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THEORETICAL FUNDAMENTALS OF QUANTITATIVE ACOUSTIC MICROSCOPY[1)]

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Consideration is given to the formation of a dependence of the output signal of the acoustic microscope on the coordinate z, which determines the mutual disposition of the acoustic lenses and the sample, in reflection and transmission modes of operation (so-called $V(z)$ — and $A(z)$ — dependences). The connection is analysed between the characteristic parameters of the $V(z)$- and the $A(z)$-dependences and the local values of the sound velocity in the sample for materials with a small shear modulus. A skimming compression wave is shown to participate in the formation of the $V(z)$-dependence. The results of this study provide a theoretical foundation for quantitative acoustic microscopy; in particular, they make it possible to asses the accuracy of the quantitative methods.

I. INTRODUCTION

Acoustic microscopy is based on the interaction of a converging acoustic beam propagating in an immersion liquid with an object to be studied, also submerged in the immersion liquid (Fig. 1). Recording the acoustic radiation after its interction with the object, one can not only visualize the microstructure of the sample but also obtain quantitative information on local physical properties of the sample in a given point [1—3]. At present quantitative methods of acoustic microscopy are being developed for plane samples whose acoustic properties vary only little at distances of the order of ultrasonic wave length. The methods of the acoustic microscopy are used to measure the values of sound velocities, acoustic attenaution, and other physical magnitudes with a resolution from dozens to decimal fractions of a micron, depending on the operation frequency of the microscope, which usually ranges from 20—30 MHz to 3 GHz [2, 4—7].

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When the acoustic microscope is used as a measuring instrument, the output signal at the transducer of the receiving lens is recorded. The value of this signal as such gives little information, and therefore the output signal is measured as a function of the distance z, the only variable which can be easily changed in the course of the experiment. In the reflection mode of operation the distance between the acoustic lens and the object is changed (Fig. 1a); in the transmission mode of operation the distance between the radiating lens and the receiving lens is changed (Fig. 1b). From the dependence of the output signal on the distance z one can determine the local viscoelastic characteristics of the sample. The physical principles of formation of such dependences, as well as their form, are different for different modes of the acoustic microscopy. To draw a distinction, we shall further denote the output signal of the microscope in the reflection mode as $V(z)$ and the output signal of the microscope in the transmission mode as $A(z)$.

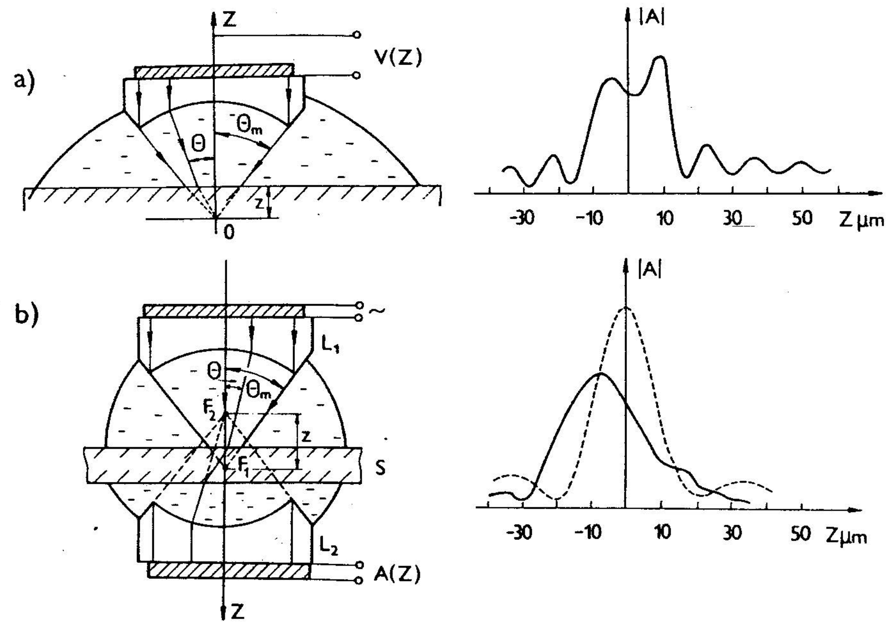

Fig. 1. Formation of an output signal of the acoustic microscope in its dependence on the coordinate z a) reflection mode of operation, b) — transmission mode of operation). Left: schemes of formation of the output signal; right: typical dependences of the signal amplitude on the coordinate z. Dashed line shows the $A(z)$ dependence in a lens system without the sample. L_1 and L_2 are radiating and receiving lenses, $z = 0$ is the position of the focal plane of the radiatings lens, S is the sample.

