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THE EFFECT OF THE INHOMOGENEITY ON A STRESS-STATE OF AN ISOTROPIC BODY WITH A CIRCULAR HOLE SUPPORTED BY AN ELASTIC RING

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The stress distribution is investigated on a contact line between a thin planar plate and a ring. This is the planar problem in the elasticity theory of inhomogeneous isotropic bodies with the circular hole supported by the elastic ring under given boundary and contact conditions.

For a given inhomogeneity (11), the complex stress function is constructed (as the solution of the boundary problems (6) and (7)): for the thin planar plate it is given by (12) and for the ring by (13). We analyse numerically three cases: a) the case when the materials of the ring and of the planar plate are similar (see Fig. 1), b) the ring is absolutely stiff (see Fig. 2), and c) the ring is absolutely flexible (see Fig. 3).

I. INTRODUCTION

In the last decades authors pay much attention to the contact problem of the theory of elasticity. This is caused by the actual requirements of modern technology and engineering.

There are many works devoted to the investigation of contact problems. We shall mention the most important monographs by N. I. Muschelishvili [5], G. N. Savin [8], I. Ya. Stayerman [14], L. A. Galin [3], I. V. Vorovich and V. A. Babeshko [2], V. A. Lomakin [4], V. S. Sarkisyan [9(1), 9(2)].

Our work follows [5] and has investigated the stress distribution on a contact line between the thin planar plate and the ring for the inhomogeneous isotropic materials under given boundary and contact conditions. We have calculated the coefficients of concentration and we construct the diagrams of stress distribu-

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tion along the contact line (thin planar plate and ring) in three different cases. The effect of inhomogeneity in all cases represents 30 ÷ 35%.

The use of composite materials (inhomogeneous, anisotropic, manycomponent etc.) allows to investigate on the basis of solved classical problems, important problems of the theory of elasticity of inhomogeneous bodies new in practice.

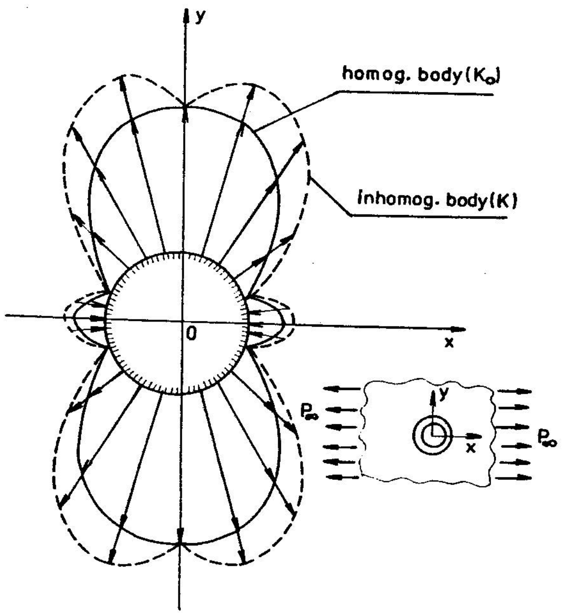

Fig. 1

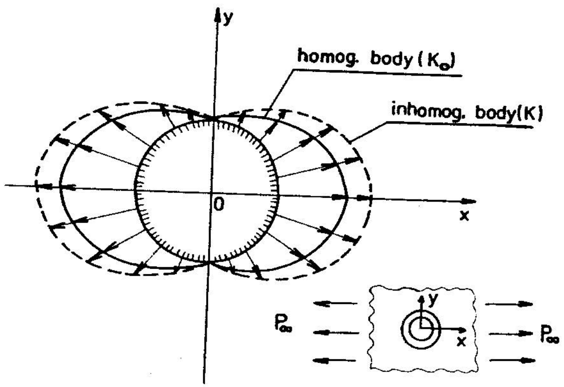

Fig. 2

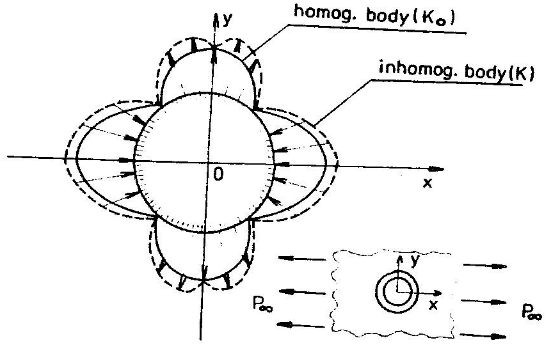

Fig. 3

II.

