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GENERALIZED k-PHOTON COHERENT STATES NTERACTING WITH AN ANHARMONIC OSCILLATOR

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We study the interaction of the generalized k-photon coherent states [2] with the nonlinear medium modelled as an anharmonic oscillator. We show that the anharmonic medium tends to revoke the squeezing of an initially squeezed k-photon coherent state at the first moments of the evolution. Nevertheless due to the periodicity of the system under consideration, the initial squeezing is completely restored later.

I. INTRODUCTION

Problems connected with the so-called squeezed states of light have been xtensively studied in quantum optics in the last decade (for review on squeezed tates see [1]). These studies were stimulated by the need for high presicion neasurements (for instance, detection of gravitational waves) and applications n communication technology.

There are two main topics in studying the squeezed states of light. The first s the problem of the generation of squeezed states of the radiation field. The second is the interaction of squeezed light with a material medium. The present paper is devoted to the second class of these problems. In particular, we will study the interaction of the generalized k-photon coherent states [2] with the nonlinear medium modelled as an anharmonic oscillator. We will show that at least for $k = 2$ the Nth order squeezing exhibits exact periodicity.

II. THE MODEL

Tanas [3] has recently considered the interaction of ordinary (Glauber's) coherent light with a nonlinear medium modelled as a nonabsorbing anharmonic oscillator with the Hamiltonian

$$\hat{H} = \omega\hat{a}^+\hat{a} + \frac{\lambda}{2}\hat{a}^{+2}\hat{a}^2. \tag{2.1}$$

He found that the light can become squeezed for large numbers of photons.

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This model has been generalized by Gerry [4] to the case of the k-photon anharmonic oscillator model with the Hamiltonian

$$\hat{H} = \omega \hat{a}^{+} \hat{a} + \frac{\lambda}{k} \hat{a}^{+k} \hat{a}^{k}. \tag{2.2}$$

Gerry has shown that this model Hamiltonian interacting with ordinary coherent light gives rise to squeezing and that the squeezing can be enhanced for a successively higher k (at least for some values of the average photon numbers).

Gerry has also analysed [5] the system described by the Hamiltonian (2.1) when at the initial moment the radiation field has been supposed to be in a squeezed SU(1,1) coherent state. He has found that the more photons are in the initial state (which also means a greater initial squeezing for SU(1,1) CS), the more rapidly the squeezing is revoked.

In the present paper we will study the system described by the Hamiltonian (2.2). As we mentioned before we are interested in the interaction of such a system with the generalized k-photon CS.

One of the possible generalizations of the ordinary CS of the harmonic oscillator has been proposed by D'Ariano et al. [6—8]. This generalization is based on the k-photon operators $\hat{A}_k$, $\hat{A}_k^{+}$ introduced by Brandt and Greenberg [9]:

$$\hat{A}_k = \hat{a}^k f(\hat{n}), \quad \hat{A}_k^{+} = f(\hat{n})\, \hat{a}^{+k}, \tag{2.3a}$$

where

$$f(\hat{n}) = \left\{ \left[\frac{\hat{n}}{k} \right] \frac{(\hat{n}-k)!}{\hat{n}!} \right\}^{1/2}, \tag{2.3b}$$

with the commutation relation

$$[\hat{A}_k, \hat{A}_k^{+}] = 1. \tag{2.4}$$

In (2.3) the function $[x]$ is defined as the greatest integer less than or equal to x. Using these operators these authors have introduced in particular the Weyl-Heisenberg group CS defined as:

$$|\xi, k\rangle = \exp(-|\xi|^2/2) \exp(\xi \hat{A}_k^{+})|0\rangle, \tag{2.5}$$

where $|0\rangle$ is the vacuum state of the harmonic oscillator.