Qualitatively the mechanism of formation of the $V(z)$- and $A(z)$-depences can be understood by resorting to the simplest ray theory considerations. In the reflection mode of operation all the specularly reflected rays come to the receiving transducer in the same phase, when the reflecting surface is disposed in the focus of the lens. When the surface shifts from the focus, the rays travelling in different directions come to the transducer in different phases. Due to interference the output signal reduces, undergoing oscillations. For an ideal reflector the $V(z)$-dependence has a usual diffraction character [8]:

$$\langle V(z)| = A\left|\frac{\sin \pi \frac{z}{\lambda} \cdot x_m}{\pi \frac{z}{\lambda} \cdot x_m}\right|, \qquad A = const, \tag{1}$$

where z denotes the shift of the reflecting surfce from the focus of the lens. The curve $V(z)$ is symmetrical, its period Δz depends on the sound velocity C in the immersion liquid and on the numerical aperture of the lens $x_m = \sin^2 \Theta_m$:

$$\Delta Z = \lambda/x_m = C/f \, . \, x_m,$$

where λ is the sonic wave length in immersion, f is the operation frequency of the microscope. The charactér of reflection at the interface of real media is more complicated. A part of the incident rays may excite waves propagating along the the surface of the sample and re-radiating back into the immersion liquid: a skimming compression wave [9], a leaky Rayleigh wave [10], leaky waveguide modes [11]. If the reflecting surface shifts from the focus towards the lens (region $z > 0$), such waves participate in the formation of the output signal of the microscope (Fig. 1a). In this case the $|V(z)|$ — dependence is asymmetrical. It has been shown both experimentally and theoretically [8, 12—15] that for many solid-state samples the formation of the $V(z)$-dependence is associated with leaky Rayleigh waves. In this case the periodicity of the $V(z)$ — curves in the region $z > 0$ is dependent on the local value of the Rayleigh wave velocity C_R

$$\Delta z = \frac{C_R}{f} \frac{1 + \cos \Theta R}{2 \, . \sin \Theta_R},$$

where $\Theta_R = \arcsin (C/C_R)$ — the so-called Rayleigh's angle [13]. The $V(z)$ — curves are used for the determination of C_R and for so called $V(z)$ — characterization (acoustic signature) of the samples [2, 4, 5, 12—15]. Today, however, there are no clear theoretical ideas concerning the shape of the $V(z)$-curves with different relationships between the acoustic properties of the sample and immersion, concerning the connection between the parameters of the $V(z)$-curves and the viscoelastic characteristics of the sample and immersion, concerning the degree of accuracy in acoustomicroscopic measurements. The role of other types of waves, besides the leaky Rayleigh wave, in the formation of the $V(z)$-dependencies, in particular, the role of the skimming wave, remains unclear.

The studies of Atalar [8, 14] remain the only studies devoted to the theoretical substantiation of quantitative methods of reflection acoustic microscopy. Atalar's studies demonstrate that the output signal of the microscope is related in an integral manner with the sound reflection coefficient at the interface of the sample and the immersion liquid; an analysis is also presented, how the role of the leaky Rayleigh waves in the formation of the $V(z)$-dependence may be taken into account. Atalar's results are usually employed for numerical calculations: the known acoustic parameters of the material are substituted into the integral formula, the $V(z)$-dependences are calculated numerically, and the results obtained are then compared with experimental curves [2, 4, 5, 8, 12—15]. Such an approach, naturally, cannot give answers to the problems set foth earlier; for solving these problems an analytical approach is required in combination with purpose-oriented numerical calculations.