We investigate the problem of the theory of elasticity of inhomogeneous isotropic planar bodies with a circular hole supported by an elastic ring under given boundary and contact conditions. This problem leads to the third order differential equation with variable coefficients:

$$\frac{\partial^3 G^{(j)}}{\partial z^2 \partial \bar{z}} + 2\mathrm{Re}\left\{A_1^{(j)}(z, \bar{z})\frac{\partial^2 G^{(j)}}{\partial z \partial \bar{z}} + A_2^{(j)}(z, \bar{z})\frac{\partial G^{(j)}}{\partial \bar{z}} + A_3^{(j)}(z, \bar{z})\frac{\partial G^{(j)}}{\partial \bar{z}}\right\} = 0, \quad (1)$$

with the boundary condition on L

$$\sigma_r^{(1)} - \mathrm{i}\tau_{r\Theta}^{(2)} = 0 \quad (2)$$

and the contact condition

$$\begin{aligned} \sigma_r^{(2)} - \mathrm{i}\tau_{r\Theta}^{(2)} &= \sigma_r^{(1)} - \mathrm{i}\tau_{r\Theta}^{(1)} \\ v_r^{(2)} - \mathrm{i}v_\Theta^{(2)} &= v_r^{(1)} - \mathrm{i}v_\Theta^{(1)}. \end{aligned} \quad (3)$$

Here L is the contour of the hole,

$G^{(j)}(z, \bar{z})$ is the complex stress function,

$A_k^{(j)}(z, \bar{z})$ $(k = 1, 2, 3)$ are complex variables depending on the inhomogeneities of the material $E^{(j)}(z, \bar{z})$ and $\nu^{(j)}(z, \bar{z})$ (the upper index $j = 1$ corresponds to the thin planar plate and $j = 2$ corresponds to the ring [8, 10(3)]).

Let us investigate the weakly inhomogeneous material [9, 10(3)]:

$$E^{(j)}(z, \bar{z}) = E_0^{(j)}[1 + \delta f_j(z, \bar{z})]; \quad |\delta f_j(z, \bar{z})| < 1; \quad 0 \leqslant \delta < 1; \quad \nu^{(j)} = \mathrm{const.} \quad (4)$$

The complex stress-function can then be written in the form

$$G^{(j)}(z, \bar{z}) = \sum_{k=0}^{\infty} \delta^k G_k^{(j)}(z, \bar{z}).$$

For the quantities $G_k^{(j)}(z, \bar{z})$ we obtain a recurrent set of boundary problems

$$L[G_0^{(j)}(z, \bar{z})] = 0, \qquad G^{(j)}(\sigma) = H^{(j)}(\sigma), \tag{6}$$

$$L[G_k^{(j)}(z, \bar{z})] = B_j[G_{k-1}^{(j)}(z, \bar{z})] + W_j[G_{k-2}^{(j)}(z, \bar{z})], \qquad G_k^{(j)}(\sigma = 0, \tag{7}$$

where

$$L[\ ] = \frac{\partial^3}{\partial z^2 \partial \bar{z}}, \quad B_j[\ ] = 2\mathrm{Re}\left\{\frac{\partial f_j}{\partial z}\frac{\partial^2}{\partial z \partial \bar{z}} + \frac{1+\nu}{4}\frac{\partial^2 f_j}{\partial z^2}\frac{\partial}{\partial \bar{z}} + \frac{1-\nu}{4}\frac{\partial^2 f_j}{\partial z \partial \bar{z}}\frac{\partial}{\partial z}\right\},$$

$$W_j[\ ] = -2\mathrm{Re}\left\{\frac{1+\nu}{4}\left(\frac{\partial f_j}{\partial z}\right)^2\frac{\partial}{\partial z} + \frac{1-\nu}{4}\frac{\partial f_j}{\partial z}\frac{\partial f_j}{\partial \bar{z}}\frac{\partial}{\partial z}\right\},$$

under the boundary conditions

$$\sigma_{rk}^{(2)} - \mathrm{i}\tau_{r\Theta k}^{(2)} = \sigma_{rk}^{(1)} - \mathrm{i}\tau_{rk\Theta}^{(1)}, \quad v_{rk}^{(2)} - \mathrm{i}v_{\Theta k}^{(2)} = v_{rk}^{(1)} - \mathrm{i}v_{\Theta k}^{(1)}, \quad (k \geqslant 0). \tag{8}$$