An alternative definition of the k-photon CS has been given by the present authors [2] — the k-photon CS is defined as an eigenvector of the operator a^k:

$$a^k |\alpha_k\rangle = \alpha_k |\alpha_k\rangle. \tag{2.6}$$

One of the possible realizations of $|\alpha_k\rangle$ is

$$|\alpha_k\rangle = N \sum_{n=0}^{\infty} \frac{\alpha_k^n}{\sqrt{(nk)!}} |nk\rangle = N \exp(\alpha_k \hat{B}_k^{+})\, |0\rangle, \tag{2.7}$$

where N is the normalization constant

$$N = h_1^{-1/2}(|\alpha|^{2/k}, k), \tag{2.8}$$

($h_1(x, k)$ is the hypergeometric function [10]). The multiphoton operator $\hat{B}_k^{+}$ is defined as:

$$\hat{B}_k^{+} = \left[\frac{\hat{n}}{k} \right] \frac{(\hat{n}-k)!}{\hat{n}!} (\hat{a}^{+})^k. \tag{2.9}$$

In what follows we will study the interaction of the nonlinear media effectively described by the Hamiltonian (2.2) with the generalized k-photon CS (2.7).

So, if we suppose the initial stae vector $|\Phi(t=0)\rangle$ to be equal to the state (2.7), then the solution of the Schrödinger equation ($\hbar = 1$):

$$\mathrm{i} \frac{\mathrm{d}}{\mathrm{d}t} |\Phi(t)\rangle = \hat{H} |\Phi(t)\rangle \tag{2.10}$$

for the state vector $|\Phi(t)\rangle$ can be found immediately:

$$\langle \Phi(t)\rangle = N \sum_{n=0}^{\infty} \frac{\alpha_k^n}{\sqrt{(nk)!}} \exp\left(-\mathrm{i} \left(\omega n k + \frac{\lambda}{k} \frac{(nk)!}{(nk-k)!} \right) t \right) \left| nk \right\rangle. \tag{2.11}$$

Now we are ready to the study of the time behaviour of the squeezing properties of the radiation field.

III. LIGHT SQUEEZING

To analyse light squeezing we introduce two hermitian time-dependent quadrature operators:

$$\hat{a}_1 = \frac{\hat{a} \exp(\mathrm{i}\omega t) + \hat{a}^{+} \exp(-\mathrm{i}\omega t)}{2}, \qquad \hat{a}_2 = \frac{\hat{a} \exp(\mathrm{i}\omega t) - \hat{a}^{+} \exp(-\mathrm{i}\omega t)}{2\mathrm{i}} \tag{3.1}$$

with the commutation relation

$$[\hat{a}_1, \hat{a}_2] = 2\mathrm{i}C \quad \text{with} \quad C = 1/4. \tag{3.2}$$

Following the idea of Hong and Mandel [11—12] we define two functions $q_i^{(N)}$ for $i = 1, 2$

$$q_i^{(N)} = \frac{\langle(\Delta\hat{a}_i)^N\rangle - (N-1)!!C^{N/2}}{(N-1)!!C^{N/2}}, \tag{3.3}$$

which measure the degree of the Nth order squeezing (for an even N) in the first (second) quadrature. The Nth order squeezing condition looks very simple:

$$q_i^{(N)} < 0 \tag{3.4}$$

and the maximum (100%) squeezing is obtained for $q_i^{(N)} = -1$.

Further, we will analyse in detail the two-photon case ($k = 2$), where the state vector $|\Phi(t)\rangle$ is given (2.11)

$$|\Phi(t)\rangle = \frac{1}{(\cosh|\alpha_2|)^{1/2}} \sum_{n=0}^{\infty} \frac{\alpha_2^n}{\sqrt{(2n)!}} \exp\left(-\mathrm{i}\left(2\omega n + \frac{\lambda}{2} 2n(2n-1)\right) t\right)|2n\rangle \tag{3.5}$$

(we will write α instead of α_2). From (3.5) it follows that

$$\langle\hat{a}_i\rangle = 0 \tag{3.6}$$

and so from (3.3) we obtain for the function $q_1^{(2)}$:

$$q_{1,2}^{(2)} = 2\langle\hat{a}^+\hat{a}\rangle \pm 2\mathrm{Re}\langle\hat{a}^2\rangle. \tag{3.7}$$

For the mean values $\langle\hat{a}^+\hat{a}\rangle$ and $\langle\hat{a}^2\rangle$ we find the expressions

$$\langle\hat{a}^+\hat{a}\rangle = |\alpha| \tanh|\alpha| = \bar{n}; \tag{3.8a}$$

(the average photon number $\bar{n}$ is an integral of motion)

$$\langle\hat{a}^2\rangle = \frac{\alpha\exp(-\mathrm{i}\tau)}{\cosh|\alpha|}\cosh(|\alpha|\exp(-2\mathrm{i}\tau)), \qquad \tau = \lambda t. \tag{3.8b}$$

