ON THE TRANSITION TEMPERATURE OF A ONE-DIMENSIONAL METALLIC STRUCTURE

JANETKA, I[1]), Bratislava

The integral equations that enable to determine the mean-field temperature of transition to ordered states of a one-dimensional metallic structure are derived. The derived equations are applicable to any one-dimensional metallic structure in which the electron-electron interaction can be expressed in terms of the dielectric function.

The equations for the temperature of transition to the states with spin-density-waves and charge-density-waves formally coincide with the well-known equation for the temperature of transition to the superconducting state. The kernels of these equations depend on the spectral density of the inverse dielectric function. On the basis of the derived equations the relation of the coupling constants of the electron gas model to the effective electron-electron interaction is established.

I. INTRODUCTION

The transition temperature is a very important parameter of solids that marks their transition to a new state. The importance of obtaining the exact value of the transition temperature on the basis of microscopic calculation is undeniable, especially for designating the properties of one-dimensional (1D) metallic structures. Depending on the sign and the size of the electron-electron interaction these structures exhibit the transitions to states showing singlet superconductivity, triplet superconductivity, charge-density-waves, and spin-density-waves. Though the interaction between the electrons is dynamic and depends on their motion, the existing methods of treating 1D metallic structures replace the dynamic interaction by a static one. The intent of this paper is to formulate the dynamical approach to the description of the ordered states in 1D metallic structures and to obtain the equations for the transition temperatures that respect the concrete form of the effective electron-electron interaction in these structures.

The mathematical description of a 1D metallic structure usually begins with the discussion on the electron gas Hamiltonian [1—4]

¹) Slovak Academy of Sciences, Institute of Electrical Engineering, Dúbravská cesta 9, 842 39 BRATISLAVA, CSFR.

$$H = \sum_{q,\alpha} \xi(q) c_\alpha^+(q) c_\alpha(q) +$$
$$+ \frac{1}{2L} \sum_{\substack{q_1,\alpha \\ q_2,\beta \\ q}} v(q) c_\alpha^+(q_1 + q) c_\beta^+(q_2 - q) c_\beta(q_2) c_\alpha(q)$$

where $c_\alpha^+(q)(c_\alpha(q))$ creates (destroys) an electron with spin projection α in the 1D state of the wave vector q and the energy $\xi(q) = \hbar^2 q^2/2m - \mu$, μ is the chemical potential, k_F is the effective electron mass, $\mu \simeq \hbar^2 k_F^2/(2m)$ is the Planck constant, k_F is the Fermi wave vector, L is the length of the structure, and $v(q)$ is the electron-electron interaction which depends on the transferred wave vector q.

As a 1D system is characterized by a Fermi surface consisting of two points $q = \pm k_F$, there exist low-energy particle-hole excitations only in the neighbourhood of $q \simeq 0$ and $q \simeq 2k_F$. This means that, in general, only the electron-electron interaction processes involving wave vector transfers in the neighbourhood of these two values play an important role. Moreover, in the simplest case one restricts oneself to the interaction processes between electrons lying near the opposite sides of the Fermi surface and neglects the interaction processes between electrons on the same side of the Fermi surface. Then, one considers only the following two kinds of the electron-electron interaction processes: the interactions between electrons on the opposite side of the Fermi surface with the $q \simeq 0$ transfers and those with the $q \simeq 2k_F$ transfers. The size of the former interactions is usually parametrized by the coupling constant g_2 and that of the latter ones by g_1. The study of the ordered states in the model with the coupling constants g_1 and g_2 is referred to as "g-ology" [4, 5].

There are four possible types of the ordered states in this model. Particle-particle pairing leads to a singlet superconducting (SS) state or to a triplet superconducting (TS) state depending on the relative spin orientation of the two particles. On the other hand, particle-hole pairing can lead to a spin-density-wave (SDW) state or to a charge-density-wave (CDW) state, likewise depending on the relative spin orientation of the particle and the hole. The mean-field theory yields the following expression for the transition temperature

$$T_c \simeq \frac{2\gamma\mu}{\pi k_B} \exp{(1/\lambda_i)}, \tag{1}$$

where $\ln \gamma = C = 0.577...$ is the Euler constant, k_B is the Boltzmann constant, and λ_i is a different combination of g_1 and g_2 for each of the different ordered states:

$$\lambda_{SS} = (g_1 + g_2) N_F/2 \tag{2.a}$$
$$\lambda_{TS} = (g_2 - g_1) N_F/2 \tag{2.b}$$

$$\lambda_{SDW} = -g_2 N_F/2, \quad (2.c)$$

$$\lambda_{CDW} = (2g_1 - g_2)N_F/2, \quad (2.d)$$

where $N_F = m/(\pi \hbar^2 k_F)$ is the density of states. Evidently, as to which of the ordered states results, depends on the relative values of g_1 and g_2 as it is always favoured that ordered state whose transition temperature is higher than all the others. Hence, the determination of g_1 and g_2, i.e., their relations to the effective electron-electron interaction is of crucial importance.

The effective electron-electron interaction $v(q, \omega)$ is, however, a complex function not only of the momentum transfer $\hbar q$ but also the energy transfer $\hbar \omega$:

$$v(q, \omega) = \frac{v(q)}{\varepsilon(q, \omega)},$$

where $\varepsilon(q, \omega)$ is the dielectric function. The inverse dielectric function $1/\varepsilon(q, \omega)$ satisfies the causality principle. Therefore, it can be written in the spectral representation of the Kramers-Kronig type which for an imaginary frequency $i\omega$ states [6, 7]

$$\frac{1}{\varepsilon(q, i\omega)} = 1 + \int_0^\infty d\Omega \frac{2\Omega}{\Omega^2 + \omega^2} \operatorname{Im}\frac{1}{\pi \varepsilon(q, \Omega)}. \quad (3)$$

As the interaction between the electrons is dynamic, the method of treating it has to be a little more subtle than that used for treating the static interaction parametrized by the coupling constants g_1 and g_2. In this paper the method developed by Kirzhnits, Maximov and Khomskii [7, 8] for the description of the superconducting state in a three-dimensional system will be applied to a 1D electron gas interacting via an interaction of the general type, which corresponds to arbitrary spatial and frequency dispersion of the dielectric function in the 1D metallic structure. The treatment presented in this paper is divided into two parts, which deal separately with particle-particle and particle-hole pairings. The stress is particularly laid upon the factorization method which depends on the type of pairing. In the third part the relation of the present method to "g-ology" is briefly outlined.

II. EQUATIONS FOR THE TRANSITION TEMPERATURE

II.1 Particle-particle pairing

The starting point of the present treatment is the equation of motion for the 1D field operator $\psi_\sigma(z, t)$ [7, 9—11]

$$\left[i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_1^2} + \mu - \sum_\gamma \times \right.$$

$$\left. \times \int_{-\infty}^\infty dz_3 \int_{-\infty}^\infty dt_3 V(z_1 - z_3, t_1 - t_3) \psi_\gamma^+(z_3, t_3) \psi_\gamma(z_3, t_3) \right] \psi_\alpha(z_1, t_1) = 0, \quad (4)$$

where $V(z, t)$ is the Fourier transform of the interaction $v(q, \omega)$. Introducing the single-particle Green's function

$$i\hbar G_{\alpha\beta}(z_1 - z_2, t_1 - t_2) = \langle T\psi_\alpha(z_1, t_1)\psi_\beta^+(z_2, t_2)\rangle,$$

where T is the time-ordering operator, and the average $\langle ... \rangle$ refers to the ground state, one finds with the help of the equation (4) that it satisfies the following equation

$$\left(i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_1^2} + \mu \right) G_{\alpha\beta}(z_1 - z_2, t_1 - t_2) =$$

$$= \delta_{\alpha\beta}\delta(z_1 - z_2)\delta(t_1 - t_2) + \left(-\frac{i}{\hbar}\right)\sum_\gamma \int_{-\infty}^{+\infty} dz_3 \int_{-\infty}^\infty dt_3 V(z_1 - z_3, t_1 - t_3) \times$$

$$\times \langle T\psi_\gamma^+(z_3, t_3)\psi_\gamma(z_3, t_3)\psi_\alpha(z_1, t_1)\psi_\beta^+(z_2, t_2)\rangle, \quad (5)$$

where $\delta_{\alpha\beta}$ is the Kronecker delta symbol of the indexes α and β, $\delta(z)$ is the Dirac delta function of the argument z.