The stress vector $\sigma_{rk}^{(j)} - \mathrm{i}\tau_{r\Theta k}^{(j)}$ and the vector of internal shifts $v_{rk}^{(j)} - \mathrm{i}v_{\Theta k}^{(j)}$ are expressed in the form

$$\sigma_{rk}^{(j)} - \mathrm{i}\tau_{r\Theta k}^{(j)} = \frac{\partial G_k^{(j)}}{\partial z} - \frac{\partial \bar{G}^{(j)}}{\partial z}\mathrm{e}^{2\mathrm{i}\Theta} = \varphi_k'^{(j)}(z) + \overline{\varphi_k'^{(j)}(z)} -$$

$$- [\bar{z}\varphi_k''^{(j)}(z) + \psi_k'^{(j)}(z)]\mathrm{e}^{2\mathrm{i}\Theta} + \frac{\partial IG_k^{(j)}}{\partial z} - \frac{\partial \overline{IG_k^{(j)}}}{\partial z}\mathrm{e}^{2\mathrm{i}\Theta},$$

$$v_{rk}^{(j)} - \mathrm{i}v_{\Theta k}^{(j)} = (u_k^{(j)} - \mathrm{i}v_k^{(j)})\mathrm{e}^{\mathrm{i}\Theta}, \tag{9}$$

$$u_k^{(j)} - \mathrm{i}v_k^{(j)} = \frac{1}{2\mu_0}[\varkappa_k^{(j)}\overline{\varphi_k^{(j)}(z)} - \bar{z}\varphi_k'^{(j)}(z) - \psi_k^{(j)}(z) + \varkappa_{k1}^{(j)}\overline{IG^{(j)}}] -$$

$$- \int \mathrm{Re}\left\{f_j(z, \bar{z})\left[\frac{\partial}{\partial \bar{z}}(u_{k-1}^{(j)} - \mathrm{i}v_{k-1}^{(j)}) + \frac{\partial}{\partial z}(u_{k-1}^{(j)} + \mathrm{i}v_{k-1}^{(j)})\right]\mathrm{d}\bar{z}\right\}, \qquad k \geqslant 1$$

$$\varkappa_k = \frac{\lambda_0 + 3\mu_0}{\lambda_0 + \mu_0}, \quad \varkappa_{k1} = \frac{\mu_0}{\lambda_0 + \mu_0},$$

For $k = 0$ they are identical with the corresponding expressions which are valid for homogeneous isotropis bodies [8].

III.

Let us take an inhomogeneous thin planar plate with a circular hole which is supported by an elastic ring under a one-axis extension in the $0x$ direction. Then

$$H^{(j)}(\sigma) = R\sigma P. \tag{10}$$

We assume the case when the inhomogeneity function $f_j(z, \bar{z})$ has the following form [10(3)]

$$f_j(z, \bar{z}) = \frac{z + \bar{z}}{(z \,.\, \bar{z})^{m+1}} \cdot R_j^{2m+1}$$

$$f_j(r, \Theta) = 2\left(\frac{R_j}{r}\right)^{2m+1} \cdot \cos\Theta \tag{11}$$

Solving the boundary problems (6) and (7) with respect to (10) and (11) we obtain the following complex stress-functions:

a) for the thin planar plate

$$G^{(1)}(z, \bar{z}) = G_0^{(1)}(z, \bar{z}) + \delta G_1^{(1)}(z, \bar{z}), \tag{12}$$

b) for the ring

$$G^{(2)}(z, \bar{z}) = G_0^{(2)}(z, \bar{z}) + \delta G_1^{(2)}(z, \bar{z}). \tag{13}$$

The unknown coefficients entering into this expression are determined from the following systems of equations:

$$\begin{aligned}
&2a_1R_1/R + \bar{b}_{-1}R/R_1 = 0\\
&2a_1 + \bar{b}_{-1} = 2A_1 + \bar{\beta}_{-1}\\
&\mu/\mu_1 \,.\, [a_1(\varkappa_1 - 1) - \bar{b}_{-1}] = A_1(\varkappa - 1) - \bar{\beta}_{-1}\\
&a_3R_1^3/R^3 + a_{-1}R/R_1 + b_{-3}R^3/R_1^3 = 0\\
&3a_3R_1^3/R^3 - \bar{a}_{-1}R/R_1 - b_1R_1/R = 0\\
&a_3 + \bar{a}_{-1} + \bar{b}_{-3} = A_3 + \bar{\alpha}_{-1} + \bar{\beta}_{-3}\\
&3a_3 - \bar{a}_{-1} - b_1 = 3A_3 - \bar{\alpha}_{-1} - \beta_1\\
&\mu/\mu_1[\varkappa_1a_3 - \bar{a}_{-1} - b_{-1-3}] = \varkappa A_3 - \bar{\alpha}_{-1} - \bar{\beta}_{-3}\\
&\mu/\mu_1[3a_3 + \varkappa_1\bar{a}_{-1} - b_1] = 3A_3 + \varkappa\bar{\alpha}_{-1} - \beta_1\\
&c_3R_1^3/R^3 + \bar{c}_{-1}R/R_1 + \bar{d}_{-3}R^3/R_1^3 = 0
\end{aligned} \tag{14a}$$

