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DYNAMICS OF POSITIVE IONS IN FRONT OF A PLANAR FLOATING WALL[1)]

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The spatial profiles of macroscopic forces on one ion and of transport quantities from an infinite fluid model of a sheath between atomic cold gas plasma and a floating planar wall are reported in dependence on the parameters A, C of ion neutral and ionization frequencies normalized by plasma frequency. Among others it is shown that at $A \gtrsim 1$, $C \lesssim 0.01$ the ion motion is controlled by a local electric field practically over the whole sheath.

I. INTRODUCTION

Though most problems of weakly ionized plasmas are in connexion with plasma wall contact, mathematical difficulties cause that so far there have been solved only the simplest space charge, steady state, usually one dimensional models without a magnetic field, consisting of electrons, singly charged positive ions and parent gas in the ground stage.

The problem was first solved by means of the Boltzmann kinetic equation for ions, for plasmas without ion neutral collisions [1]. Later the hydrodynamic approach considering inertia together with friction and creation of ions allowed to study the space charge plasma wall sheath over a wide range of plasma parameters describing collisionless as well as collisional sheaths. The application of such models is, however, restricted by assumptions concerning the ion temperature. One should take into account the original temperature of ions far from the wall and, at the same time, the rising temperature of ions towards the wall.

There have been treated hydrodynamic models assuming collisional plasmas with zero ion temperature [2, 3], collisionless plasmas with ion heating and non-zero costant gas temperature [4], collisional plasmas with ion heating and

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zero gas temperature [5] and recently collisional plasmas with constant ion temperature [6].

The models are constructed in a symmetrically finite geometry, namely either for plasmas between two parallel infinite walls [1, 4, 6] or for plasmas inside an infinite insulated cylinder [2—6], supposing that the balance between the generation and the extinction of the charged carriers is provided only by the recombination on the wall. If constant ion temperature is taken into account [2, 3, 6], the zero and the first velocity moments of the Boltzmann kinetic equation for ions are used. If ion heating takes place [4, 5], there are used the moments up to the third order and also some additional conditions necessary to close the equation system.

The here presented numerical results concern the model of an infinite plasma in contact with an infinite planar floating wall solved in [7] and analysed according to collision number in [8]. Due to the not finite dimension of the model the space recombination must be necessarily taken into account. Ion heating is also assumed there, but in a simplified way regarding [4, 5], using Wannier's formula. Then it is sufficient to describe ion dynamics similarly as in models of a constant ion temperature [2, 3, 6] only by the first two velocity moments.

Because of a fast nonlinear joining of all the physical quantities near the wall the relative roles of the particular effects acting on an ion in the plasma wall sheath are spatially varying. The presented article aims to be a contribution to the understanding of the processes yielding the ion transport properties from the plasma to a floating wall. For this purpose the spatial profiles of the macroscopic forces acting on each single ion are given in parallel with the transport quantities.

II. SYMBOLS

All the symbols used in the paper are in this section. On the left there are to be found the physical quantities, on the right the respective normalized ones in which the results are presented.

Variables

r, $\xi = r/h$ coordinate perpendicular to the wall,
v, $u = v/v_i$ ion drift velocity,
φ, $\eta = -e\varphi/kT_-$ the electric potential,
E, $\varepsilon = d\eta/d\xi$ the electric field intensity,
n_-, $x_- = n_-/n_0$ the electron number density,

n_+, $x_+ = n_+/n_0$ the ion number density,
$-$, $\Theta = n_+ v/n_0 v_i$ the ion flux density,
f, $F = fh/kT_-$ the force on an ion,
T_+, (T_+/T_-) the temperature of random ion motion in the direction perpendicular to the wall.

Parameters and constants

$-$, γ ion heating coefficient,
$-$, $\varkappa_e$ coefficient of electron reflection on the wall,
M, (M/m) the ion mass,
ν, $A = h\nu/v_i$ charge exchange and elastic scattering frequency,
α, $C = h\alpha/v_i$ direct ionisation frequency,
β, $C = hn_0\beta/v_i$ recombination rate,
$h = (\varepsilon_0 kT_0/e^2 n_0)^{1/2}$ Debye length,
$v_i = (kT_-/M)^{1/2}$ the ion sound speed,
T_- the electron temperature,
n_0 the number density of electrons, resp. ions, at an infinite distance from the wall,
m the electron mass, e the positive elementary charge,
ε_0 the permitivity of vacuum, k Boltzmann's constant,
u_+ the ion mobility, R the tube radius,
p_g and T_g the pressure and temperature of neutral gas.