Under the condition of particle-particle pairing one can factorize the average of four field operators in the Gorkov way [9—11]

$$\langle T\psi_\gamma^+(z_3, t_3)\psi_\gamma(z_3, t_3)\psi_\alpha(z_1, t_1)\psi_\beta^+(z_2, t_2)\rangle =$$

$$= -i\hbar F_{\alpha\gamma}(z_1 - z_3, t_1 - t_3)\, i\hbar F_{\gamma\beta}^+(z_3 - z_2, t_3 - t_2),$$

where the anomalous Green's functions are defined by

$$i\hbar F_{\alpha\gamma}(z_1 - z_3, t_1 - t_3) = \langle T\psi_\alpha(z_1, t_1)\psi_\gamma(z_3, t_3)\rangle$$

$$i\hbar F_{\gamma\beta}^+(z_3 - z_2, t_3 - t_2) = \langle T\psi_\gamma^+(z_3, t_3)\psi_\beta^+(z_2, t_2)\rangle.$$

A particle-particle pair can be in the singlet or triplet states in dependence on the relative spin orientation of the particles. In the singlet state the spin dependence of Green's function is as follows [8, 11]

$$G_{\alpha\beta}(z, t) = \delta_{\alpha\beta}G(z, t)$$

$$F_{\alpha\beta}(z, t) = \sigma_{\alpha\beta}F(z, t)$$

$$F_{\alpha\beta}^+(z, t) = -\sigma_{\alpha\beta}F^+(z, t),$$

where

$$\sigma = \begin{pmatrix} 0, & 1 \\ -1, & 0 \end{pmatrix}.$$

In the triplet state it is

$$G_{\alpha\beta}(z, t) = \delta_{\alpha\beta} G(z, t)$$
$$F_{\alpha\beta}(z, t) = \delta_{\alpha\beta} F(z, t)$$
$$F^+_{\alpha\beta}(z, t) = \delta_{\alpha\beta} F^+(z, t)$$

In both cases, the equation (5) takes the form

$$\left(i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z_1^2} + \mu \right) G(z_1 - z_3, t_1 - t_3) F(z_1 - z_2, t_1 - t_2) = \delta(z_1 - z_2) \delta(t_1 - t_2) -$$

$$-i\hbar \int_{-\infty}^{\infty} dz_3 \int_{-\infty}^{\infty} d_{t_3} V(z_1 - z_3, t_1 - t_3) F(z_1 - z_3, t_1 - t_3) F^+(z_3 - z_2, t_3 - t_2)$$

and its Fourier transform is

$$[\hbar\omega_1 - \xi(q_1)] g(q_1, \omega_1) =$$

$$= 1 - i\hbar \int_{-\infty}^{+\infty} \frac{dq_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} v(q_1 - q_2, \omega_1 - \omega_2) f(q_2, \omega_2) f^+(q_1, \omega_1), \quad (6)$$

where $g(q, \omega)$, $f(q, \omega)$, and $f^+(q, \omega)$ are the Fourier tranforms of $G(z, t)$, $F(z, t)$ and $F^+(z, t)$, respectively.

Proceeding along the same lines one can derive the Fourier transform of the equation for the anomalous Green's function

$$[\hbar\omega_1 + \xi(q_1)] f^+(q_1, \omega_1) =$$

$$-i\hbar \int_{-\infty}^{\infty} \frac{dq_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} v(q_1 - q_2, \omega_1 - \omega_2) f^+(q_2, \omega_2) g(q_1, \omega_1). \quad (7)$$

Retaining only the linear term in the interaction (the lowest weak-coupling approximation) one easily gets from the equations (6) and (7)

$$[\hbar^2\omega_1^2 - \xi^2(q_1)] f^+(q_1, \omega_1) =$$

$$= -i\hbar \int_{-\infty}^{\infty} \frac{dq_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} v(q_1 - q_2, \omega_1 - \omega_2) f^+(q_2, \omega_2).$$

The temperature representation from of the previous equation at the transition temperature T_c is

$$f^+(q_1, i\omega_n) =$$

$$- \frac{k_B T_c}{\hbar^2\omega_n^2 + \xi^2(q_1)} \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{\infty} \frac{dq_2}{2\pi} v(q_1 - q_2, i\omega_n - i\omega_m) f^+(q_2, i\omega_m), \quad (8)$$

where $\omega_n = (2n + 1)\pi k_B T_c/\hbar$, and n is an integer.

