acta phys. slov. 40 (1990), No. 3

GROUP-THEORETIC APPROACH OF CLASSIFICATION OF SIMILARITY SOLUTIONS FOR AN UNSTEADY FLUID FLOW THROUGH A POROUS MEDIUM

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A group-theoretic approach of classification of similarity solutions in natural convection for an unsteady flow of a viscous incompressible fluid through a porous medium is considered. Two cases are considered: (i) the permeability of the medium is constant and (ii) the permeability is a function of the space and time coordinates. It is shown that similarity solutions are possible in both cases.

I. INTRODUCTION

The study of similarity solutions of partial differential equations based on the concepts of continuous transformation groups has been treated extensively by Birkhoff [1], Morgan [2], Michal [3] and Chakraborty [4] have also paid attention to this field. The main object of this method is to reduce by one the number of independent variables in some system of partial differential equations.

In the present paper we will find whether the similarity solution of the differential equations involved in the problems of the unsteady flow of a viscous fluid through a porous medium of given permeability is possible. We discuss here two different cases: the first is that the permeability of the porous medium is constant and in the second the permeability is assumed to be a function of the spacae coordinates and time.

II. CASE I: PERMEABILITY CONSTANT

As an application of the group-theoretic approach we consider the case when the fluid flows through a porous medium of permeability K. The relevant equations in usual non-dimensional forms are

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$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + T - \frac{1}{K}u, \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + \frac{GE}{R^2}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

where u, v are the velocity components, T is the temperature, P_r the Prandle number, G the Grashoff number, E the Eckert number and R is the Reynold number.

We assume that the permeability K is constant.

Let us now introduce the following continuous one-parameter group of transformations [2]:

$$\bar{t} = t + \nu_1 a,\ \ \bar{x} = e^{\nu_2 a}x,\ \ \bar{y} = y + \nu_3 a,\ \ \bar{u} = e^{\nu_4 a}u,\ \ \bar{v} = e^{\nu_5 a}v,\ \ \bar{T} = e^{\nu_6 a}T. \tag{4}$$

Then the expression

$$\Phi_1 \equiv \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} - T + \frac{1}{K}u$$

is constant conformally invariant under the transformation (4) if

$$\nu_2 = \nu_4 = \nu_6 \quad \text{and} \quad \nu_5 = 0. \tag{5}$$

The expression

$$\Phi_2 \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

is an absolute invariant under the transformation (4) under the conditions (5). Also the expression

$$\Phi_3 \equiv \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - \frac{GE}{R^2}\left(\frac{\partial u}{\partial y}\right)^2,$$

to be constant conformally invariant of transformation (4) satisfying (5), is that the dissipation term must be zero.

The expressions Φ_1, Φ_2 and Φ_3 can be expressed in terms of the variables η_1, η_2 and F_1, F_2, F_3 and their derivatives, where the variables η_1 and η_2 are such that they are absolute invariants of the subgroup of the transformation of the independent variables x, y, t. From the nature of the transformation group (4) under the conditions (5) we choose η_1 and η_2 as follows:

$$\eta_1 = e^{-t}x^{\nu_1/\nu_2}, \quad \eta_2 = e^{-y}x^{\nu_3/\nu_2}. \tag{6}$$

Now

$$\bar{\eta}_1 = e^{-\bar{t}}\bar{x}^{\nu_1/\nu_2} = e^{-(t+\nu_1 a)}e^{\nu_1 a}x^{\nu_1/\nu_2} = e^{-t}x^{\nu_1/\nu_2} = \eta_1, \tag{7}$$

$$\bar{\eta}_2 = e^{-\bar{y}}\bar{x}^{\nu_3/\nu_2} = e^{-(y+\nu_3 a)}e^{\nu_3 a}x^{\nu_3/\nu_2} = e^{-y}x^{\nu_3/\nu_2} = \eta_2.$$

Hence η_1 and η_2 are functionally independent absolute invariants of the transformation group

$$\bar{t} = t + \nu_1 a,\ \ \bar{x} = e^{\nu_2 a}x,\ \ \bar{y} = y + \nu_3 a,\ \ \bar{u} = e^{\nu_2 a}u,\ \ \bar{v} = v,\ \ \bar{T} = e^{\nu_2 a}T. \tag{8}$$