$$
\begin{aligned}
&2c_1R_1/R + \bar{d}_{-1}R/R_1 = 0\\
&3c_3R_1^3/R^3 - \bar{c}_{-1}R/R_1 - d_1R_1/R = 0\\
&c_2 + \bar{d}_{-2} = 2B_{20}/PR\\
&\varkappa_1^{(2)}\bar{c}_{-1}R/R_1 + 3c_3R_1^3/R - d_1R_1/R = 0\\
&\mu_0^{(0)}/\mu_0^{(2)}[-d_0 + 2c_2] = 4(B_{21} + A_{21})/PR\\
&c_1R_1/R(\varkappa_1^{(2)} - 1) + \bar{d}_{-1}R/R_1 = 0\\
&\mu_0^{(1)}/\mu_0^{(2)}[\varkappa_1^{(2)}c_2 - \bar{d}_{-2}] = 4(B_{22} + A_{21})/PR\\
&\varkappa_1^{(2)}c_3R_1^3/R^3 + \bar{c}_{-1}R/R_1 + \bar{d}_{-3}R^3/R_1^3 = 0.
\end{aligned}
\tag{14b}
$$

Solving this system of equations we obtain

$$
\begin{aligned}
&c_{-1} = c_1 = c_3 = d_{-3} = d_{-1} = d_1 = 0\\
&d_0 = \left[\frac{2(B_{22} - A_{22})\mu_0^2}{\mu_0^{(2)}(1 + \varkappa_1^{(2)})} - \frac{(B_{21} - A_{21})\mu_0^{(2)}}{\mu_0^{(1)}} + \frac{B_{20}}{(1 + \varkappa_1^{(2)})}\cdot\frac{1}{PR},\right.\\
&d_{-2} = \frac{1}{(1 + \varkappa_1^{(2)})n^2PR}\left[\frac{\varkappa_1^{(2)}B_{20}}{2} - B_{22} + A_{22}\right],\\
&c_2 = \frac{n^2}{(1 - \varkappa_1^{(2)})PR}\left[\frac{B_{20}}{2} + \frac{(B_{22} - A_{22})\mu_0^{(2)}}{\mu_0^{(1)}}\right],
\end{aligned}
\tag{15}
$$

where $n = R/R_1$, and A_{21}, A_{22}, B_{20}, B_{21}, B_{22} are complicated expressions depending on elasticity properties.

The components of the stress and the shifts are expressed as follows [10(3)]:

a) for the thin planar plate

$$
\sigma_r^{(1)} = \frac{P}{2}\left[\left(1 - \frac{1}{2}\beta_{-1}\frac{R^2}{r^2}\right) + \left(1 - 2\alpha_{-1}\frac{R^2}{r^2} - \frac{3}{2}\beta_{-3}\frac{R^4}{r^4}\right)\cos 2\Theta\right] +
$$
$$
+ \delta(M_1\cos 3\Theta + M_2\cos\Theta),
$$

$$
\sigma_\Theta^{(1)} = \frac{P}{2}\left[\left(1 + \frac{\beta_{-1}}{2}\frac{R^2}{r^2}\right) - \left(1 - \frac{3}{2}\beta_{-3}\frac{R^4}{r^4}\right)\cos 2\Theta\right] + \delta(M_3\cos 3\Theta + M_4\cos\Theta),
$$

$$
\tau_{r\Theta}^{(1)} = \frac{P}{2}\left(1 + \alpha_{-1}\frac{R^2}{r^2} + \frac{3}{2}\beta_{-3}\frac{R^4}{r^4}\right)\sin 2\Theta + \delta(M_5\sin 3\Theta + M_6\sin\Theta),
$$