The introduction of the spectral representation $\varphi(q, \Omega)$

$$f^+(q, i\omega_n) = \int_{-\infty}^{\infty} d\Omega \frac{\varphi(q, \Omega)}{i\omega_n - \Omega}$$

enables to perform the frequency sum is the equation (8) explicitly [7, 8]. Introducing a new unknown function

$$\Phi(q) = \int_0^{\infty} \frac{d\hbar\Omega}{\pi} [\varphi(q, \Omega) - \varphi(q, -\Omega)] |\xi(q)|$$

and making tedious arrangements [7, 8], one arrives at an equation of the BCS type

$$\Phi(q_1) = - \int_{-\infty}^{\infty} \frac{dq_2}{2\pi} \frac{th \frac{\xi(q_2)}{2k_B T_c}}{2\xi(q_2)} K(q_1, q_2). \Phi(q_2) \quad (9.a)$$

with

$$K(q_1, q_2) =$$

$$= v(q_1 - q_2) \left[1 + 2 \int_0^{\infty} \frac{d\hbar\Omega}{\hbar\Omega + |\xi(q_1)| + |\xi(q_2)|} Im \frac{1}{\pi\epsilon(\Omega, q_1 - q_2)} \right]. \quad (9.b)$$

The integral equation (9.a) with the kernel (9.b) is the 1D form of the equation obtained by Kirzhnits, Maximov and Khomskii [7, 8], who showed that $\Phi(q)$ corresponds to the gap function.

For the SS state the function $\Phi(q)$ is even and the equations (9) can be written as

$$\Phi_{SS}(q_1) = - \int_0^{\infty} \frac{dq_2}{2\pi} \frac{th \frac{\xi(q_2)}{2k_B T_c}}{2\xi(q_2)} K_{SS}(q_1, q_2) \Phi_{SS}(q_2) \quad (10.a)$$

with

$$K_{SS}(q_1, q_2) =$$

$$= v(q_1 - q_2)\left[1 + 2\int_0^\infty \frac{d\hbar\Omega}{\hbar\Omega + |\xi(q_1)| + |\xi(q_2)|}\, \text{Im}\, \frac{1}{\pi\varepsilon(\Omega, q_1 - q_2)}\right] +$$

$$+ v(q_1 + q_2)\left[1 + 2\int_0^\infty \frac{d\hbar\Omega}{\hbar\Omega + |\xi(q_1)| + |\xi(q_2)|}\, \text{Im}\, \frac{1}{\pi\varepsilon(\Omega, q_1 - q_2)}\right]. \qquad (10.b)$$

On the other hand, for the TS state the function $\Phi(q)$ is odd

$$\Phi_{TS}(q_1) = -\int_0^\infty \frac{dq_2}{2\pi}\, \frac{\text{th}\,\frac{\xi(q_2)}{2k_B T_c}}{2\xi(q_2)}\, K_{TS}(q_1, q_2)\, \Phi_{TS}(q_2)$$

$$K_{TS}(q_1, q_2) = \qquad (11.a)$$

$$= v(q_1 - q_2)\left[1 + 2\int_0^\infty \frac{d\hbar\Omega}{\hbar\Omega + |\xi(q_1)| + |\xi(q_2)|}\, \text{Im}\, \frac{1}{\pi\varepsilon(\Omega, q_1 - q_2)}\right] -$$

$$- v(q_1 + q_2)\left[1 + 2\int_0^\infty \frac{d\hbar\Omega}{\hbar\Omega + |\xi(q_1)| + |\xi(q_2)|}\, \text{Im}\, \frac{1}{\pi\varepsilon(\Omega, q_1 + q_2)}\right]. \qquad (11.b)$$

The temperatures of transition to the SS and TS states are the eigenvalues of the equations (10) and (11), respectively.