III. THE PHYSICAL MODEL AND DYNAMICS OF IONS

Singly charged positive ions in a parent atomic gas of the ground stage, zero temperature and constant number density move from homogeneous undisturbed plasma through an electrostatic field

$$E = -\frac{d\varphi}{dr} \qquad \frac{dE}{dr} = \frac{e}{\varepsilon_0}(n_+ - n_-) \tag{1}$$

$$n_- = n_0 \exp(-e\varphi/kT_-)$$

to a planar floating wall determined by condition [9, 10]

$$n_+ v = \frac{1-\varkappa_e}{1+\varkappa_e}\frac{1}{2} n_- \left(\frac{8kT_-}{\pi m}\right)^{1/2}. \tag{2}$$

In the space charge sheath an effective generation of ions occurs given by a difference of single stage ionizations and recombinations

$$\frac{\mathrm{d}(n_+ v)}{\mathrm{d}r} = \alpha n_- - \beta n_- n_+ \tag{3}$$

according to which under the condition in infinity

$$\beta = \alpha / n_0.$$

At the same time the ions are supposed to suffer charge exchange and elastic collisions providing together with ionisations a mechanism of ion heating. The rising ion temperature T_+ is assumed to be described by Wannier's formula [11]

$$kT_+ = kT_g + \gamma M v^2, \quad \gamma \in \langle 0, 1 \rangle, \qquad T_g = 0. \tag{4}$$

To describe the dynamics of ions the first velocity moment is used for the case of cold ($\gamma = 0$) and heated ($\gamma \neq 0$) ions, respectively [7]:

For reasons of a cold gas assumption ($T_g = 0$) the original movement force acting on each ion in the direction towards the wall is only that of an electric field

$$f_E = eE. \tag{5}$$

At the same time the ion is decelerated towards the wall by a friction force due to ion collisions with neutrals

$$f_F = \nu M v \tag{6}$$

and by a creation force of cold ions generated near the wall

$$f_C = \alpha M v n_- / n_+, \tag{7}$$

while recombinations (3) do not create any force [12].

If ion heating (4) takes place, $\gamma \neq 0$, there are acting on each ion besides the forces (5)—(7) also the accelerating force of diffusion

$$f_D = -\gamma \frac{M_{v^2}}{n_+} \frac{\mathrm{d}n_+}{\mathrm{d}r} \tag{8}$$

and the retarding for of thermodiffusion

$$f_T = 2\gamma M v \frac{\mathrm{d}v}{\mathrm{d}r}. \tag{9}$$

The forces (5)—(9) results in the inertia force on one ion

$$f_I = M v \frac{\mathrm{d}v}{\mathrm{d}r} \tag{10}$$

$$f_I = f_E + f_D - (f_F + f_C + f_T)$$

of a cold gas heated ion approximation.

IV. RESULTS AND DISCUSSION

IV. 1 Models comparison

As there is known only one condition at the wall (2), all models [1—7] were solved as an initial task with starting values reached by power series expansions near the values in the infinite [7], resp. at the axis [1—6] of the plasma.

The disparate scales of infinite and finite models cause that the only comparable values of such models are those at the floating wall (index w) (Tables 1, 2). However, there should be noted a difference of the parameters A, C as, instead of the normalization constant n_0, in finite symmetrical models the value n_{-0}, $n_{-0} < n_{+0}$, of the electron number density from the mid plane resp. axis,

Table 1

Values of normalized potential η_w and drift velocity u_w at the floating wall of $\varkappa_e = 1/3$ (2) for argon plasmas; p — planar geometry, c — cylindrical geometry, t — gas temperature $T_g/T_- = 0.1$, h — heated ions.

		[7] p ($\gamma = 0$)		[7] ph ($\gamma = 1$)		[4] pth		[4] cth		[5] ch		[3] c	
	A/C	0.1	0.1	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01
η_w	1	6.94	8.08	6.99	8.09	—	—	—	—	6.87	—	6.74	7.79
	0	6.06	6.23	6.26	6.46	5.10	5.31	5.90	5.73	5.86	5.69	5.73	5.59
u_w	1	1.27	0.936	1.18	0.903	—	—	—	—	0.955	—	0.994	0.76
	0	3.18	3.34	2.31	2.43	3.08	3.22	3.19	3.27	3.17	3.25	3.10	3.19

Table 2

Values of normalized potential η_w at the same parameters as in Table 1 but for mercury plasmas. p, c — planar and cylindrical geometry, h — heated ions, k — kinetic theory.