Quantitative investigations in the transmission mode of operation are based on different principles. We have proposed a method using the dependence of the output signal A on the distance between the radiating lens and the receiving lens [6, 7]. The method is based on shifting the focus of the converging beam due to refraction during the passage of the beam through the plane sample (Fig. 1b). In the absence of the sample the signal is maximum, when the foci of the lenses coincide (confocal system). As the receiving lens shifts, the signal drops rapidly, undergoing oscillations, due to interference effects. If an object shaped as a plane-parallel plate is placed in the path of the focused beam, the rays falling at the angle 0, after passing through the plate, will be collected at a point on the acoustic axis, shifted with respect to the focus of the radiating lens for the distance:

$$\Delta z = d\,.(1 - \tan\alpha/\tan\Theta), \qquad (2)$$

where d is the thickness of the plate; α is the angle of refraction, defined by the condition $\sin\alpha = (C_L/C)\sin\Theta$, C is the velocity of sound in immersion, C_L is the velocity of longitudinal sound in the sample. If the aperture of the lens Θ is small, the tangents in (2) may be replaced by sinuses (paraxial approximation). Then the displacement of the convergence points of the rays does not depend on the angle of incidence: after the passage through the plate the beam remains focused. There arise a new position of the focus and, accordingly, a new position of the maximum on the curve $|A(z)|$. The shift of the maximum:

$$\Delta z = d\,.(1 - C_L/C) \qquad (3)$$

depends on the local value of sound velocity in the sample. By masuring the shift, it is possible to obtain the value C in a given point of the sample. A change in the absolute value of the $|A(z)|$ maximum after the introducing of the sample determines the local coefficient of transmission. This makes it possible to assess the local absorption in the sample. The possibility of measuring sound velocities and attenuation coefficients by such a method was confirmed experimentally on polymer and biopolymer films, collagen fibers, and other object [7].

Paraxial approximation, however, is applicable only to the beams with a small aperture ($\Theta_m \lesssim 10°$—$15°$). In acoustic microscopes, as a rule, wide-aperture lenses are employed ($\Theta_m \sim 30°$—$60°$). In such systems focused beams, when passing through the sample, are subject to considerable aberrations. The focal region of the beam widens strongly, and the application of the proposed method requires a substantiation. The present paper is devoted to an analysis of the theoretical fundamentals of the quantitative acoustic microscopy, at least for definite classes of objects.

It is convenient to describe the formation of the output signal within the framework of concepts of the spatial radiation spectrum [8]. The field of the radiating lens is represented as a set of plane waves having the same frequency ω but different directions of propagation. The amplitudes $U(\boldsymbol{k})$ of waves with different wave vectors $\boldsymbol{k} = \left\{k_x, k_y, k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}\right\}$ constitute the spatial spectrum of focused radiation. As the radiation propagates in a homogeneous medium over the distance z, the spectrum becomes transformed due to the multiplication by the phase factor $\exp\{ik_z z\}$; upon reflection from the interface it is multiplied by the reflection coefficient $R(k_x, k_y)$; upon passage through the plane-parallel sample it is multiplied by the transmission coefficient $T(k_x, k_y)$. As a result, a set of plane waves with different k_x and k_y falls onto the receiving lens. Each such wave creates its own signal at the receiving transducer, this signal being defined, apart from the amplitude and the incident wave phase values, by the response characteristic of the receiving lens $U_2(k_x, k_y)$. The value $U_2(k_x, k_y)$ is a signal created at the receiving transducer by the plane wave with the wave vector $\{k_x, k_y, k_z\}$ having a unit amplitude and a zero phase in the focus of the receiving lens.

In the reflection mode of operation the dependence of the output signal V on the shift z of the sample surface from the focus of the acoustic lens is expressed in an integral manner [8] through the reflection coefficient $R(k_x, k_y)$:

$$V(z) = \iint_{-\infty}^{\infty} U_1(k_x, k_y) U_2(k_x, k_y) R(k_x, k_y)\, e^{ik_z z}\, dk_x\, dk_y \qquad (4)$$

The function $P(k_x k_y) = U_1 U_2$ is an aperture function of the lens; its shape can be found theoretically for an ideal lens [1] and experimentally for real lenses [16].