We now proceed to find the functions F_1, F_2 and F_3. We have

$$F_i(\eta_1, \eta_2) = g_i(x, y, t;\ u, v, T), \tag{9}$$

where g_i $(i = 1, 2, 3)$ are functionally independent absolute invariants of the transformation group (7). We choose g_i as follows:

$$g_1 = ue^t x^{-(\nu_1+\nu_2)/\nu_2},$$
$$g_2 = ve^t e^{-(\nu_1/\nu_3)y}, \tag{10}$$
$$g_3 = Te^y x^{-(\nu_2+\nu_3)/\nu_2}.$$

It may be seen that g_i are absolute invariants of the group of transformation (8). From (9) and (10) we have

$$u = e^{-t}x^{(\nu_1+\nu_2)/\nu_2}F_1(\eta_1, \eta_2),$$
$$v = e^{-t}e^{(\nu_1/\nu_3)y}F_2(\eta_1, \eta_2), \tag{11}$$
$$T = e^{-y}x^{(\nu_2+\nu_3)/\nu_2}F_3(\eta_1, \eta_2).$$

Using (11), the expressions for Φ_i $(i = 1, 2, 3)$ take the forms

$$\Phi_1 \equiv \eta_1\left[F_1 + \eta_1\frac{\partial F_1}{\partial \eta_1}\right] - \eta_1^2 F_1\left[\left(1 + \frac{\nu_1}{\nu_2}\right)F_1 + \eta_1\frac{\partial F_1}{\partial \eta_1} + \eta_2\frac{\partial F_1}{\partial \eta_2}\right] +$$
$$+ \eta_1^2\eta_2^{1-\frac{\nu_1}{\nu_3}}F_2\frac{\partial F_1}{\partial \eta_2} + \eta_1\eta_2\left[\frac{\partial F_1}{\partial \eta_2} + \eta_2\frac{\partial^2 F_1}{\partial \eta_2^2}\right] + \eta_2 F_3 - \frac{1}{K}\eta_1 F_1 = 0, \tag{12}$$

$$\Phi_2 \equiv \left(1 + \frac{\nu_1}{\nu_2}\right)\eta_1 F_1 + \eta_1^2\frac{\partial F_1}{\partial \eta_1} + \eta_1\eta_2\frac{\partial F_1}{\partial \eta_2} + \eta_1\eta_2 e^{-\frac{\nu_1}{\nu_3}}\left[\frac{\nu_1}{\nu_3}F_2 - \eta_2\frac{\partial F_2}{\partial \eta_2}\right] = 0, \tag{13}$$

$$\Phi_3 \equiv \eta_1\eta_2\frac{\partial F_3}{\partial \eta_1} - \eta_1 F_1\left[\left(1 + \frac{\nu_3}{\nu_2}\right)\eta_2 F_3 + \eta_2\left(\frac{\nu_1}{\nu_2}\eta_1\frac{\partial F_3}{\partial \eta_1} + \frac{\nu_3}{\nu_2}\eta_2\frac{\partial F_3}{\partial \eta_2}\right)\right] +$$
$$+ \eta_1\eta_2^{1-\nu_1/\nu_3}F_2\left[F_3 - \eta_2\frac{\partial F_3}{\partial \eta_2}\right] + \frac{1}{P_r}\eta_2\left[F_3 + 3\eta_2\frac{\partial F_3}{\partial \eta_2} + \eta_2^2\frac{\partial^2 F_3}{\partial \eta_2^2}\right] = 0. \tag{14}$$

In the expressions for Φ_i ($i = 1, 2, 3$), the independent variables are η_1, η_2 and the dependent variables are F_1, F_2, F_3.