II. 2 Particle-hole pairing

In this case again, the starting point is the equation of motion for the 1D field operator but written in this form

$$\left[i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial z_1} + \hbar k_F\right)^2 + \mu - \right.$$

Then, the equation for the single-particle Green's function defined by

$$i\hbar G_{\alpha\beta}(z_1 - z_2, t_1 - t_2) = \langle T\, e^{-ik_F z_1} \psi_\alpha(z_1, t_1)\, \psi_\beta^+(z_2, t_2)\, e^{ik_F z_2}\rangle$$

$$\left[i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial z_1} + \hbar k_F\right)^2 + \mu\right] G_{\alpha\beta}(z_1 - z_2, t_1 - t_2) =$$

$$-\sum_\gamma \int_{-\infty}^\infty dz_3 \int_{-\infty}^\infty dt_3\, V(z_1 - z_3, t_1 - t_3)\, \psi_\gamma^+(z_3, t_3)\, \psi_\gamma(z_3, t_3)\left]e^{-ik_F z_1}\psi_\alpha(z_1, t_1)\right]e^{-ik_F z_1}\psi_\alpha(z_1, t_1) = 0.$$

$$= \delta_{\alpha\beta}\delta(z_1 - z_2)\delta(t_1 - t_2) + \left(-\frac{i}{\hbar}\right)\sum_\gamma \int_{-\infty}^\infty dz_3 \int_{-\infty}^\infty dt_3\, V(z_1 - z_3, t_1 - t_3)\, e^{ik_F z_2} \times$$

$$\times \langle T\, e^{-ik_F z_1} \psi_\gamma^+(z_3, t_3)\, \psi_\gamma(z_3, t_3)\, \psi_\alpha(z_1, t_1)\, \psi_\beta^+(z_2, t_2)\, e^{ik_F z_2}\rangle. \qquad (12)$$

Under the condition of particle-hole pairing the following factorization of the Hartree-Fock type leads to the proper description of the transition

$$\langle T\, e^{-ik_F z_1} \psi_\gamma^+(z_3, t_3)\, \psi_\gamma(z_3, t_3)\, \psi_\alpha(z_1, t_1)\, \psi_\beta^+(z_2, t_2)\, e^{ik_F z_2}\rangle =$$

$$= i\hbar G_{\alpha\gamma}(z_1 - z_3, t_1 - t_3)\, i\hbar F_{\gamma\beta}^+(z_3 - z_2, t_3 - t_2)\, e^{2ik_F(z_3 - z_1)} -$$

$$- i\hbar F_{\gamma\gamma}(0, -0)\, i\hbar F_{\alpha\beta}^+(z_1 - z_2, t_1 - t_2)\, e^{2ik_F(z_3 - z_1)},$$

where anomalous Green's functions are given by

$$i\hbar F_{\alpha\gamma}(z_1 - z_3, t_1 - t_3) = \langle T\, e^{-ik_F z_1} \psi_\alpha(z_1, t_1)\, \psi_\gamma^+(z_3, t_3)\, e^{-ik_F z_3}\rangle$$

$$i\hbar F_{\gamma\beta}^+(z_3 - z_2, t_3 - t_2) = \langle T\, e^{ik_F z_3} \psi_\gamma^+(z_3, t_3)\, \psi_\beta^+(z_2, t_2)\, e^{ik_F z_2}\rangle.$$

However, particle-hole pairs consist of particles and holes lying near the opposite side of the Fermi surface. Hence, the Fourier transforms of anomalous Green's functions are supposed to be non-zero only for the wave vectors q from the interval $-k_F/2 \leqslant q \leqslant k_F/2$.

Again, there exist two possible relative spin orientations of a particle and a hole. In the SDW state (the triplet state of the particle-hole pair) the spin dependence of Green's function is

$$G_{\alpha\beta}(z, t) = \delta_{\alpha\beta} G(z, t)$$
$$F_{\alpha\beta}(z, t) = \sigma_{\alpha\beta} F(z, t)$$
$$F_{\alpha\beta}^+(z, t) = -\sigma_{\alpha\beta} F^+(z, t)$$

and in the CDW state (the singlet state of the particle-hole pair) it is

$$G_{\alpha\beta}(z, t) = \delta_{\alpha\beta} G(z, t)$$
$$F_{\alpha\beta}(z, t) = \delta_{\alpha\beta} F(z, t)$$
$$F_{\alpha\beta}^+(z, t) = \delta_{\alpha\beta} F^+(z, t).$$