	[7] p ($\gamma = 0$]		[7] ph ($\gamma = 1$)		[1] pk		[2] c	
A/C	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01
1	7.1	8.25	7.15	8.25	—	—	8.02	9.13
0	6.2	6.37	6.4	6.6	5.94	6.16	6.58	6.41

occurs. From this point of view a reasonable comparison between the infinite and the finite models is possible only at the parameters A, C, where in the centre $(n_{+0} - n_{-0}) \ll n_{-0}$ (Tab. 3).

Table 3
Values of $(n_{+0} - n_{-0})/n_{-0} \times 100\%$ on the axis, taken from [3]

A/C	0.1	0.01
0	10.4	1.01
1	1.46	0.015

Comparing the floating potentials (Tabs. 1, 2), there is seen among others the geometry influence at $A = 0$, as $\eta(C)$ is decreasing in planar geometry, increasing in a cylindrical one. If we follow the floating potentials in parallel with the ion drift velocities at the wall (Tab. 1), the influence of the volume processes at the wall is evident. For example:
at $A = 0$ there is $u_w^2 < 2\eta_w$;
$\eta_w(A)$ is increasing while $u_w(A)$ is decreasing:
at $A = 0$ $u_w(C)$ is decreasing, at $A = 1$ it is increasing.

Comparing the ion drift velocities at $A = 1$ both models for heated ions [5, 7] yield smaller values of u_w than the models for cold ions [3, 7]. Comparing u_w at $A = 0$ the values from the heated ion approximation [5] are higher than those from the cold ion approximation [3], while our models [7] give opposite results. The reason of [7] is evidently based on the not suitable use of Wannier's formula (4) at $A = 0$, $C \leqslant 0.1$, as the ion neutral collisions are absent there and the number of ionisations in the sheath is small [8].

IV. 2 Macroscopic forces on an ion

The typical curves of normalized forces for collisional ($A = 1$) and nearly collisionless ($A = 0.01$) sheaths [8], respectively, are drawn in Figures 1, 2 as functions of the normalized relative coordinate ξ_{Hg}. The index Hg denotes that the zero value

$$\xi_{Hg} = 0$$

belongs to the fully absorbing wall (2) $\varkappa_e = 0$ in mercury plasma.

In Figs. 1 the normalized forces on a cold ion ($\gamma = 0$) are shown. The upper value belongs, of course, to the electric force F_E (5). F_E depends, like the space charge sheath thickness [8], more on the parameter C than A.

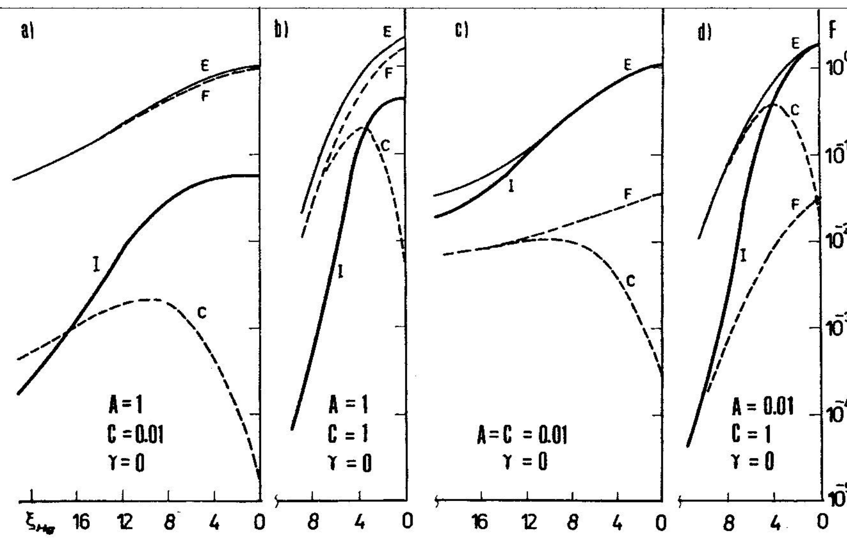

Fig. 1. Normalized spatial profiles of macroscopic forces $F(\xi)$ on one ion for cold ions ($\gamma = 0$) and various A, C parameter couples;
——— E: the accelerating force of the electric field, – – – – F, C: the decelerating forces of friction and creation, ——— I: the resulting inertia force.