Using the above variables, we set up a new group of transformation as follows:

$$\bar{\eta}_1 = b^{\varepsilon_1}\eta_1,\ \bar{\eta}_2 = b^{\varepsilon_2}\eta_2,\ \bar{F}_1 = b^{\varepsilon_3}F_1,\ \bar{F}_2 = b^{\varepsilon_4}F_2,\ \bar{F}_3 = b^{\varepsilon_5}F_3. \tag{15}$$

Under the transformation (15), the expression for $\bar{\Phi}_1$ becomes

$$\bar{\Phi}_1 \equiv \bar{\eta}_1\left[\bar{F}_1 + \bar{\eta}_1\frac{\partial \bar{F}_1}{\partial \bar{\eta}_1}\right] - \bar{\eta}_1^2\bar{F}_1\left[\left(1+\frac{\nu_1}{\nu_2}\right)\bar{F}_1 + \bar{\eta}_1\frac{\partial \bar{F}_1}{\partial \bar{\eta}_1} + \bar{\eta}_2\frac{\partial \bar{F}_1}{\partial \bar{\eta}_2}\right] +$$

$$+ \bar{\eta}_1^2\bar{\eta}_2^{1-\nu_1/\nu_3}\bar{F}_2\frac{\partial \bar{F}_1}{\partial \bar{\eta}_2} + \bar{\eta}_1\bar{\eta}_2\left[\frac{\partial \bar{F}_1}{\partial \bar{\eta}_2} + \bar{\eta}_2\frac{\partial^2 \bar{F}_1}{\partial \bar{\eta}_2^2}\right] + \bar{\eta}_2\bar{F}_3 - \frac{1}{K}\bar{\eta}_1\bar{F}_1 =$$

$$= b^{\varepsilon_1+\varepsilon_3}\eta_1\left[F_1 + \eta_1\frac{\partial F_1}{\partial \eta_1}\right] - b^{2(\varepsilon_1+\varepsilon_3)}\eta_1^2F_1\left[\left(1+\frac{\nu_1}{\nu_2}\right)F_1 + \eta_1\frac{\partial F_1}{\partial \eta_1} + \eta_2\frac{\partial F_1}{\partial \eta_2}\right] +$$

$$+ b^{\varepsilon_1+\varepsilon_4-\frac{\nu_1}{\nu_3}\varepsilon_2}\eta_1^2\eta_2^{1-\frac{\nu_1}{\nu_3}}F_2\frac{\partial F_1}{\partial \eta_2} + b^{\varepsilon_1+\varepsilon_3}\eta_1\eta_2\left[\frac{\partial F_1}{\partial \eta_2} + \eta_2\frac{\partial^2 F_1}{\partial \eta_2^2}\right] +$$

$$+ b^{\varepsilon_2+\varepsilon_5}\eta_2F_3 - \frac{1}{K}b^{\varepsilon_1+\varepsilon_3}\eta_1F_1.$$

Φ_1 will be an absolute invariant under the transformation group (15) if

$$\varepsilon_3 = -\varepsilon_1,\ \varepsilon_5 = -\varepsilon_2,\ \varepsilon_4 = \frac{\nu_1}{\nu_3}\varepsilon_2 - \varepsilon_1.$$

Hence the transformation group (15) reduces to

$$\bar{\eta}_1 = b^{\varepsilon_1}\eta_1,\ \bar{\eta}_2 = b^{\varepsilon_2}\eta_2,\ \bar{F}_1 = b^{\varepsilon_1}F_1,\ \bar{F}_2 = b^{(\nu_1/\nu_3)\varepsilon_2-\varepsilon_1}F_2,\ \bar{F}_3 = b^{-\varepsilon_2}F_3. \tag{16}$$

It is seen that the expressions Φ_2 and Φ_3 remain absolute invariants under the transformations (16).

Let us now proceed to express η_1 and η_2 by means of a single variable η as follows:

$$\eta = \eta_1^{-\frac{\varepsilon_2}{\varepsilon_1}}\eta_2. \tag{17}$$

It may be verified that η is an absolute invariant under the transformation (16).

We now choose the dependent variables $H_i(\eta)$ ($i = 1, 2, 3$) as follows:

$$F_1 = \eta_2^{-\varepsilon_2/\varepsilon_1}H_1(\eta),$$
$$F_2 = \eta_2^{(\nu_1/\nu_3-\varepsilon_1/\varepsilon_3)}H_2(\eta), \tag{18}$$
$$F_3 = \eta_1^{-\varepsilon_2/\varepsilon_1}H_3(\eta).$$