In a theoretical analysis one can resort to the simplest approximation for P, assuming that the focused beam is an ensemble of waves with the same amplitude and phase, propagating along directions lying within the angular aperture of the lens. Similarly, the response function is dependent but little on the angle within the angular aperture Θ_m. Under these assumptions the aperture function is constant at the angles Θ smaller than Θ_m and is equal to zero beyond the angular aperture of the lens (so-called pupil function) [1, 8]. For isotropic samples and acoustic lenses with small apertures $\Theta_m \lesssim 30°$—$40°$ equation (4) is simplified:

$$V(z) = A \int_0^{x_m} R(x)\, e^{ik_z x}\, dx, \tag{5}$$

where $x = \sin^2 \Theta$, $x_m \sin^2 \Theta_m$, A is a constant. Under the same assumptions concerning the isotropism of the sample and the shape of the aperture function the dependence of the output signal of the microscope in the transmission mode of operation on the distance z between the foci of acoustic lenses (Fig. 1b) is expressed through the transmission coefficient $T(\Theta, d)$ (d being the sample thickness) in the following manner:

$$A(z) = B \int_0^{\Theta_m} T(\Theta, d)\, e^{ik(z-d)\cos\Theta} \sin\Theta\, d\Theta, \tag{6}$$

where B is a constant.

Proceeding from equations (5) and (6), we shall consider the formation of the $V(z)$- and $A(z)$-dependences for an interesting class of materials: samples with a small shear modulus G (biological tissues and cells, polymer and biopolymers etc.). The assumption of G being small allows one to simplify the expressions for the reflection and transmission coefficients and to obtain analytical expressions for the $V(z)$-and $A(z)$-dependences. Furthermore an investigation of the $V(z)$-dependences for the samples with small shear modulus makes it possible to solve the principally important problem concerning the contribution of the skimming wave to the formation of the output signal of the microscope operating in the reflection mode. The major results of our analysis are presented below.

II. REFLECTION MODE OF OPERATION

The character of angular dependence of the reflection coefficient

$$R(x) = \frac{\sqrt{1-x} - \varrho\sqrt{x_L - x}}{\sqrt{1-x} + \varrho\sqrt{x_L - x}}, \qquad \varrho = \varrho_i/\varrho_s$$

($x_L = C^2/C_L^2$) depends on the relationship between C and C_L. When the sound

velocity in immersion is greater than in the sample ($C > C_L$), the reflection coefficient is real and total internal reflection is absent. Since $x_L > 1$ and $x_m \ll 1$, for $R(x)$ linear approximation may be used:

$$R(x) = R_0 - \beta x,$$

where $R_0 = R(\Theta = 0°) = \frac{1-y}{1+y}$ is the reflection coefficient at normal incidence; $y = \frac{\varrho_i C}{\varrho_s C_L}$ is the immersion and the sample impedance ratio; ϱ_i and ϱ_s are the immersion and the sample densities; $\beta = \frac{y}{(1+y)^2} \cdot (1 - C_L^2/C^2)$. The output signal amplitude measured in the experiment is

$$V(z) = A\left\{a\frac{\sin\xi}{\xi^2} + \frac{1}{\xi^2} + \frac{\sin\xi}{\xi^3}\left(\frac{\sin\xi}{\xi} - 2.\cos\xi\right)\right\}^{1/2} \tag{7}$$

here A is a constant, $\xi = \frac{1}{2}k_z x_m$, symmetrical with respect to the point $z = 0$. The curve $V(z)$ is a sequence of alternating minima and decreasing maxima (Fig. 2). The distance between the successive minima (maxima)

$$\Delta z = \lambda x_m = \frac{f}{c} x_m \tag{8}$$

depends on the wave length in immersion and does not depend on the properties of the object. On the contrary, the shape of the curve $V(z)$ which is determined by the value of the parameter

$$a = \frac{R_0(R_0 - \beta x_m)}{(\beta x_m/2)^2} \sim R(\Theta = 0°) \,.\, R(\Theta = \Theta_m)$$

