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THEORETICAL AND NUMERICAL ANALYSIS OF LIGHT DIFFRACTION ON ACOUSTIC PULSES IN BRAGG REGION[1)]

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The paper presents a brief theoretical description and numerical analysis of diffraction of a Gauss light beam on acoustic pulses in the Bragg region. The dependence enabled us to calculate the intensity of the diffracted light as a function of diffraction angle and time was obtained. The numerical analysis was carried out for different values of acoustic wave velocity, acoustic wave attenuation, carrier frequency of the wave, the Gauss distribution parameter of the light beam and different envelopes of the acoustic pulse. Based on the results of the numerical analysis conclusions concerning the possibilities of the practical use of that interaction were formulated.

I. INTRODUCTION

An interaction of light with acoustic pulses is one of the most interesting problems in acousto-optics because of the possibility of its practical use. For instance, it can be the basis for the elaboration of an acoustic pulse analyser [1]. The majority of the paper engaged in that problem concern the interaction in the Raman-Nath region [2—5]. In [6] preliminary theoretical considerations of the light diffraction on acoustic pulses in the Bragg region were presented. The aim of this paper is a brief theoretical description of the interaction of the Gauss light beam with acoustic pulses in an isotropic, dielectric medium in the Bragg region and the presentation of the results of numerical analysis carried out on the basis of that description.

II. THEORETICAL DESCRIPTION

The theoretical analysis is based upon the work of Bhatia and Noble [7]. The propagation of the electro-magnetic wave in an isotropic, non-magnetic,

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dielectric medium can be described by the integral-differential equation [8]:

$$\boldsymbol{E}'(\boldsymbol{r}, t) = \boldsymbol{E}^{(i)} + \frac{1}{\varepsilon_0}\iiint_{V'} \text{rot rot}\left\{\alpha N \frac{\boldsymbol{E}'(\boldsymbol{r}', t - R/c)}{R}\right\} dV' \qquad (1)$$

where $R = |\boldsymbol{r} - \boldsymbol{r}'|$; $\boldsymbol{r}', \boldsymbol{r}$ — vectors, describing the position of the points inside the area filled with a dielectric V' and outside it, respectively; $\boldsymbol{E}^{(i)}$ — electric field intensity vector of the incident light wave; $\boldsymbol{E}'(\boldsymbol{r}, t)$ — vector of effective electric field intensity at the point $\boldsymbol{r}$, at the time t; N — particle's concentration in the medium; α — its polarizability; ε_0 — permitivity of free space; c — light velocity.

According to the phenomenological theory of the elasto-optical effect for the isotropic medium, its electric impenetrability is

$$\varkappa = \frac{1}{\varepsilon} = \varkappa_0 + \Delta\varkappa = \varkappa_0 + p\zeta, \qquad (2)$$

where ε — permitivity of the medium; $\varkappa_0$ — electric impenetrability of the medium with no disturbance; $\Delta\varkappa$ — change of the impenetrability connected with a deformation in the medium; p — its effective photo-elastic constant of it; ζ — deformation connected with acoustic wave propagating in the medium. Taking into consideration the connection between the electric field intensity of the incident light wave and the effective electric field intensity and using the Lorentz—Lorenz formula one can obtain the equation for the electric field intensity of the light wave, arising as a result of an acousto-optic interaction, in the form

$$\boldsymbol{E}^{(\zeta)} = -\frac{1}{\varepsilon_0}\iiint_{V'} \text{rot rot}\left\{p\zeta \frac{\boldsymbol{E}'(\boldsymbol{r}', t - R/c)}{R}\right\} dV'. \qquad (3)$$

Further considerations were made concerning the geometry of the interaction shown in fig. 1. Let the acoustic wave propagate along the x axis in the layer of L thickness and have the constant intensity in its cross-section. Let the light wave run at the angle Θ to the z axis and let it not be limited in the direction of the y axis. Let its electric field intensity be described by the Gauss distribution in the direction perpendicular to the y axis and its direction of propagation. Then, the deformation connected with the acoustic wave

$$\zeta = A \exp(-\beta x) F(x - Vt) \cos[K(x - Vt)], \qquad (4)$$

where A — amplitude of deformation; β — acoustic wave attenuation coefficient; K — wave number of acoustic wave; V — its velocity; $F(x - Vt)$ — function describing an envelope of the acoustic wave in the interaction area.