Using (17) and (16), the equations (12) to (14) reduce to

$$\left[\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 - \varepsilon_2\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1'\right] - H_1\left[\left(1+\frac{\nu_1}{\nu_2}\right)\eta^{-2\frac{\varepsilon_1}{\varepsilon_2}}H_1 - \frac{\varepsilon_2}{\varepsilon_1}\eta^{1-2\frac{\varepsilon_1}{\varepsilon_2}}H_1' - \frac{\varepsilon_1}{\varepsilon_2}\eta^{-2\frac{\varepsilon_1}{\varepsilon_2}}H_1 +\right.$$

$$\left. + \eta^{1-2\frac{\varepsilon_1}{\varepsilon_2}}H_1'\right] + H_2\left[-\frac{\varepsilon_1}{\varepsilon_2}\eta^{-2\frac{\varepsilon_1}{\varepsilon_2}}H_1 + \eta^{1-2\frac{\varepsilon_1}{\varepsilon_2}}H_1'\right] - \frac{\varepsilon_1}{\varepsilon_2}\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 + \eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1' +$$

$$+ \frac{\varepsilon_1}{\varepsilon_2}\left(1+\frac{\varepsilon_1}{\varepsilon_2}\right)\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 - \frac{\varepsilon_1}{\varepsilon_2}\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1' + \eta^{2-\frac{\varepsilon_1}{\varepsilon_2}}H_1'' + \eta H_3 - \frac{1}{K}\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 = 0, \tag{19}$$

$$\left(1+\frac{\nu_1}{\nu_3}\right)\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 - \frac{\varepsilon_2}{\varepsilon_1}\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1' - \frac{\varepsilon_1}{\varepsilon_2}\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_1 + \eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1' + \frac{\nu_1}{\nu_3}\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_2 -$$

$$-\left[\left(\frac{\nu_1}{\nu_3} - \frac{\varepsilon_1}{\varepsilon_2}\right)\eta^{-\frac{\varepsilon_1}{\varepsilon_2}}H_2 + \eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_2'\right] = 0, \tag{20}$$

$$\frac{1}{P_r}[\eta H_3 + 3\eta^2H_3' + \eta^3H_3''] + H_2\left[\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_3 + \eta^{2-\frac{\varepsilon_1}{\varepsilon_2}}H_3'\right] - \frac{\varepsilon_2}{\varepsilon_1}[\eta H_3 + \eta^2H_3'] -$$

$$-\left(1+\frac{\nu_3}{\nu_2}\right)\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_1H_3 - \frac{\nu_3}{\nu_2}\eta^{2-\frac{\varepsilon_1}{\varepsilon_2}}H_1H_3' + \frac{\varepsilon_2}{\varepsilon_1}\cdot\frac{\nu_1}{\nu_2}\left[\eta^{1-\frac{\varepsilon_1}{\varepsilon_2}}H_3 + \eta^{2-\frac{\varepsilon_1}{\varepsilon_2}}H_3'\right]H_1 = 0. \tag{21}$$

Thus we find that the equations (1), (2) and (3) can be reduced to the ordinary differential equations (19), (20), (21), respectively. Hence there exists a similarity solutions for such a problem. The boundary conditions can be fixed up for H_i ($i = 1, 2, 3$) and the equations (19) to (21) can be solved by using these boundary conditions.

III. CASE II: PERMEABILITY IS VARIABLE

Let us now consider the case when the permeability K is a function of the space coordinate x and time t. To find the similarity solution of the equations (1) to (3), we introduce the following continuous parameter group of transformations:

$$\bar{u} = a^pu,\ \bar{v} = a^qv,\ \bar{T} = a^rT,\ \bar{x} = a^mx,\ \bar{y} = a^ny,\ \bar{t} = a^st,\ \bar{K} = a^lK \tag{22}$$

Then the expression Φ_1 is constant conformally invariant under the transformation group if

$$p = m - 2n,\ q = -n,\ r = m - 4n,\ s = 2n,\ l = 2n. \tag{23}$$

Thus the group of transformations (22) becomes

$$\bar{u} = A^{1-\frac{2n}{m}} u,\quad \bar{v} = A^{-\frac{n}{m}} v,\quad \bar{x} = Ax,\quad \bar{y} = A^{\frac{n}{m}} y,\quad \bar{t} = A^{\frac{2n}{m}} t,$$

$$\bar{T} = A^{1-\frac{4n}{m}} T,\quad \bar{K} = A^{\frac{2n}{m}} K, \tag{24}$$

where $A = a^m$.