is dependent on the properties of the object. If within the angular aperture of the lens the reflection coefficient does not change its sign and the angle of zero reflection (acoustic analogue of Brewster's angle) is absent, $a > 0$. In this case the curve $V(z)$ has a clearly pronounced main maximum at $z = 0$ (curve 1, Fig. 2). As a diminishes, the maxima of the curve $V(z)$ decline, whereas the minima rise (curve 2, Fig. 2); at $a = 0$ the secondary maxima and minima merge (curve 3). The parameter a becomes negative when the angle of zero reflection proves to be within the angular aperture of the lens. In so far as within the aperture the reflection coefficient changes its sign, a part of the waves shaping the reflected beam is phase-shifted through π. With the interference of the signals created by the reflected waves at the transducer, the main maximum dimnishes because of this shift. A minimum is formed in its place, and two symmetrical maxima arise nearby (curve 4, Fig. 2). Furthermore, in the place of secondary maxima there originate minima and vice versa. When $R_0 = \frac{1}{2}\beta x_m$, the minimum at $z = 0$ drops down to zero.

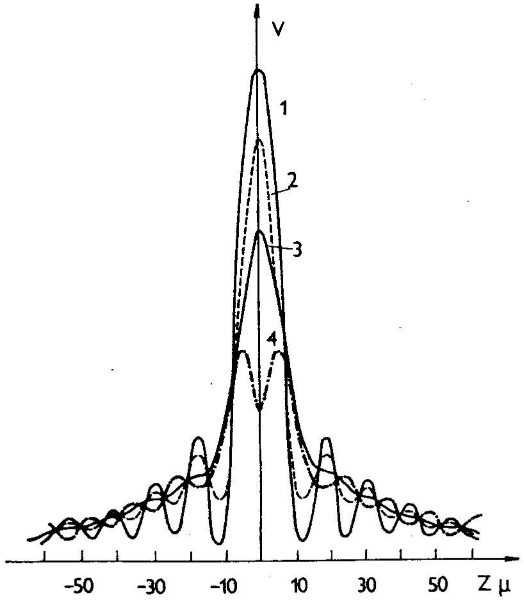

Fig. 2. The dependence of the output signal amplitude V of the acoustic microscope in the reflection mode of operation on the shift of z in the case when total reflection is absent ($C_L < C$). Numerical calculation for $\Theta_m = 30°$, $x_m = 0.25$, $\lambda = 3$ μm. The curves differ in the value of parameter a: $1 — a = 100$; $2 — a = 10$; $3 — a = 0$; $4 — a = -2.25$.

When the sound velocity in the sample is greater than the velocity in immersion ($C_L > C$), for angles $\Theta > \Theta_L = \arcsin(C/C_L)$ a total reflection takes place: the coefficient of reflection becomes complex and its modulus becomes equal to 1. If $\Theta > \Theta_m$, the formation of the $V(z)$ — dependence proceeds as in the case of $C_L < C$, discussed above. When the critical angle Θ_L is found within the aperture of the lens, the skimming wave participates in the formation of the output signal and the curve $|V(z)|$ loses its symmetry (Fig. 3). Figs. 3 and 4 show the results of numerical calculations of the $V(z)$ — dependences for different values of the ratio $\varrho = \varrho_i/\varrho_s$ of the immersion and the sample densities (Fig. 3) and for different positions of the critical angle Θ_L within the angular aperture

of the lens (Fig. 4). When $\varrho \ll 1$, the curve $V(z)$ remains symmetrical, its period is equal to $\Delta z_m = \lambda/x_m$ and does not depend on the properties of the sample (curve 1, Fig. 3). As the immersion density increasses, the amplitude of the