$$
v_r^{(1)} = \frac{PR}{8\mu_0^{(1)}}\left\{\left[\varkappa_0^{(1)} - 1)\frac{r}{R} + \beta_{-1}\frac{R}{r}\right] + \left[2\frac{r}{R} + \alpha_{-1}(\varkappa_0^{(1)} + 1)\frac{r}{R} + \right.\right.
$$
$$
\left.\left. + \beta_{-3}\frac{R^3}{r^3}\right]\cos 2\Theta\right\} + \frac{\delta}{2\mu_0^{(1)}}(M_7\cos\Theta + M_8\cos 3\Theta + M_9\cos 5\Theta + M_{10}\cos 7\Theta),
$$

$$
v_\Theta^{(1)} = \frac{PR}{8\mu_0^{(1)}}\left[-\frac{2r}{R} - \alpha_{-1}(\varkappa_0^{(1)} - 1)\frac{R}{r} + \beta_{-3}\frac{R^3}{r^3}\right]\sin 2\Theta +
$$
$$
+ \frac{\delta}{2\mu_0^{(1)}}(M_{11}\sin\Theta + M_{12}\sin 3\Theta + M_{13}\sin 5\Theta + M_{14}\sin 7\Theta),
$$

b) for the ring

$$
\sigma_\Theta^{(2)} = \frac{P}{2}\left[\left(a_1 + \frac{b_{-1}}{2}\frac{R^2}{r^2}\right) - \left(\frac{b_1}{2} - 6a_3\frac{r^2}{R^2} - \frac{3}{2}b_{-3}\frac{R^4}{r^4}\right)\cos 2\Theta\right] +
$$
$$
+ \delta(N_1\cos 3\Theta + N_2\cos\Theta),
$$

$$
\sigma_r^{(2)} = \frac{P}{2}\left[\left(a_1 - \frac{b_{-1}}{2}\frac{R^2}{r^2}\right) + \left(\frac{b_1}{2} - 2a_{-1}\frac{R^2}{r^2} - \frac{3}{2}b_{-3}\frac{R^4}{r^4}\right)\cos 2\Theta\right] +
$$
$$
+ \delta(N_3\cos 3\Theta + N_4\cos\Theta),
$$

$$
\tau_{r\Theta}^{(2)} = \frac{P}{2}\left[3a_3\frac{r^2}{R^2} - \frac{b_1}{2} - \frac{a_{-1}R^2}{r^2} - \frac{3}{2}b_{-3}\frac{R^4}{r^4}\right]\sin 2\Theta + \delta(N_5\sin 3\Theta + N_6\sin\Theta),
$$

$$
v_r^{(2)} = \frac{PR}{8\mu_0^{(2)}}\left\{\left[a_1(\varkappa_1^{(2)} - 1)\frac{r}{R} + b_{-1}\frac{R}{r}\right] + \left[a_3(\varkappa_1^{(2)} - 3)\frac{r^3}{R^3} + b_1\frac{r}{R} + \right.\right.
$$
$$
\left.\left. + a_{-1}(\varkappa_1^{(2)} + 1)\frac{R}{r} + b_{-3}\frac{R^3}{r^3}\right]\cos 2\Theta\right\} + \frac{\delta PR}{8\mu_0^{(2)}}(N_7\cos\Theta +
$$
$$
+ N_8\cos 3\Theta + N_9\cos 5\Theta + N_{10}\cos 7\Theta),
$$

$$
v_\Theta^{(2)} = \frac{PR}{8\mu_0^{(2)}}\left[a_3(\varkappa_1^{(2)} + 3)\frac{r^3}{R^3} - b_1\frac{r}{R} - a_{-1}(\varkappa_1^{(2)} - 1)\frac{R}{r} + b_{-3}\frac{R^3}{r^3}\right]\sin 2\Theta +
$$
$$
+ \frac{\delta PR}{8\mu_0^{(2)}}(N_{11}\sin\Theta + N_{12}\sin 3\Theta + N_{13}\sin 5\Theta + N_{14}\sin 7\Theta),
$$

where M_i and N_i ($i = 1, 2, \ldots, 14$) are complicated expressions depending on constants of elasticity.