Then, the equation (12) takes the form

$$\left[i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial z_1} + \hbar k_F\right)^2 + \mu\right] G(z_1 - z_2, t_1 - t_2) = \delta(z_1 - z_2)\delta(t_1 - t_2) +$$

$$+ i\hbar\int_{-\infty}^\infty dz_3 \int_{-\infty}^\infty dt_3\, V(z_1 - z_3, t_1 - t_3)\, F(z_1 - z_3, t_1 - t_3)\, F^+(z_3 - z_2, t_3 - t_2) -$$

$$-2i\hbar\Theta \int_{-\infty}^{\infty} dz_3 \int_{-\infty}^{\infty} dt_3\, V(z_1 - z_3, t_1 - t_3) F(0, -0) \times$$

$$\times F^+(z_1 - z_3, t_1 - t_2)\, e^{2ik_F(z_3 - z_1)},$$

where Θ is equal to zero for the SDW state and 1 for the CDW state. The Fourier transform of the previous equation is

$$[\hbar\omega_1 - \xi(q_1 + k_F)]\, g(q_1, \omega_1) =$$

$$= 1 + i\hbar \int_{-k_F/2}^{k_F/2} \frac{dq_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} [v(q_1 - q_2, \omega_1 - \omega_2) -$$

$$- 2\Theta v(2k_F, 0)\, e^{i\omega_2 0}]\, f(q_2, \omega_2)\, f^+(q_1, \omega_1),$$ (13)

where $g(q, \omega)$, $f(q, \omega)$ and $f^+(q, \omega)$ are the Fourier transforms of $G(z, t)$, $F(z, t)$ and $F^+(z, t)$, respectively.

In a similar way, one finds

$$[\hbar\omega_1 - \xi(q_1 - k_F)]\, f^+(q_1, \omega_1) =$$

$$= i\hbar \int_{-k_F/2}^{k_F/2} \frac{dq_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} [v(q_1 - q_2, \omega_1 - \omega_2) -$$

$$- 2\Theta v(2k_F, 0)\, e^{i\omega_2 0}]\, f^+(q_2, \omega_2)\, g(q_1, \omega_1).$$ (14)

The two equations (13) and (14) yield the equation for $f^+(q, \omega)$, which can be written into the temperature representation form

$$f^+(q_1, i\omega_n) = - \frac{k_B T_c}{[i\hbar\omega_n - \xi(q_1 + k_F)][i\hbar\omega_n - \xi(q_1 - k_F)]} \times$$

$$\times \sum_{m=-\infty}^{\infty} \int_{-k_F/2}^{k_F/2} \frac{dq_2}{2\pi} [v(q_1 - q_2, i\omega_n - i\omega_m) - 2\Theta v(2k_F, 0)]\, f^+(q_2, i\omega_m).$$

To simplify the treatment of 1D systems, one usually linearizes the electronic spectrum [3]

$$\xi(q + k_F) \simeq \eta(q) \qquad \xi(q - k_F) \simeq -\eta(q),$$

where $\eta(q) = \hbar^2 k_F q/m$. This is a quite reasonable approximation in the finite region of the width k_F around the point $q = 0$.

Then, the basic equation for the determination of the transition temperature is of the same form as the equation (8)

$$f^+(q_1, i\omega_n) = \frac{k_B T_c}{\hbar^2 \omega_n^2 + \eta^2(q_1)} \times$$

Therefore, one can simply state that the temperature of transition to the SDW state is to determined as the eigenvalue of the following equation

$$\Phi_{SDW}(q_1) = \int_{-k_F/2}^{k_F/2} \frac{dq_2}{2\pi} \frac{\mathrm{th}\frac{\eta(q_2)}{2k_B T_c}}{2\eta(q_2)} K_{SDW}(q_1, q_2) \Phi_{SDW}(q_2)$$ (15.a)

where

$$K_{SDW}(q_1, q_2) = v(q_1 - q_2) \times$$

$$\times \left[1 + 2 \int_0^{\infty} \frac{d\hbar\Omega}{\hbar\Omega + |\eta(q_1)| + |\eta(q_2)|} \, \mathrm{Im} \frac{1}{\pi\varepsilon(\Omega, q_1 - q_2)}\right]$$ (15.b)