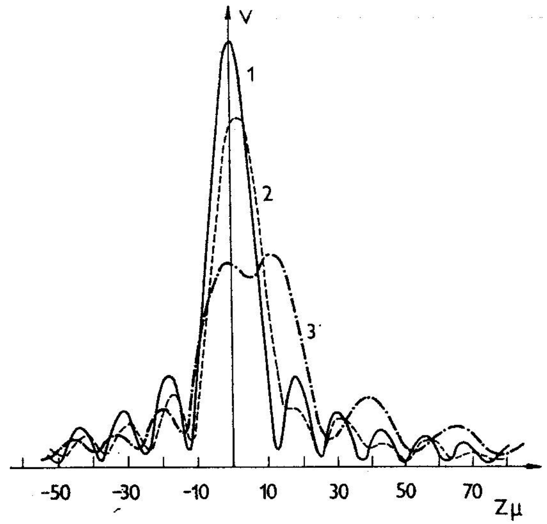

Fig. 3. The dependence of the output signal amplitude V on the shift of z in the case when total reflection takes place at the interface of the sample and the immersion liquid ($C_L > C$). Numerical calculation for $\Theta_m = 30°$, $x_m = 0.25$, $x_L = 0.125$, $\lambda = 3$ μm. Different curves correspond to different density rations: 1 — $\varrho = 0.1$, 2 — $\varrho = 1$; 3 — $\varrho = 5.5$.

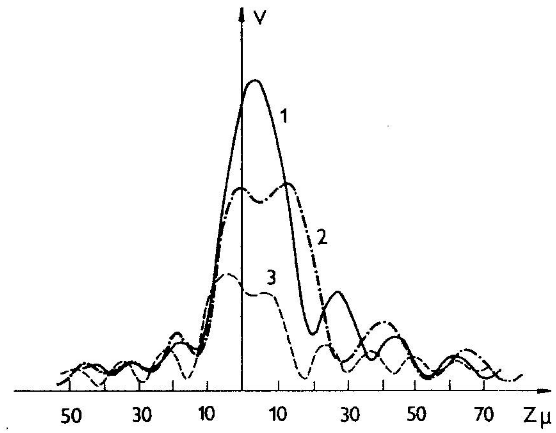

Fig. 4. $V(z)$ — dependences for different longitudinal sound velocities in the sample: 1 — $x_L = 0.067$, 2 — $x_L = 0.125$, 3 — $x_L = 0.24$. Numerical calculation for $x_m = 0.25$, $\varrho = 5.5$, $\lambda = 3$ μm.

skimming wave in the liquid grows and, correspondingly, the asymmetry of the $V(z)$ — curves becomes greater: the structure of the main maximum becomes distorted and in the region of the positive z (the object moves towards the lens) the period of the disposition of the minima and secondar maxima changes (curves 2 and 3, Fig. 3). The numerical calculations are confirmed by an analysis of the asymptotic behaviour of the $V(z)$ — dependences at a large z, where an oscillating dependence of the signal amplitude on the distance is observed. At $kz \gg 1$:

$$V(z) = \frac{A}{kz}\{R_0 - e^{-i\varphi_0} . e^{ikzx_m}\} + 4e^{-i\frac{\pi}{4}} f(z)\, e^{ikzx_L} \tag{9}$$

where

$$\varphi_0 = 2.\arctan(\varrho\sqrt{x_m - x_L}) \text{ and } f(kz) = \int_0^{\infty} \frac{\sqrt{s}\,e^{-kzs}}{1 + i\varrho^2 s} ds. \text{ At } \varrho < 1 \; f(kz) \sim \frac{1}{(kz)^{3/2}}$$

and the second term in equation (9) may be neglected. Then the dependence of the signal amplitude on z is expressed with sufficient accuracy as

$$|V(z)| \simeq A\frac{1}{kz}\sqrt{1 + R_0^2 - 2R_0 \cos(kzx_m - \varphi_0)}. \tag{10}$$

Its periodicity is determined by the value Δz_m, which follows also from the numerical calculations. For heavy immersions ($3 \lesssim \varrho \lesssim 15$) the function $f(kz)$ in the range of values $z \sim (5 \div 40)\lambda$ is close to a hyperbola: $f(kz) \sim \frac{1}{kz}$. In this case the dependence of the output signal amplitude on the distance z:

$$|V(z)| \sim \frac{1}{kz}\{A_0 + A_1 \cos(kzx_m + \varphi_1) + A_2 \cos(k_2 x_L + \varphi_2) +$$
$$+ A_3 \cos(kz(x_m - x_L) + \varphi_3)\}^{1/2} \tag{11}$$