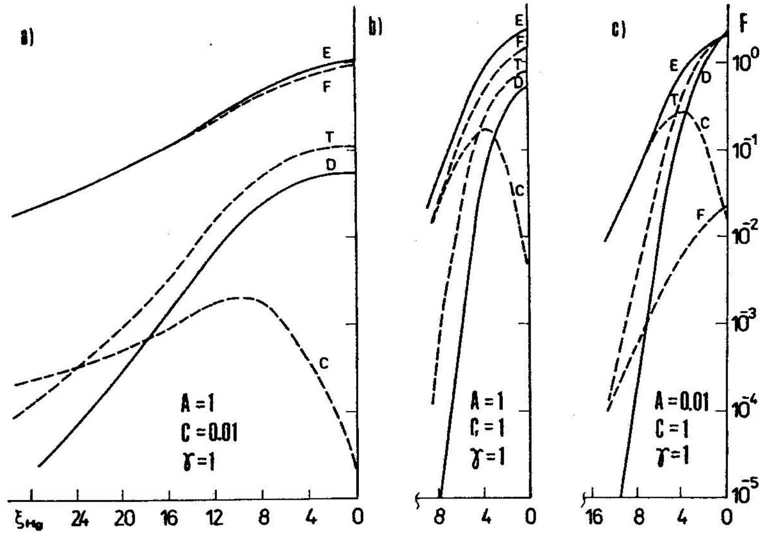

Fig. 2. Normalized spatial profiles $F(\xi)$ of macroscopic forces on one ion for supremely heated ions ($\gamma = 1$) and various A, C parameter couples;
——— E, D: the accelerating electric and diffusion forces, – – – – F, C, T: the decelerating friction, creation and thermodiffusion forces.

Figure la shows that at the parameter couple $(A, C) = (1, 0.01)$, in the whole plasma wall space the electric force F_E practically equals the friction force F_F (6) so that the drift velocity of ions is controlled only by the local value of the electric field and, as it is supposed in Wannier's theory [11],

$$\varepsilon / u \doteq A. \tag{11}$$

(Considering smaller C, $(A, C) = (1, 0.001)$, it becomes closer).

On the other hand (Fig. 1c), at $A = C = 0.01$ the electric force F_E is converted near the wall into the inertia force F_I (10). However, as expected (Tab. 1), the ion free fall occurs only over a finite distance from the wall[1]. Therefore also at small collision parameters A, C there must be considered a space charge plasma wall model with both inertia (10) and collision forces (6, 7).

In Figs. 1b, 1d the force curves at $A = C = 1$, and $(A, C) = (0.01, 1)$ respectively, are given to show the trends of force shifting by a rising C in relation with $C = 0.01$ (Figs. 1a, 1c). From the practical point of view the last parameters are out of interest.

In Figs. 2 there are plotted the normalized forces (5)—(9) for heated ion approximation (4) at the upper value for γ, $\gamma = 1$, representing the hypothetical supremely heated case. The inertia force F_I (10) is not given separately, as $F_I = F_T/2$ (9). Also the case of small values of A and C, $A = C = 0.01$, is absent in relation to Figs. 1 because similarly as at $A = 0$ (part IV.1) there is less than one ion neutral and ionization collision during the ion transit through the space charge sheath [8] and due to this Wannier's formula (2) is not admissible.

The forces of diffusion F_D (8) and thermodiffusion F_T (9) are rising towards the wall (Figs. 2). However, the collision number analysis [8] shows that just near the wall the collisions vanish (see the ion transit time through the last Debye length in front of the wall in [8] — Tab. 4) e.g., the mechanism of ion heating falls off there and Wannier's formula yields, without regard to the supreme value of γ used, an overestimation of F_T and F_D. In fact, F_T should be expected to have a peak between the plasma and the wall.

Comparing Figs. 1 with Figs. 2 there are no essential differences between the shapes of respective F_E, F_F, F_C and F_I profiles for the case of cold and supremely heated ions.

1) This occurs approximately from the point where the normalized drift velocity achieves the value $u \doteq 1$ predicted by Bohm's criterion as a border of a collisionless space charge sheath, and which also determines the border of the quasineutral presheath theory [13]. The problem of joining both theories [1] appears from our model at $u = 1$, $\eta = 1.22$, while according to [13] at $\eta = 0.822$.