Assuming that the angle Θ is small, the electric field intensity of the incident light wave is described by the approximate dependence

$$\boldsymbol{E}^{(i)}(\boldsymbol{r}, t) = \boldsymbol{e} E_0 \exp(-x^2/w^2) \exp[\text{j}(kz \cos\Theta - kx \sin\Theta - \omega t)], \qquad (5)$$

where $\boldsymbol{e}$ — polarization vector of the incident light wave, $|\boldsymbol{e}| = 1$; E_0 — amplitude of the electric field intensity of that wave; k — its wave number; ω — its frequency; w — Gauss distribution parameter; $\text{j} = (-1)^{1/2}$.

We have assumed that the diffracted light wave propagates at the angle φ towards the z axis.

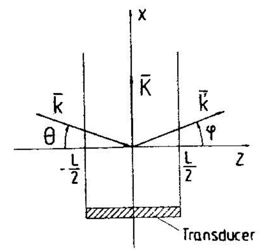


Fig. 1. Geometry of interaction.

In our considerations we assumed that the frequency dependence of β causes changes of the function $F(x - Vt)$, i.e. changes of the envelope of the acoustic wave, which propagates in the medium. However, we assumed that the dimension of the interaction area in the direction of the acoustic wave propagation is sufficiently small to assume that $F(x - Vt)$ is constant in its area. That assumption is equivalent to the neglection of the frequency dependence of β when we describe the acoustic wave in the interaction area only.

Substituting (4) and (5) into the formula (3), neglecting the factors in which the quantity $V/c \ll 1$ occurs and taking into account that no integrand is a function of y' we obtained

$$\boldsymbol{E}^{(\zeta)} = -\frac{pAE_0}{\varepsilon_0}\int_{x'}\int_{z'} \text{rotrot}\,\{\boldsymbol{e}\exp[-(\beta x' + x'^2/w^2)] F(x' - Vt) \times$$
$$\times \cos[K(x' - Vt)] \exp[\text{j}(kz' \cos\Theta - kx' \sin\Theta - \omega t + kR)]\}\, dx'\, dz'. \qquad (6)$$

Equation (6) can be much more simplified if we assume the light wave polatization vector $\boldsymbol{e}$ to be parallel to the y axis. Moreover, when we took into account that under the experimental conditions $\lambda \ll R$ (λ — light wave length) and $w \ll d$ (d — distance between the interaction area and the place, where diffraction pattern is observed), we obtained after passing to the scalar notation

$$E^{(s)} = -\frac{pAE_0k^2 \exp(jkd)}{\varepsilon_0 \quad d} \exp(-j\omega t) \int_{-\infty}^{\infty} \exp[-(\beta x' + x'^2/w^2)] \times$$
$$\times F(x' - Vt) \cos[K(x' - Vt)] \exp[jkx' (\sin\Theta + \sin\varphi)]\, dx' \times$$
$$\times \int_{-\infty}^{\infty} \exp[jkz'(\cos\Theta - \cos\varphi)]\, dz'. \quad (7)$$

Assuming that the condition $(K^2L/k) \gg 1$ is fulfilled (the Bragg region), taking into account the geometry of interaction assumed above and making some mathematical transforms we obtained an expression for the electric field intensity of the diffracted light wave

$$E_d = D \exp(-j\eta) \int_{-\infty}^{\infty} \exp[-\beta x' + x'^2/w^2)]\, F(x' - Vt) \exp(j\gamma x')\, dx' \quad (8)$$

where the following destinations were made

$$D = -\frac{pAE_0Lk^2}{2\varepsilon_0 d} \frac{\sin[k(\cos\Theta - \cos\varphi)L/2]}{k(\cos\Theta - \cos\varphi)L/2};$$
$$\eta = (\omega + \Omega)t - kd;$$
$$\Omega = KV; \qquad \gamma = K - k(\sin\Theta + \sin\varphi).$$

Based on (8) and making the most of the connection between the electric field intensity of the light wave E_d and average light intensity I_d of it

$$I_d = .5(\varepsilon_0/\mu_0)^{1/2} |E_d|^2 \quad (9)$$

after some mathematical transformation one can get

$$I_d = .5(\varepsilon_0/\mu_0)^{1/2} D^2 \left\{ \left[\int_{-\infty}^{\infty} \exp[-(\beta x' + x'^2/w^2)]\, F(x' - Vt) . \cos\gamma x'\, dx' \right]^2 + \right.$$
$$\left. + \left[\int_{-\infty}^{\infty} \exp[-(\beta x' + x'^2/w^2)]\, F(x' - Vt) \sin\gamma x'\, dx' \right]^2 \right\}. \quad (10)$$

