THE CHANGE OF THE FFIRST ADIABATIC INVARIANT OF A CHARGED PARTICLE IN THE DIPOLAR GEOMAGNETIC FIELD WITH D_{st} VARIATION

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The fluctuations of the magnetic moment of a charged particle orbital motion in a geomagnetic dipolar field disturbed during magnetic storms are studied. The relation between the relative change of the magnetic moment of particle mirroring in opposite hemispheres during one half of the bounce period in a perturbed and an unperturbed field estimated theoretically is analysed by numerical calculations of particle trajectories. A satisfactory agreement is found as regards the description of the change of the first adiabatic invariant, adequate to the dipolar field.

I. INTRODUCTION

The first adiabatic invariant of tharged particle motion in the magnetic field is the magnetic moment of its orbital motion μ. The stability of the motion of particles in magnetic traps including geomagnetic ones is controlled by the degree of conservation of an invariant μ during manifold bounces of the particle along the field line between the mirror points. The character of the motion depends on the parameter of the nonadiabaticity $\chi = \varrho/R_c$, where ϱ is the gyroradius of the particle and R_c is the curvature radius of the field line. For the dipolar field the value $\chi = 3p/qBM$, where p is the impulse of the particle, q — its charge, B the magnetic field magnitude and M is the dipolar moment. For a low $\chi(\chi \ll 1)$ the conservation of μ is satisfied. The adiabatic invariancy of μ in the general case means that the magnetic moment is slightly changing in infinite time. This is connected with the possibility of a superposition of small fluctuations of μ during manifold oscillations of the bouncing particle. These superpositions $\Delta\mu$ may have a statistical character [1]. The random changes of μ lead to the smoothing of the mirror points along the guiding field line. This may lead as a consequence to the entrance of a part of the particle population into the loss cone. For the description of losses of particles it is necessary to

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know the corresponding diffusion coefficient on μ, which is determined through the value of $\Delta\mu$ related to the half period of the latitude bounces.

In our earlier paper [2] we have shown the change of the shape of the loss cone for particles trapped or quasitrapped in a dipolar magnetic field when the D_{st} variation representing the distortion of the geomagnetic field by the ring current is present. Here we estimate on the basis of trajectory computations and of the theory the value $\Delta\mu$ in the dipolar geomagnetic trap distorted in the same way during the main phase of the a geomagnetic storm. The distorted field means a quiet geomagnetic field plus the D_{st} index field which is in the equatorial plane directed antiparallel to the main field [3]. During the magnetic storm a depression of the magnetic field is formed resulting in an increase of nonadiabatic effects of the radiation belt particles [4]. The duration of the D_{st} perturbation is usually longer than the periods of cyclic motion of the particle, thus we shall assume the D_{st} variation independent of time and we characterize it by the amplitude of disturbance h. Further, we assume for simplicity that the additional field h is uniform. In that case the sum field in the cartesian coordinate system with the centre in the dipole will differ from the usual dipolar one [5] in the z-component

$$B_{sx} = B_x = -\frac{3Mxz}{R^5}, \quad B_{sy} = B_y = -\frac{3Myz}{R^5}, \quad B_{sz} = -\frac{M}{R^5}(3z^2 - R^2) - h, \qquad (1)$$

where $R^2 = x^2 + y^2 + z^2$, the axis $z \uparrow\uparrow B_{se}$, the index "$e$" means that the corresponding value is taken on the equator ($z = 0$) and B_s is the sum of the field ($B_{se} = B_e - h$).

II. THEORETICAL ESTIMATES

We shall find the expression for $\Delta\mu$ in the described field B_s. Under real conditions it can be ralated to the particles "created" during the storms of the geomagnetic field, when during some time interval the described model is realized and the betatron effects may be neglected. The case of particles trapped before the substorm, when changes of all parameters in the system "particle-magnetic field" should be taken into account, will be discussed separately. We shall make the computation of the value $\Delta\mu$ by the scheme given in paper [6]. The initial expression for the determination of $\Delta\mu$ is the following [7]:

$$\Delta\mu = -Re\int_{l_1}^{l_2} \frac{v_\perp}{B_s R_{sc}}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)e^{i\varphi}\frac{dl}{v_\parallel}, \quad \varphi = \varphi_0 - \frac{\omega_e}{B_{se}}\int_0^l B_s \frac{dl}{v_\parallel}, \qquad (2)$$

where $v_\parallel$, $v_\perp$ are the parallel and the perpendicular component of the velocity vector related to the sum field B_s, l is the coordinate along the field line measured from the equator, l_1 and l_2 are mirror points, φ_0 is the phase of the particle measured in the same way as in [2], R_{sc} is the curvature radius of the field line and ω is the gyrofrequency of the particle. The magnetic field is described by the expression

$$B_s = \frac{M}{R^3}\left[\left(1 - \frac{hR^3}{M}\right)^2 + 3\left(1 + 2\frac{hR^3}{M}\right)\cos^2\Theta\right]^{1/2}, \qquad (3)$$

where $\cos\Theta = z/R$. In the magnetic field (3) the equation of the field line has the approximate form [4]

$$R \simeq \left[R_e \sin^2\Theta\left(1 + \frac{b}{2}\sin^6\Theta\right)\right] \Big/ \left[1 + \frac{b}{2}\right], \qquad (4)$$

where $b = h/B_e$.