and to the CDW state of the equation

$$\Phi_{CDW}(q_1) = \int_{-k_F/2}^{k_F/2} \frac{dq_2}{2\pi} \frac{\mathrm{th}\frac{\eta(q_2)}{k_B T_c}}{2\eta(q_2)} K_{CDW}(q_1, q_2) \Phi_{CDW}(q_2)$$ (16.a)

$$K_{CDW}(q_1, q_2) = v(q_1 - q_2) \times$$

$$\times \left[1 + 2 \int_0^{\infty} \frac{d\hbar\Omega}{\hbar\Omega + |\eta(q_1)| + |\eta(q_2)|} \, \mathrm{Im} \frac{1}{\pi\varepsilon(\Omega, q_1 - q_2)}\right.$$

$$\left. - 2v(2k_F)\left[1 + 2 \int_0^{\infty} \frac{d\Omega}{\Omega} \, \mathrm{Im} \frac{1}{\pi\varepsilon(\Omega, 2k_F)}\right]\right].$$ (16.b)

The term $v(2k_F, 0) = v(2k_F)/\varepsilon(2k_F, 0)$ has been written using the equation (3).

II. 3 The relation to "g-ology"

To get approximate expressions for the coupling constants g_1 and g_2 in terms of the dielectric function, one first takes into account that the integrals in the equations (10.a) and (11.a) are dominated by the contribution of the integrand at $q_1 = k_F$. Because of this, in a finite region around this point one can make substitutions

$$\Phi_{SS}(q) \to \Phi_{SS}(k_F) \qquad \Phi_{TS}(q) \to \Phi_{TS}(k_F)$$

$$K_{SS}(q_1, q_2) \to \lim_{q \to 0} K_{SS}(k_F, k_F + q) = g_2 + g_1$$

where

$$K_{TS}(q_1, q_2) \to \lim_{q \to 0} K_{TS}(k_F, k_F + q) = g_2 - g_1$$

$$g_2 = \lim_{q \to 0} v(q) \left[1 + 2 \int_0^\infty d\hbar\Omega \frac{1}{\hbar\Omega + |\eta(q)|} \, \text{Im} \, \frac{1}{\pi\varepsilon(\Omega, q)} \right]$$

(17)

$$g_1 = v(2k_F, 0).$$

The width of the finite region in which such substitutions are supposed to be valid is again chosen as k_F. Because of the incomplete screening of the interaction (17), there can be a singularity in g_2, calculated as the limit of the expression mean value of this expression calculated over the interval $-k_F/2 \leqslant q \leqslant k_F/2$.

The equations (10.a) and (11.a) then reduce to the following equations

where

$$1 = -(g_2 + g_1) I \qquad 1 = -(g_2 - g_1) I,$$

$$I = \int_{k_F/2}^{3k_F/2} \frac{dq}{2\pi} \frac{\text{th} \dfrac{\xi(q)}{2k_B T_c}}{2\xi(q)} \approx \frac{m}{2\pi\hbar^2 k_F} \ln\left(\frac{2\gamma\mu}{\pi k_B T_c}\right),$$

which lead to the transition temperature (1) with λ_{SS} and λ_{TS} given in (2.a) and (2.b), respectively.

Similarly, the integrals in the equations (15.a) and (16.a) are dominated by the contribution of the integrand at $q_1 = 0$. After substitutions

$$\Phi_{SDW}(q) \to \Phi_{SDW}(0) \qquad \Phi_{CDW}(q) \to \Phi_{CDW}(0)$$

$$K_{SDW}(q_1, q_2) \to \lim_{q \to 0} K_{SDW}(0, q) = g_2$$

$$K_{CDW}(q_1, q_2) \to \lim_{q \to 0} K_{CDW}(0, q) = g_2 - 2g_1$$

the equations (15.a) and (16.a) get the form

$$1 = g_2 J \qquad 1 = (g_2 - 2g_1) J,$$

where

$$J = \int_{-k_F/2}^{k_F/2} \frac{dq}{2\pi} \frac{\text{th} \dfrac{\eta(q)}{2k_B T}}{2\eta(q)} \approx \frac{m}{2\pi\hbar^2 k_F} \ln\left(\frac{2\gamma\mu}{k_B T}\right)$$

giving again the transition temperature (1) with λ_{SDW} and λ_{CDW} of the same form as in (2.c) and (2.d), respectively. The resolution of the integral equations (10),

(11), (15), (16) with the kernel of the general form is a complicated mathematical problem well-known from the theory of superconductivity. Solving the integral equation of this type, one usually approximates the kernel with a constant that is nonzero only in the region of a finite width. This method of solving was presented in this subsection. The procedures that lead to the solutions of these integral equations with the kernel of the other form can be found in [7, 12, 13].