The equation (10) constitutes the starting-point for carrying out a numerical analysis of light diffraction on acoustic pulses. It enables to calculate the deflected light wave intensity as a function of the diffraction angle φ and time t. What is important about formula (10) is the fact that no limits have been imposed on the function $F(x - Vt)$, i.e. the acoustic wave can have an arbitrary envelope (modulated continuous wave, train of acoustic pulses or a single acoustic pulse).

III. NUMERICAL ANALYSIS

The following data were considered as a starting-point of the numerical analysis of light diffraction on acoustic pulses in the Bragg region.

1. Light wave:
   $\lambda = 514.5$ nm, $\quad w = 1$ mm, $\quad \Theta = 1°8'29''$.

The angle Θ was chosen equal to the Bragg angle for carrier frequency.

2. Acoustic wave:
   $\Omega/2\pi = 460$ MHz, $\quad L = 4$ mm.

The envelope of the acoustic pulse was shown in fig. 2.

3. Parameters of the medium:
   $V = 5960$ m/s, $\quad \beta = 28.8\ m^{-1}$.

The light wave length was taken equal to the length of light of the argon laser. The parameters of the medium meet the parameters of the fused quartz. The ratio of the deflected light intensity at a given angle φ and time t to the maximum value of that intensity I/I_{max} was calculated numerically. The value of φ was changed every 5″ in the interval of φ, where I/I_{max} was different from zero. The value of t was changed every 0.05 μs. The points obtained in the calculations were joined by straight lines on the diagrams. Calculation results obtained for these data have been presented in fig. 3. The maximum value of I/I_{max} was

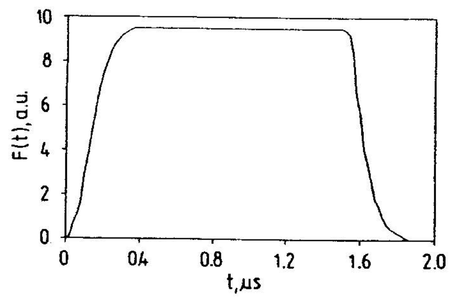


Fig. 2. Envelope of the acoustic pulse.

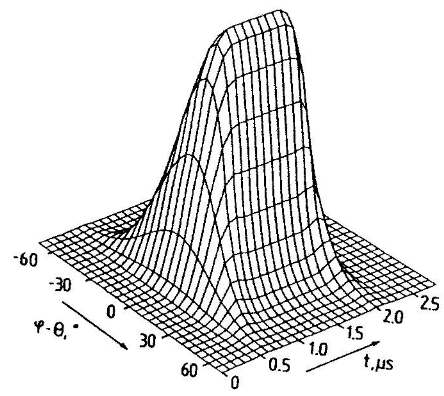
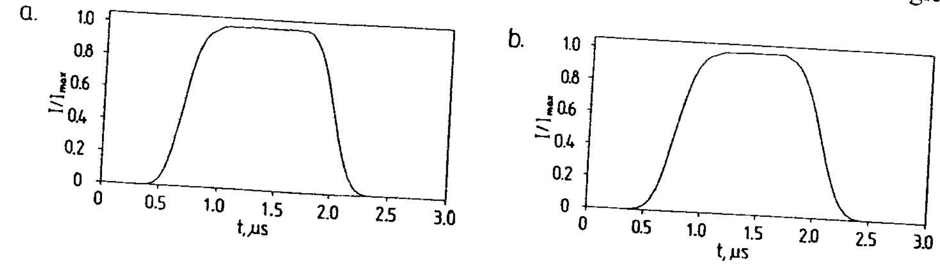


Fig. 3. Diffracted light intensity as a function of diffraction angle and time.

obtained for $\varphi = \Theta$. The diagram shows that during the transition of the acoustic pulse edges through the light beam the light was deflected in a broader interval of the diffraction angles than during the transition of the central part of that pulse through the light beam. The interaction of light with the acoustic pulse during the time when there is a flat part of it in the interaction area only, meets the diffraction of light on the continuous acoustic wave. The broadening is connected with the interaction of the light wave with the harmonic components connected with the pulse edges. It is greater for pulses with steeper edges. The light beam interacts with components of the acoustic pulse of different frequencies and it causes the deflection of light at angles φ different from Θ.