The integration of (2) is carried out by the method described in [6]. We approximate the function (2) as the sum in the degrees of l near $B_s = 0$ and we confine ourselves to the derivatives of the second order. Then the zero of the function $B_s(l)$ will be the imaginary value $\tilde{l}$ near which the following expansion is valid

$$B_s \simeq B_{se}\left(1 + \frac{B_{se}^\parallel}{B_{se}} \cdot \frac{l^2}{2}\right) \simeq B_{se}^\parallel \tilde{l}(l - \tilde{l}), \qquad (5)$$

where

$$B_{se} \simeq B_e(1 - b); \quad \frac{B_{se}^\parallel}{B_{se}} \simeq \frac{9}{R_e^2}(1 + 3b); \quad \tilde{l} = -i\left(\frac{2B_{se}}{B_{se}^\parallel}\right).$$

Substituting (5) into the equation (2) we have as a result

$$\left(\frac{\Delta\mu}{\mu_e}\right)_s \simeq 0.74\,\frac{14 - \sin^2\alpha_e}{\sin 2\alpha_e} \cdot \cos\varphi_0 . \exp\left[-\frac{3\psi(\alpha_e)}{\chi}\left(1 - \frac{5}{2}b\right)\right]$$

$$\psi(\alpha_e) = \frac{1}{3\sqrt{2}\sin^2\alpha_e}\left(\frac{1 + \sin^2\alpha_e}{\sin\alpha_e} \cdot \ln\frac{1 + \sin\alpha_e}{\cos\alpha_e} - 1\right) \qquad (6)$$

$$\chi_e \simeq 5.04 \times 10^{-5} L^2 pc,$$

where $L = R_e/R_E$, pc is measured in MeV, R_E is the radius of the Earth and α_e is the equatorial pitch angle. A further task is to evaluate with the help of numerical calculations of the integration of particle motion in the magnetic field B_s the correctness of the analytical solution (6).

III. TRAJECTORY COMPUTATIONS

The motion of the particle in the magnetic field (1) is described by the Lorenz equation in the cartesian system of coordinates

$$\ddot{x} = a(\dot{y}B_z - \dot{z}B_y), \quad \ddot{y} = a(\dot{z}B_x - \dot{x}B_z), \quad \ddot{z} = a(\dot{x}B_y - \dot{y}B_x), \tag{7}$$

where $a = q/m$, m is the mass of the particle. The initial conditions are set in the equatorial plane ($z = 0$) and are determined by the point of injection $R_0(x_0, 0, 0)$, the velocity v_0, the pitch-angle α_e and by the phase φ_0 defined as the angle between the meridional plane and $v_\perp$ measured in the sense of the particle gyration. To the initial phase $\varphi_0 = 0$ there corresponds the point $R_e(x_0, 0)$. The initial velocity $\mathbf{v}_0$ is given in the values of α and $\varphi(v_0 \sin\alpha_e \cos\varphi_0, v_0 \sin\alpha_e \sin\varphi_0, v_0 \cos\alpha_e)$. The numerical integration of (7) was performed by the Runge-Kutta method of the fourth order. Because of the axial symmetry the accuracy was controlled both according to the conservation of v_0^2 and according to the generalized impulse of the particle having the form in the spherical coordinates

$$P_\varphi = mR^2 \sin^2\Theta\dot{\varphi} + qR\sin\Theta A_{s\varphi} = \text{const}, \tag{8}$$

where $A_{s\varphi}$ is the vector potential of the sum field. In our case

$$A_{s\varphi} = \frac{M\sin\Theta}{R^2} + \frac{hR\sin\Theta}{2}. \tag{9}$$

From equations (8) and (9) it follows that in the equatorial plane

$$\tilde{R}_e \sin\alpha_e \sin\varphi_0 + \frac{1}{R_e}\left(1 + \frac{b}{2}\right) = 2\gamma_1, \tag{10}$$

where $\tilde{R}_e = R_e//C_{st}$, $C_{st} = \left(\frac{Mq}{mV}\right)^{1/2}$ is Störmer's unit of length and the constant γ_1 is the Störmer length for the case $b = 0$. The expression (10) could be used for adjusting the initial conditions related to one tube of force ($\gamma_1(R_e, \alpha_e, \varphi_0) = \text{const}$).