(A_i and φ_i are constants, $i = 1, 2, 3$) is a superposition of oscillating dependences with different spatial periods:

$$\Delta z_m = \lambda/x_m, \quad \Delta z_L = \lambda/x_L, \tag{12}$$
$$\Delta z_{m-L} = \lambda/(x_m - x_L).$$

The results of numerical calculations demonstrate that the curves $|V(z)|$ with different behaviour are observed, including two-periodic ones (curve 2, Fig. 3). Nevertheless, a general rule exists: the period of pronounced oscillations (the distance between the most proximate minima) is equal to the least of the periods

Δz_L and Δz_{m-L}. This result is illustrated by the curves $|V(z)|$, calculated for different x_L but with x_m fixed (Fig. 4). The analysis has shown that the use of heavy immersions, for instance, a liquid, metals, makes it possible to employ thes $V(z)$ — curves for the determination of the velocity of the longitudinal sound in samples.

III. TRANSMISSION MODE OF OPERATION

Let us consider the simplest version of the $A(z)$-method, applicable to samples for which the transmitted beam is formed as a result of a single passage of sound through the sample. It it necessary that an incident beam should not excite transverse waves in the sample, and the refracted longitudinal waves should not experience repeated reflections at the boundaries of the sample. Biological tissues, polymer and biopolymer films, etc. may serve as examples of such objects if water is used as the immersion. For the sake of simplicity we shall also assume that the attenuation of longitudinal waves passing through the sample is small. In this case the transmission coefficient is equal to the phase factor corresponding to the change in the wave phase upon its single passage through the sample:

$$T(\Theta, d) = \exp\{i\Phi(\Theta, d)\}, \quad \Phi(\Theta, d) = kd\sqrt{C^2/C_L^2 - \sin^2\Theta}. \tag{13}$$

Substituting this expression into equation (6) and expanding the phase $\Phi(\Theta, d)$ within the angular aperture Θ_m in powers of the minor parameter $x' = 1 - \cos\Theta$, down to the terms x^2 inclusive, we shall write down $A(z)$ as

$$A(z) = \frac{B}{b}\exp(i\Phi_0)\int_a^{a+bx'_m} \exp\left\{i\pi\frac{S^2}{2}\operatorname{sign}(1 - C_L/C)\right\} ds, \tag{14}$$

where

$$x'_m = 1 - \cos\Theta_m, \quad b = \left\{\frac{1}{\pi}kd\frac{C_L}{C}\cdot\left|1 - \frac{C_L^2}{C^2}\right|\right\}^{1/2}$$

$$a(z) = \left\{\frac{1}{\pi}kd\frac{C}{C_L}\Big/\left|1 - \frac{C_L^2}{C^2}\right|\right\}^{1/2}\cdot(1 - C_L/C - z/d)\operatorname{sign}\left(1 - \frac{C_L}{C}\right)$$

and

$$\Phi_0 = k[z - d(1 - C/C_L)] - \frac{1}{2}\cdot kd\cdot(1 - C_L/C - z/d)^2\Big/\left[\frac{C_L}{C}\left(1 - \frac{C_L^2}{C^2}\right)\right].$$

The amplitude of the output signal A as a function of z is expressed through Fresnel's integrals $C(y)$ and $S(y)$ of the arguments

$$y_p(z) = a(z) + bx'_m,$$

$$yQ(z) = a(z)$$

$$|A(z, d)| = \frac{B}{b} \cdot \{[C(y_p) - C(y_Q)]^2 + [S(y_p) - S(y_Q)]^2\}^{1/2}. \quad (15)$$