It can be easily verified that the expression Φ_2 is a constant conformally invariant under the group of transformations (24). The expression Φ_3 is constant conformally invariant under the transformations (24) provided we neglect the dissipation term.

We can reduce the three independent variables x, y and t to two independent variables η_1 and η_2, which are absolute invariants under the transformation (24) such that the expressions Φ_i $(i = 1, 2, 3)$ in the independent variables x, y and t and the dependent variables u, v, T and the known function K can be reduced to expressions in the independent variables η_1 and η_2 and the dependent variables F_1, F_2, F_3 which are functions of η_1 and η_2 and in terms of a known function $F_4(\eta_1, \eta_2)$.

We choose the variables η_1 and η_2 as follows:

$$\eta_1 = tx^{-\frac{2n}{m}},\quad \eta_2 = yx^{-\frac{n}{m}}. \tag{25}$$

It can be verified that η_1 and η_2 are absolute invariants under the group of transformations (24).

We can also find out the following absolute invariants involving the dependent variables:

$$g_1 = ux^{\frac{2n}{m}-1},\quad g_2 = vx^{\frac{n}{m}},\quad g_3 = Tx^{\frac{4n}{m}-1},\quad g_4 = \frac{1}{K}x^{\frac{n}{m}}. \tag{26}$$

It can be verified that g_i $(i = 1, 2, 3, 4)$ are functionally independent.

We can define the dependent variables as follows:

$$u = F_1(\eta_1, \eta_2)x^{-\frac{2n}{m}+1},\quad v = F_2(\eta_1, \eta_2)x^{-\frac{n}{m}},$$

$$T = F_3(\eta_1, \eta_2)x^{-\frac{4n}{m}+1},\quad \frac{1}{K} = F_4(\eta_1, \eta_2)x^{-\frac{2n}{m}}. \tag{27}$$

Using the transformations (27) and the definitions of η_1 and η_2 given by (25) in the equation $\Phi_1 = 0$ we get

$$\eta_1\,\partial F_1/\partial\eta_1 - F_1\left[\frac{2n}{m}\eta_1^2\frac{\partial F_1}{\partial\eta_1} + \frac{n}{m}\eta_1\eta_2\frac{\partial F_1}{\partial\eta_2} + \left(\frac{2n}{m} - 1\right)\eta_1 F_1\right] + \eta_1 F_2\frac{\partial F_1}{\partial\eta_2} - \eta_1\frac{\partial^2 F_1}{\partial\eta_2^2} - \eta_1 F_3 + \eta_1 F_1 F_4 = 0. \tag{28}$$

Similarly, the equations $\Phi_2 = 0$ and $\Phi_3 = 0$ are respectively transformed to

$$\left(1 - \frac{2n}{m}\right)F_1 - \frac{2n}{m}\eta_1\frac{\partial F_1}{\partial\eta_1} - \frac{n}{m}\eta_2\frac{\partial F_1}{\partial\eta_2} = 0, \tag{29}$$

$$\frac{\partial F_3}{\partial\eta_1} - F_1\left[\left(\frac{4n}{m} - 1\right)F_3 + \frac{2n}{m}\eta_1\frac{\partial F_3}{\partial\eta_1} + \frac{n}{m}\eta_2\frac{\partial F_3}{\partial\eta_2}\right] + F_2\frac{\partial F_3}{\partial\eta_2} - \frac{1}{P_r}\frac{\partial^2 F_3}{\partial\eta_2^2} = 0. \tag{30}$$

The independent variables in (28) to (30) are η_1 and η_2, while the dependent variables are F_1, F_2, F_3 and the new known function is F_4.