IV. 3 Transport quantities

In Figures 3—5 the spatial profiles of normalized drift velocities, concentrations and flux densities of ions are presented a) at $C = 0.1$ for various values of the normalized collision frequency A, b) at $A = 0.1$ for various values of the normalized ionization frequency C, c) at $C/A = 1$ for various $A = C$. The behaviour of transport quantities shown in the presented graphs is typical also for the regimes of parameters discussed above. The seen phenomena become clear when we follow the forces on one ion (Figs. 1, 2).

Figures 3 show that the spatial curves of normalized drift velocities $u(\xi)$ are for various A (Fig. 3a) mutually divergent towards the wall, which is understandable as at higher A mostly the friction force F_F grows to a detriment of the inertia force F_I (Figs. 1a, 1c).

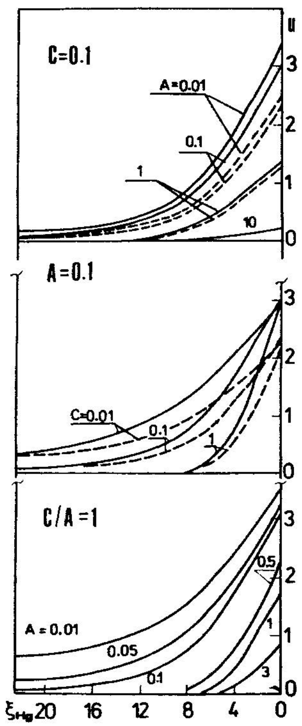

Fig. 3. Normalized spatial profiles of ion drift velocities $u(\xi)$, a) for constant $C = 0.1$ and various A, b) constant $A = 0.1$ and various C, c) various $A/C = 1$:
—— $\gamma = 0$, ---- $\gamma = 1$.

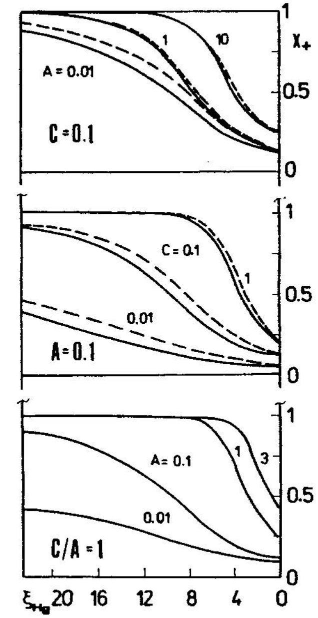

Fig. 4. Normalized spatial profiles of ion concentrations $x_+(\xi)$ at similar parameter couples as in Fig. 3;
—— $\gamma = 0$, ---- $\gamma = 1$.

In turn, for various C (Fig. 3b) the spatial curves $u(\xi)$ are mutually convergent towards the wall and for smaller C even cross in front of the wall (compare with $u_w(C)$ values in Tab. 1 at $A = 0$, $A = 1$ respectively). The reason of convergence is firstly in the fact that the higher the C, the higher the F_E at the wall, while the sheath thickness becomes narrower (Figs. 1a, 1b and 1c, 1d).

At a parallel change of a relatively high $A = C$ (Fig. 3c) the $u(\xi)$ curves are divergent, at a relatively small $A = C$ they are convergent.

Figs. 4 show that the increasing A and C, respectively increase the ion number densities in space but the changes by A are smaller.

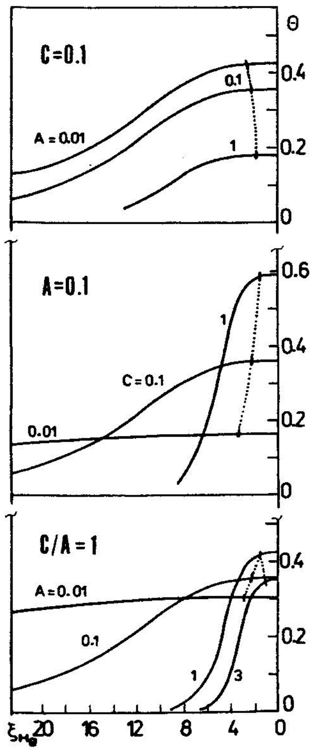

Fig. 5. Normalized spatial profiles of ion flux densities $\Theta(\xi)$ at the same parameter couples as in Fig. 4; the marked points denote the position of the floating wall in hydrogen plasma.