IV. EXAMPLES

a) The material of the ring and of the thin planar plate is the same. The concentration coefficient is determined from the relation

$$K = \frac{\sigma_\Theta}{P}$$

under the following elasticity constants

$$a_{-1} = 2/n^2, \ a_1 = 1, \ a_3 = 0, \ b_3 = -2/n^4$$

$$b_{-1} = 2/n^2, \ b_1 = 2, \ \alpha_{-1} = 2/n^2, \ \beta_{-1} = 2/n^2, \ \beta_{-3} = -2/n^4.$$

Putting

$$\mu^{(1)} = \mu^{(2)} = 8.1 \times 10^5 \ \mathrm{kg\,cm}^2, \ \varkappa_1^{(1)} = \varkappa_1^{(2)} = 2.08; \ \nu = 0.3; \ n_2 = r/R_1, \ n_1 = n/n_2$$

we obtain the following Table 1 ($n = 1, 2; n_2 = 1, 2$):

b) The absolutely stiff ring.

Taking for the elasticity constants the following values

$$\alpha_{-1} = -2/\varkappa, \quad \beta_{-1} = 1 - \varkappa, \quad \beta_{-3} = 2/\varkappa$$

we obtain the following Table 2 ($n = 1.\ 2; n_2 = 1.\ 2$):

c) The absolutely flexible ring.

Taking the elasticity constants as

$$a_3 = a_1 = a_{-1} = b_1 = b_{-1} = b_{-3} = 0, \quad \beta_{-1} = 2, \quad \beta_{-3} = -2, \quad \alpha_{-1} = 2$$

we obtain the Table 3 ($n = 1.\ 2; n_2 = 1.\ 2$):

We note that the numerical calculations were performed on the computer EC-1045.

1. In the case of a one-axis extension in the direction $0x$ the maximal stress for inhomogeneous bodies is in the points $\Theta = \pm\frac{5\pi}{12} + \pi n$ when the contour of the thin planar plate is strained by the absolutely stiff ring. This should be compared with the values $\Theta = \pm\frac{\pi}{2} + \pi n$ valid for homogeneous bodies.

2. In the cases a) and c) the zone of extension ($\sigma_\Theta > 0$) is concentrated in the $0y$ direction and the zone of suppression ($\sigma_\Theta < 0$) is in the $0x$ direction, both for homogeneous and inhomogeneous bodies. In the case b) the zone of extension ($\sigma_\Theta > 0$) is concentrated in the $0x$ direction.

3. The size of these zones of stress-distribution depends on the value of the small physical parameter.

4. When the stress inhomogeneity is present, then the stress on a contact line between the thin planar plate and the ring increases by $30 \div 35\%$.

Table 1

Θ	0°	15°	30°	45°	60°	75°	90°
$K_0(\delta = 0)$	−0.376	−0.212	0.236	0.847	1.459	1.907	2.071
$K(\delta = 0.2)$	−0.486	−0.275	0.306	1.101	1.899	2.477	2.071

Table 2

Θ	0°	15°	30°	45°	60°	75°	90°
$K_0(\delta = 0)$	0.4512	0.4215	0.3406	0.2300	0.1194	0.0385	0.0090
$K(\delta = 0.2)$	0.5807	0.5433	0.4400	0.2981	0.1550	0.0501	0.0090

Table 3

Θ	0°	15°	30°	45°	60°	75°	90°
$K_0(\delta = 0$	−0.3008	−0.2605	−0.1504	−0.1415	0.1504	0.2605	0.3008
$K(\delta = 0.2)$	−0.3871	−0.3358	−0.1943	−0.1832	0.1943	0.3358	0.3008

V. CONCLUSION

Our results have indicated that the use of composite materials may be of great advantage in modern engineering and technology.

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ВЛИЯНИЕ НЕОДНОРОДНОСТИ НА НАПРЯЖЕННОЕ СОСТОЯНИЕ ИЗОТРОПНЫХ ТЕЛ С КРУГОВЫМ ОТВЕРСТИЕМ, ПОДКРЕПЛЕННЫХ УПРУГИМ КОЛЬЦОМ

В работе исследуется распределение напряжений на контактной линии пластина — кольцо для неоднородных изотропных сред при определенных граничных условиях и условиях сопряжения.

Рассматривается плоская задача теории упругости неоднородных изотропных тел с круговым отверстием, подкрепленных упругим кольцом при определенных граничных условиях и условиях сопряжения. Данная задача решается при одноосном растяжении по направлению оси $0x$. Для данной неоднородности (11) получается решение задачи, т.е. решение краевой задачи (6), (7) — комплексная функция напряжений (12) для пластинки и (13) для кольца. Численно решены три случая, когда: а) материал кольца и пластинки одинаковы (Рис. 1), б) абсолютно жесткое кольцо (Рис. 2); в) абсолютно гибкое кольцо (Рис. 3).