Next, the analysis was continued for different value of β, V, Ω, w and for an acoustic pulse envelope with a shorter plateau. Changes of β with other constant parameters do not change the diffraction pattern. A decrease of the acoustic wave velocity V gives in the result an increase of the increasing and the decreasing times of the light pulse. Moreover, the plateau of its pulse becomes shorter and vanishes with a further decrease of V. At the same time the angle



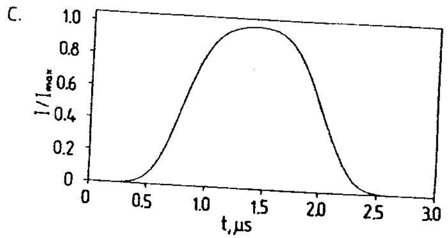


Fig. 4. Diffracted light intensity dependence on t for $\varphi = \Theta$ and $\beta = 0$. a. $V = 5960$ m/s, b. $V = 4500$ m/s, c. $V = 3000$ m/s.

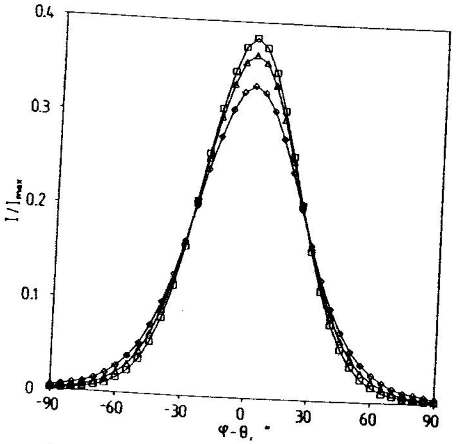
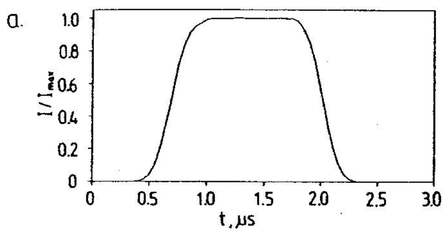


Fig. 5. Diffracted light intensity as a function of φ for the time corresponding to the maximum angle broadening and $\beta = 0$. □ — $V = 5960$ m/s, △ — $V = 4500$ m/s, ◇ — $V = 3000$ m/s.

broadening connected with the pulse edges increases. These effects are visible in fig. 4—5. Changes of $\Omega/2\pi$ from 460 MHz to 160 MHz do not change the diffraction pattern, except the value of the angle φ at which the maximum of I/I_{max} is observed. It is connected with the frequency dependence of Θ. Another parameter changed in that analysis is w. The increase of w causes the decrease of interval of the diffraction angles in which the light beam is deflected. At the same time it is connected with an increase of the increasing and the decreasing times of the light pulse and the shortening of its plateau, as shown in fig. 6—7. Calculations for fig. 6—7 were carried out for $\beta = 0$. The last changing parameter is the length of the plateau of the acoustic pulse envelope. Its shortening causes the shortening of the plateau of the light pulse, and its disappearance in the final result (fig. 8).



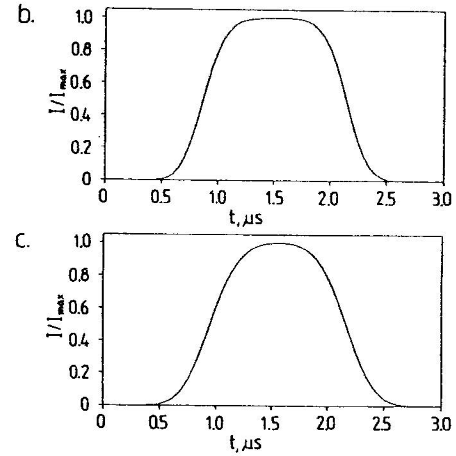


Fig. 6. Diffracted light intensity dependence on t for $\varphi = \Theta$ and $\beta = 0$. a. $w = 1.0$ mm, b. $w = 1.5$ mm, c. $w = 2.0$ mm.