Now we introduce a new transformation group as follows:

$$\bar{\eta}_1 = b^{\delta_1}\eta_1,\quad \bar{\eta}_2 = b^{\delta_2}\eta_2,\quad \bar{F}_1 = b^{\delta_3}F_1,\quad \bar{F}_2 = b^{\delta_4}F_2,\quad \bar{F}_3 = b^{\delta_5}F_3,\quad \bar{F}_4 = b^{\delta_6}F_4. \tag{31}$$

Under the transformations (31), the left hand side of the equation (28) is constant conformally invariant provided

$$\delta_1 = 2\delta_2,\quad \delta_3 = -2\delta_2,\quad \delta_4 = -\delta_2,\quad \delta_5 = -4\delta_2,\quad \delta_6 = -2\delta_2. \tag{32}$$

Then the transformations (31) under the conditions (32) become

$$\eta_1 = b^{2\delta_2}\eta_1,\quad \bar{\eta}_2 = b^{\delta_2}\eta_2,\quad \bar{F}_1 = b^{-2\beta_2}F_1,\quad \bar{F}_2 = b^{-\delta_2}F_2,$$

$$\bar{F}_3 = b^{-4\delta_2}F_3,\quad \bar{F}_4 = b^{-2\delta_2}F_4. \tag{33}$$

It can be seen that the left-hand sides of the equations (29) and (30) are constant conformally invariant under the transformations (33).

We next proceed to find the absolute invariants of the independent variables under the transformations group (33). It can be easily verified that the transformation of the independent variables η_1, η_2 given by

$$\eta = \eta_1^{-\frac{1}{2}}\eta_2 \tag{34}$$

is an absolute invariant of the group of transformations (33). Absolute invariants containing the dependent variables are given by

$$h_1 = F_1\eta_1,\quad h_2 = F_2\eta_1^{\frac{1}{2}},\quad h_3 = F_3\eta_1^2,\quad h_4 = F_4\eta_1. \tag{35}$$

Equations (28) to (30) can now be reduced to ordinary differential equations if we express the dependent variables and the known functions as follows:

$$F_1 = \eta_1^{-1}H_1(\eta),\quad F_2 = \eta^{-\frac{1}{2}}H_2(\eta),\quad F_3 = \eta_1^{-2}H_3(\eta),\quad f_4 = \eta_1^{-1}H_4(\eta). \tag{36}$$

It can be easily verified that h_i $(i = 1, 2, 3, 4)$ are functionally independent. Using the transformation (36) and the definition (34) for η in equations (28) to (30) we obtain

$$\left(H_1 + \frac{1}{2}\eta H_1'\right) - H_1^2 - H_2 H_1' + H_1'' + H_3 - H_1 H_4 = 0, \tag{37}$$

$$H_1 + H_2' = 0, \tag{38}$$

$$\left(2H_3 + \frac{1}{2}\eta H_3'\right) - H_1 H_3 - H_2 H_3' + \frac{1}{P_r} H_3'' = 0. \tag{39}$$

Thus the equations (37) to (39) are ordinary differential equations of the independent variables H_1, H_2 and H_3 and these variables are functions of the dependent variable η. The function H_4 is involved in equation (37) as a known function.

Hence, we can conclude that since the equations $\Phi_i = 0$ $(i = 1, 2, 3)$ are reduced to ordinary differential equations, a similarity solution in this case is possible. The boundary conditions for $H_i (i = 1, 2, 3)$ can be fixed up at this stage and the equations (37) to (39) can be solved for H_i by using these boundary conditions.

Let us consider the free convection flow. Then the boundary conditions are

$$u = v = 0, \quad T = T_\omega \quad \text{at} \quad y = 0,$$

and u, $T \to 0$ as $y \to \infty$.
The boundary conditions under the above group of transformations reduce to

$$H_1 = H_2 = 0 \quad \text{at} \quad \eta = 0$$

and H_1, $H_3 \to 0$ as $\eta \to \infty$,
where $T_\omega = t^{-2}x$.

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Received December 21st, 1988
Accepted for publication January 18th, 1989

ТЕОРЕТИКО-ГРУППОВОЙ ПОДХОД ДЛЯ КЛАССИФИКАЦИИ ПОДОБИЯ РЕШЕНИЙ ДЛЯ НЕСТАЦИОНАРНЫХ ТОКОВ ЧЕРЕЗ ПОРИСТУЮ СРЕДУ

Работа посвящена теоретико-групповому подходу для классификации подобия решений в естественной конвекции нестационарного потока вязкой несжимаемой жидкости через пористую среду. Учтены два случая: 1) проницаемость среды равна постоянной и 2) проницаемость является функцией пространственно-временных координат. показано, что решения подобия возможны в обоих случаях.