is the specific heat ratio P is the Prandtl number and

$$L_1 = \mu\left(\frac{\pi}{2p\varrho}\right)^{\frac{1}{2}}$$

is the mean free path. We shall take h and f as constants.

Also, since the walls of the channel are non-conducting, the boundary conditions for the magnetic field are

$$b = 0 \text{ at } \eta = \pm 1. \tag{22}$$

III. SOLUTIONS OF THE PROBLEM

Integrating the equation (18) w.r.t. η, we get

$$\frac{1}{P_m}\frac{db}{d\eta} + u = \text{constant} = C_3. \tag{23}$$

Eliminating u and b from (17), (19) and (23) we get

$$\frac{d^4t}{d\eta^4} - (F + M^2)\frac{d^2t}{d\eta^2} + (M^2F + R_a)t = C_4, \tag{24}$$

where

$$C_4 = M^2C_3 - C_2.$$

From equation (24) we get the solution for $t(\eta)$ by using the boundary conditions (21) and also obtain the solutions for $u(\eta)$ and $b(\eta)$ from (19) and (23) by using the boundary conditions (20) and (22).
These solutions are

$$t(\eta) = A_1\cosh K_1\eta + A_2\cosh K_2\eta + \frac{C_4}{M^2F + R_a}, \tag{25}$$

$$u(\eta) = (F - K_1^2)A_1\cosh K_1\eta + (F - K_2^2)A_2\cosh K_2\eta + \frac{FC_4}{M^2F + R_a}, \tag{26}$$

$$\frac{b(\eta)}{P_m} = \frac{F - K_1^2}{K_1}A_1(\eta\sinh K_1 - \sinh K_1\eta) + \frac{F - K_2^2}{K_2}A_2(\eta\sinh K_2 - \sinh K_2\eta), \tag{27}$$

where

$$K_1, K_2 = \left[\frac{1}{2}(F + M^2) \pm \frac{1}{2}\{(F - M^2)^2 - 4R_a\}^{\frac{1}{2}}\right]^{\frac{1}{2}},$$

$$A_1 = -\frac{C_4}{(M^2F + R_a)\Lambda}[(F - K_2^2)(\cosh K_2 + hK_2\sinh K_2) - F(\cosh K_2 + fK_2\sinh K_2)], \tag{28}$$

$$A_2 = \frac{C_4}{(M^2F + R_a)\Lambda}[(F - K_1^2)(\cosh K_1 + hK_1\sinh K_1) - F(\cosh K_1 + fK_1\sinh K_1)],$$

$$\Lambda = (F - K_2^2)(\cosh K_2 + hK_2\sinh K_2)(\cosh K_1 + fK_1\sinh K_1) - (F - K_1^2)(\cosh K_1 + hK_1\sinh K_1)(\cosh K_2 + fK_2\sinh K_2).$$

The non-dimensional flow rate $\tilde{\omega}$ and the heat transfer coefficient h_t(at the wall $\eta = 1$) due to thermal conduction are given by

$$\tilde{\omega} = \int_{-1}^{1} u\,d\eta = 2\left[A_1/K_1(F - K_1^2)\sinh K_1 + A_2/K_2(F - K_2^2)\sinh K_2 + \frac{C_4}{M^2F + R_a}\right] \tag{29}$$

and

$$h_t = -(dt/d\eta)_{\eta=1} = A_1K_1\sinh K_1 + A_2K_2\sinh K_2. \tag{30}$$

IV. RESULTS AND DISCUSSION

It is seen that the radiation tends to increase the rate of heat transport by causing the increase of temperature of the gas. Thus the radiation effect reduces the influence of natural convection by reducing the temperature difference between the fluid and the channel walls. By taking $R_a = 1$, $M^2 = 10$, $C_4 = 1$, $h = 0.2$, $f = 0.5$, this effect has been shown in figure 1. Due to this effect, the velocity $u(\eta)$ at a point also decreases with increase of η. This is shown in

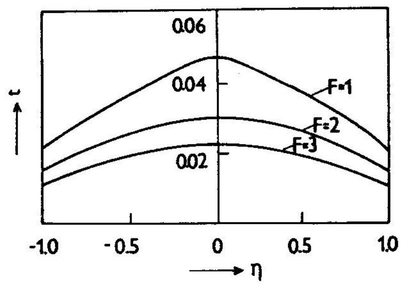

Fig. 1.

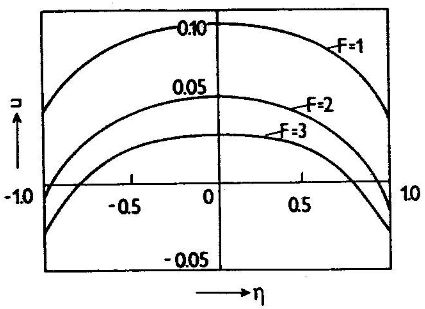

Fig. 2.

figure 2. In the case of the induced magnetic field (Fig. 3) it is seen that the field first increases with the increase of F up to $\eta = 0$ from $\eta = -1$ and then decreases with F and is zero again at $\eta = 1$.

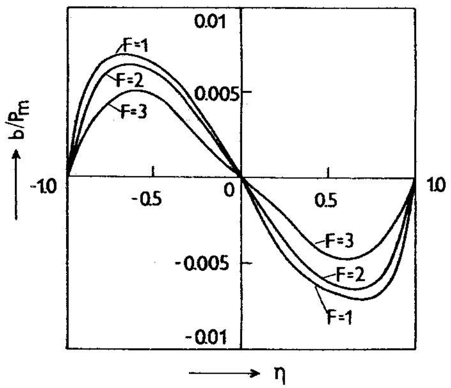

Fig. 3.

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ГИДРОДИНАМИЧЕСКАЯ ПЕРЕДАЧА РАДИАЦИОННО ОЧИЩЕННОГО ГАЗА В ВЕРТИКАЛЬНОМ КАНАЛЕ

Изучены граничные условия влияния радиации на свободный и вынужденный проток электрически проводимого газа вертикальным каналом, обеспеченный поперечным магнитным полем с ламинарным протоком и температурным перепадом. Получены выражения скорости, индуцированного магнитного поля, температуры, скорости протока и коэффициента передачи тепла. Влияние радиации на первые три величины привидится в графической форме.