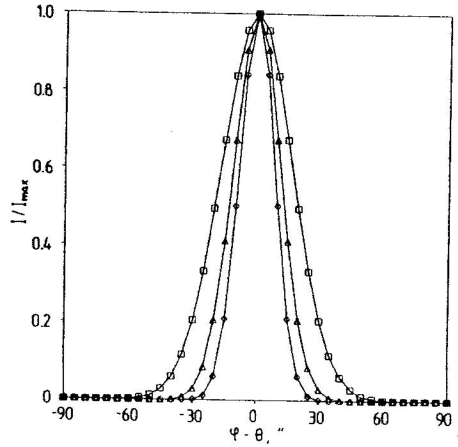


Fig. 7. Diffracted light intensity as a function of φ for the time corresponding to light pulse plateau and $\beta = 0$. □ — $w = 1.0$ mm, △ — $w = 1.5$ mm, ◇ — $w = 2.0$ mm.

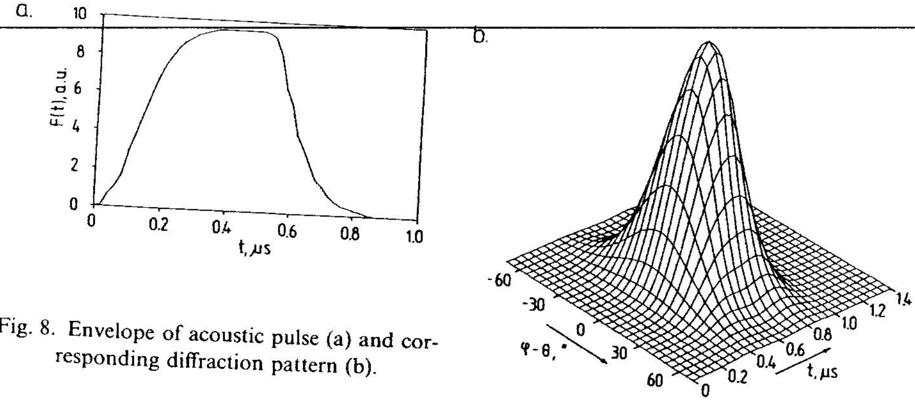


Fig. 8. Envelope of acoustic pulse (a) and corresponding diffraction pattern (b).

IV. CONCLUSIONS

The theoretical description of light diffraction on acoustic pulses presented in chapter II enabled the numerical analysis of such an interaction. Based on the results of that analysis a few conclusions were drawn: The main is that the diffraction pattern depends on three groups of parameters, i.e. parameters of the light wave, the acoustic wave and the medium in which the interaction occurs. Therefore it seems possible to use light diffraction on acoustic pulses in practice. The alteration of parameters of the interaction results in changes of the dependence of the diffracted light intensity on the diffraction angle and time. So, investigations of the diffraction pattern should give a possibility to determine a chosen parameter if the other parameters remain constant. As it can be seen in fig. 4 and fig. 6 the decrease of V and the increase of w cause similar changes in the dependence of I/I_{max} on t at $\varphi = \Theta$. On the other hand, these alternations cause opposite changes in the angle width of the diffraction pattern (fig. 5 and fig. 7). Thus, if one wants to make the most of the light diffraction on an acoustic wave for the determination of the parameters of that interaction, one has to investigate the dependence of the diffracted light intensity on φ and t. The most interesting possibility to utilize that interaction is the determination of the acoustic wave envelope $F(x - Vt)$ on the basis of measurements of I/I_{max} as a function of φ and t, when the others parameters are known. The acquittance of $F(x - Vt)$ at the definite interaction area gives the possibility to analyse the propagation of the acoustic wave in the medium. It could be used to construct an analyser of acoustic pulse parameters, based on the acousto-optic interaction.

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ТЕОРЕТИЧЕСКИЙ И ЧИСЛЕННЫЙ АНАЛИЗ ДИФФРАКЦИИ СВЕТА НА АКУСТИЧЕСКИХ ИМПУЛЬСАХ В БРЕГГОВСКОМ РЕЖИМЕ

В работе представлено короткое теоретическое описание и численный анализ диффракции светового пучка с гауссовым сечением на акустических импульсах в брэгговском режиме. Получена зависимость позволяющая вычислить интенсивность диффрагированного света, как функцию угла диффракции и времени. Проведен численный анализ для различных значений скорости звуковой волны, ее затухания, несущей частоты, коэффициента гауссового распределения для светового пучка и различных огибающих акустического импульса. На основе результатов численного анализа сформулированы выводы, касающиеся возможности практического применения этого взаимодействия.