In trajectory computations there was evaluated the change of the magnetic moment of the particle orbit $\mu = mv_\perp^2/2B_s$ during one half of the bounce motion, i.e. during its one pass between the mirror points. The integration (7) was performed from the equator to the mirror point ($z > 0$) and then after the change $a \to -a$ and $V_0 \to -V_0$ the trajectory of the particle was computed from the equator to the conjugated point ($z < 0$). Such a procedure is equivalent to the direct and the backward integration (in time) of equations (7) from one point on the equator. This way is suitable because the particle moving from one mirror point to another crosses the equatorial plane with the necessary parameters

$R(x_0, 0)$, α_e, φ_0. The primary aim of the trajectory computation was the task to find the value of K by means of the value

$$y = \ln\left[\left(\frac{\Delta\mu}{\mu}\right)_{b\neq 0} \Big/ \left(\frac{\Delta\mu}{\mu}\right)_{b=0}\right] = \frac{3\psi K}{\chi_e}\cdot b. \tag{11}$$

In this notation

$$\Delta\mu(v_0, b, \alpha_e, \varphi_0, x_0) = \mu_1(B_{sm_1}) - \mu_2(B_{sm_2}), \tag{12}$$

where B_{sm} is the field in the mirror point. According to the theoretical estimate (6)

$$K = \frac{\chi_e}{3\psi}\cdot\frac{y}{b} = 2.5. \tag{13}$$

IV. RESULTS AND CONCLUSION

The theoretical dependence of y on b for three selected triplets of values χ_e, α_e and φ_0 is shown in Fig. 1 by straight lines. The experimental data which denote the results of trajectory computations are presented in Fig. 1 as discrete points. The computations were performed for three energies of protons, namely 50, 200 and 600 MeV and several L parameters below $L = 3$. The D_{st} variation was set for given initial parameters with the step 50 nT from zero up to 400 nT.

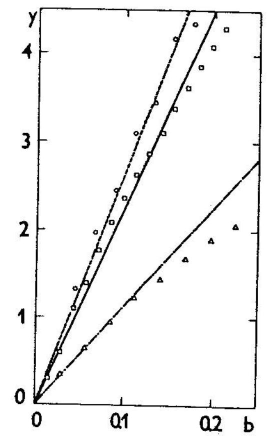

Fig. 1. Results of computed values y (defined by (11) dependent) on b ($b = h/B_e$, see text). Three sets of data are displayed: $E = 50$ MeV, $L = 2.4$, $\chi/3\psi = 0.095$, $\alpha = 4.01°$, $\varphi = 140°$ (0); $E = 50$ MeV, $L = 2.6$, $\chi/3\psi = 0.112$, $\alpha = 4.01°$, $\varphi = 140°$ (□); $E = 200$ MeV, $L = 2.6$, $\chi/3\psi = 0.219$, $\alpha = 20°$, $\varphi = 56°$ (△). The corresponding theoretical dependencies are shown as straight lines. The decrease of the computed values from the theoretical ones is apparent for larger $\chi/3\psi$ and large values of D_{st}.

From Fig. 1 we can see that the computed values are placed in the neighbourhood of their corresponding theoretical lines. Some dispersion of the computed points is connected with the method of determination of $\Delta\mu$ according to formulae (12). It is possible to select such initial conditions for which the fluctuations of μ around the mirror points are periodical and minimal. In such cases the difference between the average values of μ_1 and μ_2 will be quite well described by expression (12). If the quasiperiodicity of the motion is strong, which is e.g., the case of large α_e, then it is practically impossible to compute $\Delta\mu$. From (11) and Fig. 1 it follows that between the theoretical and the experimental values $\Delta\mu$ (*b*) and *K* an agreement is acceptable.

The D_{st} — field may sufficiently enchance the changes $\Delta\mu$ and consequently it may lead to an enhanced dispersion of the mirror points in *R* This effect may cause the breakdown of the conditions necessary for the finity of the motion and to the quasitrapping of the particles. These questions however, belong to a different sphere of interest.

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ИЗМЕНЕНИЕ ПЕРВОГО АДИАБАТИЧЕСКОГО ИНВАРИАНТА ЗАРЯЖЕНОЙ ЧАСТИЦЫ В ДИПОЛЬНОМ ГЕОМАГНЕТИЧЕСКОМ ПОЛЕ В ЗАВИСИМОСТИ ОТ D_{st} ВАРИАЦИЙ

В работе изучаются флуктуации магнитного момента орбитального движения заряжных частиц в дипольном поле нарушеном магнитными бурями. С применением нумерических вычислений, теоретически анализируется зависимость относительных изменений магнитного момента частицы, отображеной в обратном полушарии во время полупериода каука, от их траекторий в искаженном и не искаженном полях. Приемлемое согласие получено когда первый адиабатический инвариант соответствует дипольному полю.