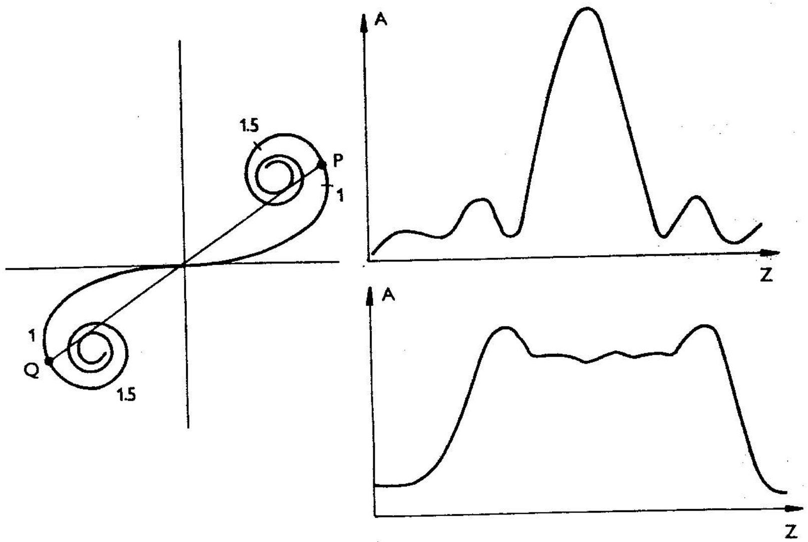

Fig. 5. Formation of the $A(z)$ — dependence for the transmission mode of operation. Left: geometrical determination of the output signal amplitude with the help of Cornu's spiral; right: typical $A(z)$ — dependences: a — thin sample, b — thick sample.

To investigate the character of the $A(z)$-dependence, we shall use geometrical interpretation (15) based on Cornu's spiral (Fig. 5). Plotting the values of the arguments y_p and y_Q along Cornu's spiral taking into account the signs, we obtain points P and Q. The length of the segment PQ is equal to the amplitude of the signal $|A|$ normalized for the value $\frac{|B|}{b}$. The distance between the points P and Q along Cornu's spiral, equal to bx'_m is independent of the coordinate z; as z changes, a change occurs only in the position of the points P and Q, but not in the length of the curve between them. The curve $|A(z)|$ is symmetrical with

respect to the value $z = z_0$, at which P and Q are disposed at equal distances along Cornu's spiral from its origin but at the opposite sides from it:

$$y_p(z_0) = -y_Q(z_0). \quad (16)$$

Condition (16) defines the coordinate of the point of symmetry:

$$z_0 = d\left\{1 - \frac{C_L}{C} + \frac{1}{2} \cdot \frac{C_L}{C}\left(1 - \frac{C_L^2}{C^2}\right)x'_m\right\}. \quad (17)$$

As a rule, the thicknesses of the samples satisfy the condition

$$d < 3\lambda/[x'^2_m|1 - C_L/C|]. \quad (18)$$

For them $bx'_m < 2.44$; the length of the intercept PQ is maximum when the position of the points P and Q is symmetrical with respect to the origin of the spiral and it decreases rapidly when the symmetry is disturbed and the curvilinear intercept PQ is wound on the left-hand or the right-hand helix of the spiral. The curve $|A(z)|$ has a clear-cut main maximum at the point of symmetry $z = z_0$ and decreasing maxima (Fig. 5a). On the whole, this curve is close to the diffraction curve which originates in paraxial approximation, but with a refined position of the main maximum in accordance with (17). For thicker samples, for which inequality (18) does not hold, there originates on the curve $|A(z)|$ in the vicinity of $z = z_0$ on both sides of this value a wide region of considerable output signal values (Fig. 5b). The width of the region is comparable with its shift with respect to the focus of the confocal system. However, an investigation of such thick samples in the transmission acoustic microscopy is unrealistic because of both small working distance between the lenses and strong attenuation of ultrasound in the sample.

The results presented above constitute a theoretical foundation for the application of the transmission acoustic microscopy in quantitative measurements of the local velocity of sound for a large class of materials.

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ТЕОРЕТИЧЕСКИЕ ОСНОВЫ КВАНТИТАТИВНОЙ АКУСТИЧЕСКОЙ МИКРОСКОПИИ

Обсуждается возникновение зависимости входящего сигнала акустического микроскопа от координаты z, которая определяет взаимное расположение акустических линз и образца в отражающей и пропускающей модах эксплуатации (так называемых $V(z)$ и $A(z)$зависимостях). Исследуется связь между характеристическими параметрами $V(z)$и $A(z)$ зависимостей и локальными значениями скорости звука в образце материалов с маленьким модулем сдвига. Результаты предлагаемой работы дают возможность теоретического обоснования квантитативной акустической микроскопии; в частности, они позволяют повысить точность квантитативных методов.