Choosing the phase φ of the complex parameter α to be equal to π we finally derive for the function $q_1^{(2)}$ the following expression

$$q_1^{(2)}(\tau) = 2|\alpha|\,(\tanh|\alpha| - \frac{1}{2\cosh|\alpha|}(\exp(|\alpha|\cos 2\tau)\cos(\tau + |\alpha|\sin 2\tau) +$$

$$+ \exp(-|\alpha|\cos 2\tau)\cos(\tau - |\alpha|\sin 2\tau)). \tag{3.9}$$

The function $q_1^{(2)}(\tau)$ is periodical, with the period $T = 2\pi/\lambda$. Moreover one can easily find that the functions $q_1^{(2)}(\tau)$ and $q_2^{(2)}(\tau)$ are mutually shifted by the half period:

$$q_2^{(2)}(\tau) = q_1^{(2)}(\tau + \pi). \tag{3.10}$$

From (3.9) it follows that the maximum squeezing ($\sim 56\%$) in the first quadrature can be obtained at $\tau = 0$ for $|\alpha| \sim 0.68$ (or, which is the same, for $\bar{n} \cong 0.40$). The last is in contrast with the standard 2-photon coherent state (squeezed state), when degree of squeezing increases with the intensity of the radiation field ($q_1^{(2)} \to -1$ with $\bar{n} \to \infty$).

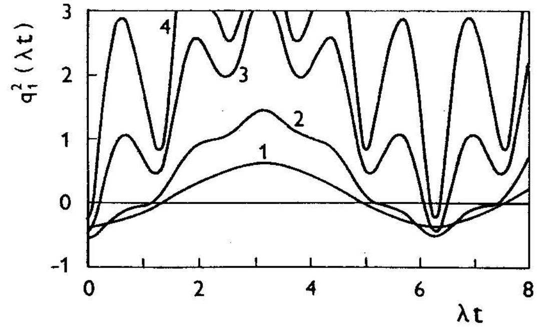

Fig. 1 — Time evolution of $q_1^{(2)}(t)$ plotted for various values of $|\alpha|$. Line 1 is for $|\alpha| = 0.25$, line 2 for $|\alpha| = 0.5$, line 3 for $|\alpha| = 1.0$ and line 4 for $|\alpha| = 1.5$.

During the first moments of the evolution the initialsqueezing in the first quadrature became revoked, but then at $t = T$ it is completely restored. This periodical restoration of the initial squeezing is the most characteristic feature of the function $q_1^{(2)}(\tau)$ in the present model. In fig. 1 the time evolution of the function $q_1^{(2)}(\tau)$ is plotted for various values of $|\alpha|$.

IV. DISCUSSION AND CONCLUSIONS

In the present paper we have analysed in detail generalized two-photon coherent states interacting with nonlinear medium via two photon processes. We have shown that the second order squeezing exhibits an exact periodicity. It can be also shown that the function $q_1^{(4)}(\tau)$ describing the fourth order squeezing in the first quadrature (for details see [2]) as well as the functions $q_i^{(N)}(\tau)$ describing the Nth order squeezing are periodical $q_i^{(N)}(\tau) = q_i^{(N)}(\tau + T)$. So we can conclude that in spite of the fact that the squeezing is revoked in the first moments of the evolution, the initial squeezing properties become restored at $t = T$.

The k-photon CS interacting with nonlinear medium via k-photon processes will be studied elsewhere.

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ОБОБЩЕННЫЕ κ-ФОТОННЫЕ КОГЕРЕНТНЫЕ СОСТОЯНИЯ, ВЗАИМОДЕЙСТВУЮЩИЕ С АНГАРМОНИЧЕСКИМ ОСЦИЛЛЯТОРОМ

В работе изучено взаимодействие обобщенных κ-фотонных когерентных состояний [2] с нелинейной средой, которая представлена в виде ангармонического осциллятора. Показано, что ангармоническая среда на первом этапе эволюции уничтожает свойства сжатия первоначального κ-фотонного когерентного состояния. Несмотря на это, благодаря периодичности изучаемой системы, первоначальное сжатие полностью восстанавливается в последующие моменты времени.