Similarly as in [5] for $A \neq 0$, the ion heating (4) sets the drift velocities to lower (Figs. 3a, 3b) and the concentrations to higher (Figs. 4a, 4b) values, but the differences at smaller A, resp. C, are, for the already mentioned reason of vanishing collisions and the supreme value of γ, overestimated here.

Finally Figs. 5 show, as may be expected, that the ion flux densities become at increasing A smaller while they grow at increasing C. The dependences are underlined in Figs. 5 by points marking the fully absorbing wall ($\varkappa_e = 0$ (2)) in hydrogen.

V. CONCLUSION

Since the dimensionless numerical silutions are common for any atomic gas and the floating wall position of any atomic gas lies between the wall of hydrogen and mercury [7] (Figs. 5) the presented results concern any atomic gas.

Though the way of ion heating inclusion by Wannier's formula (4) is not absolutely correct the comparison of Figs. 1 and 2 gives a basis to suppose the conclusions regarding Figs. 1 (part IV.2) to be suitable also for a more satisfying description of ion heating.

At $A > 1$, $C \lesssim 0.01$ (Figs. 1a, 2a) a plasma wall model without ion inertia force (10) may be built up (11) eliminating the problem of the original ion temperature inclusion [6] in the model.

For practical purposes the relations between the dimensionless and the physical parameters are important. A simple estimation at $A \gtrsim 1$, $C \lesssim 0.01$ is possible:

The assumption of dominant direct ionization demands to satisfy the relation [14]

$$|e\varphi_w| > e\varphi_i$$

(φ_i the ionization potential). From this it follows that $kT_- > e\varphi_i/\eta_w$, Table 4.

Table 4

Values for various atomic plasmas:

$kT_- = e\varphi_i/\eta_w$ in [eV] at $\eta_w(\varkappa_e = 0,\ A = 1,\ C = 0.01)$ [7], $K_A[\mathrm{m}^{-3/2}\,\mathrm{Pa}^{-1}\,\mathrm{K}]$, $K_B[\mathrm{m}^2\,\mathrm{Pa}^2\,\mathrm{K}^{-3}]$

	H	He	Ar	Kr	Xe	Hg
kT_-	2.1	3.5	1.9	0.51	1.4	1.8
K_A	1.2	0.94	2.1	2.3	2.8	6
K_B	8	31	6.4	5.1	3	0.75

At the same time from (11) we have

$$v = e/M\mu_+$$

and for α we can use the relation

$$\alpha = D_a(2.4/R)^2, \quad D_a = \mu_+ kT_-/e,$$

resulting from the quasineutral presheath theory [13] without volume recombination when $C \ll A$ and the momentum transfer equation has the form $f_E + f_D = f_F, f_D = (kT_+/n_+)\mathrm{d}n_+\mathrm{d}r$. Simultaneously there is valid for the mobility μ_+

$$\mu_+ = \frac{760.133}{273}\,\mu_0\,\frac{T_g}{p_g}.$$

Substituting all these into the dimension-less parameters

$$A = \nu\left(\frac{\varepsilon_0 M}{e^2 n_0}\right)^{1/2} \quad \text{and} \quad C/A = \alpha/\nu,$$

by the values of μ_0 [15] we get the approximative relations

$$A = 108 K_A n_0^{-1/2} p_g / T_g,$$

$$C/A = 10^{-13} K_B T_- (T_g / R p_g)^2,$$

K_A, K_B (Table 4). Inserting n_0, p_g, T_g, T_- and R into A, C/A the dimensionless parameters $A \gtrsim 1$, $C \lesssim 0.01$ conform to the macroscopis parameters of the positive column of the glow discharge.

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ДИНАМИКА ПОЛОЖИТЕЛЬНЫХ ИОНОВ ПЕРЕД ПЛОСКОЙ ИЗОЛИРОВАННОЙ СТЕНОЙ

В работе предложены пространственные профили макроскопических сил, приходящихся на один ион, и пространственные профили транспортных величин ионов, полученные по бесконечной флюидной модели слоя между плазмой с атомным холодным газом и плоской стеной на изолированном потенциале, для различных значений A и C частоты ион-нейтральных и ионизационных столкновений, определенных частотой плазмы. Кроме того, указано, что при A $\gtrsim$ 1, C $\lesssim$ 0,01 движение ионов определено локальной величиной электрического поля.