III. DISCUSSION

A strictly 1D electron system cannot undergo phase transition at any finite temperature [14, 15] as thermal and quantum fluctuations prevent the development of a long-range order [1—5, 15—19]. However, real materials which consist of a collection of weakly coupled 1D structures exhibit phase transition. The theoretical explanation of the properties of such materials has usually its origin in the mean-field description of single 1D structures, i.e., in the description that does not take into account the effects of fluctuations. It is due to the fact that the coupling among 1D structures partially suppresses the effects of fluctuations, e.g., a small degree of the coupling is sufficient to bring the actual transition temperature close to its mean-field value [4, 5, 20, 21].

An example of such a mean-field treatment of a single 1D metallic structure was presented in the previous section. On the basis of the derived equations one can determine the ordered state whose transition temperature is higher than all the others and whose tendency to develop in the corresponding real material is, therefore, prevalent. Particle-particle pairing and particle-hole pairing were, however, treated separately without taking into account their competition. Because of this insufficient treatment the calculated transition temperature can be regarded as a reliable mean-field value only if one type of pairing is highly dominant over the other, since only in this case the use of a single-channel calculation rather than a coupled two-channel approach is fully justified.

There is another subtle point of the present treatment which deserves to be mentioned, namely, vertex corrections to the electron-electron vertex. However, the vertex corrections can be neglected in the system in which the weak-coupling approximation is valid [4, 7, 8]. In any case, the problem to treat the vertex corrections properly can be related to the calculation of the effective electron-electron interaction.

On the other hand, the derived equations are suitable for the determination of the transition temperature of a wide class of 1D metallic structures including those in which electrons interact via the interaction with space and time dispersion, damping, etc. Perhaps the simplest metallic structure of this class is the 1D electron system in which the bare electron-electron Coulomb interaction $v(q)$ is screened by the dielectric function of the following form [22]

$$\varepsilon(q, \omega) = 1 - v(q)\Lambda(q, \omega),$$

where $\Lambda(q, \omega)$ is the 1D Lindhard function. The bare electron-electron Coulomb interaction $v(q)$ has the logarithmic singularity as $q \to 0$ expressing the long-range nature of the Coulomb repulsion. The proper long-wavelength and high-frequency behaviour (plasmons) of the electrons is, however, described by the screened electron-electron interaction

$$v(q, \omega) = \frac{v(q)}{1 - v(q)\Lambda(q, \omega)}$$

Clearly, the question which of the possible ordered states is favoured in this 1D electron system can be answered with the help of the derived equations. This matter is very interesting, particularly for high-temperature superconductivity, as there are opinions [23, 24] that the plasmon mechanism of superconductivity could be responsible for the high critical temperatures in the layered perovskites.

So far the plasmon mechanism of superconductivity has been studied only in two-and three-dimensional systems [23—26]. The problem whether plasmons can mediate such a strong attractive electron-electron interaction leading to superconductivity also in the 1D electron system will be treated in a forthcoming paper.

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О ТЕМПЕРАТУРЕ ПЕРЕХОДА ОДНОМЕРНОЙ МЕТАЛЛИЧЕСКОЙ СТРУКТУРЫ

Получены интегральные уравнения, которые в приближении среднего поля позволяют определить температуру перехода к упорядоченному состоянию одномерной металлической структуры. Полученные уравнения могут быть применены к одномерной металлической структуре, в которой эффективное электрон-электронное взаимодействие может быть выражено в терминах диэлектрической функции.

Уравнения для температуры перехода к состояниям с волнами спиновой плотности и волнами зарядовой плотности формально совпадают с хорошо известными уравнения для температуры перехода к сверхпроводящему состоянию. Ядра этих уравнений зависят от спектральной плотности обратной диэлектрической функции. На основе полученных уравнений установлена связь констант связи модели электронного газа и эффективного электрон-электронного взаимодействия.