

## NEW METHOD OF ANALYSIS OF SEMICONDUCTORS PLASMA PARAMETERS

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The paper presents the simulation and the analysis of microwave experiments used for the excitation of helicon waves in semiconductor plasma. An equivalent of the experimental setup and the results of the computer simulation based on this equivalent circuit are presented. From the computer simulation a new experimental procedure is proposed. This procedure allows to abstract the desired plasma parameters from the measured change of the signal as a function of the external magnetic field even in the case when the range of the external magnetic field is not large enough to get two or more interference patterns of the measured signal which were necessary for the analysis in the previous works.

### 1. INTRODUCTION

Helicon waves are well-known circularly polarized transversal electromagnetic waves propagating in one-component magnetoactive or semiconductor plasma [1—4]. The orientation of the circular polarization defines an ordinary or extraordinary helicon. Due to its characteristic parameters [1, 2] the ordinary helicon is strongly damped and because of that practically does not propagate in the plasma. On the other hand the damping of the extraordinary helicon is weak, which allows the experimental investigation of its properties. The orientation of the circular polarization of the extraordinary helicon is the same as the orientation of cyclotron propagation of the charge carriers. The dispersion equation for the extraordinary helicon (further only helicon) is [1, 2]

$$k^+(\omega) = \frac{k_0 \omega_p}{\sqrt{\omega \omega_c}} \left( 1 - i \frac{\nu}{2\omega_c} \right), \quad (1)$$

where  $k_0 = \omega/c$  is the wavenumber in the vacuum,  $\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}}$  is the plasma frequency,  $\omega_c = eB/m$  is the cyclotron frequency,  $\nu$  is the collision frequency of

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the charge carriers,  $n$  is the concentration of the charge carriers,  $e$  the charge,  $\epsilon_0$  the permittivity of the vacuum,  $B$  the induction of the external magnetic field and  $m$  is the mass of the carriers. From (1) it follows that the necessary condition of the weak damping is a small value of the collision to the cyclotron frequency ratio, which can easily be obtained in high mobility materials at low temperatures (low  $\nu$ ) and high external magnetic fields (large  $\omega_c$ ).

## II. EXPERIMENT AND ITS SIMULATION

One of the standard methods of the investigation of properties of semiconductor materials by means of excited helicon waves is the measurement of the transmitted microwave signal through the semiconductor sample placed in an external magnetic field. Helicon waves are slow (relatively to the speed of light) waves with the velocity depending on the amplitude of the external magnetic field. Thus the change of the magnetic field significantly changes not only the amplitude, but also the phase of the microwave signal passing through the semiconductor sample. The evaluation of such measurement gives information on the mobility and concentration of the major charge carriers in the sample. To enhance the sensitivity usually setup is shown in Fig. 1. It is similar to that described in [5], but without the possibility to use a superconducting magnet. The experimental results obtained from the analysis of the signal at the detector D are comparable with results obtained in [6].

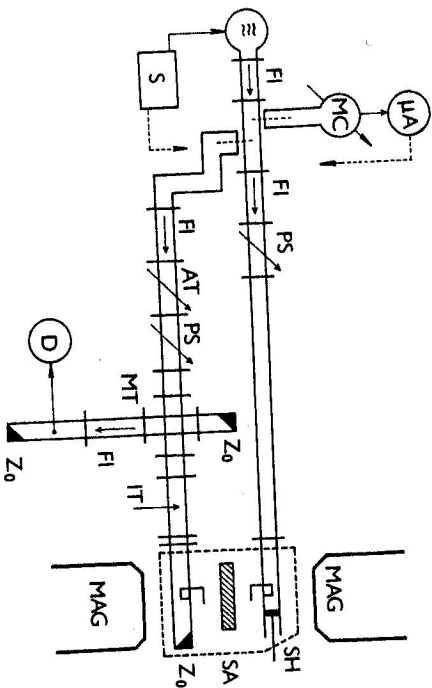


Fig. 1: Microwave bridge. S — source, FI — ferrite isolator, PS — phase shifter, AT — attenuator, MT — magic T,  $Z_0$  — characteristic impedance, D — detector, IT — impedance transformer, SH — moving short, SA — sample, MAG — magnet, MC — microwave cavity

The aim of the present paper is, starting from the equivalent circuit of the microwave bridge in Fig. 1, to analyse it and using computer simulation to infer from the voltage dependence on the detector D the parameters of the investigated semiconductor plasma.

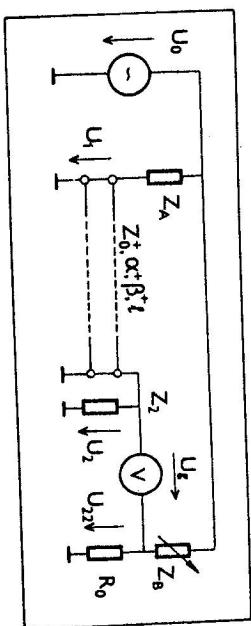


Fig. 2: Equivalent circuit of the microwave bridge

The equivalent circuit of the microwave bridge in Fig. 1 is in Fig. 2. The equivalent network contains two arms. In one of them are the impedances  $Z_B$  and  $R_0$  and in the other the impedance  $Z_A$  and a transmission line simulating the investigated sample. The external magnetic field is included in the parameters of the transmission line. Expressing the collision frequency in (1) as

$$\mu B = \frac{\omega_c}{\nu}, \quad (2)$$

where  $\mu$  is the mobility and taking the signal frequency  $f = 9200$  MHz we obtain for the phase constant

$$\alpha^+ = 1.08 \times 10^{-7} \cdot \sqrt{\frac{n}{B}} \quad (3)$$

the damping constant

$$\beta^+ = \frac{0.54 \times 10^{-7} n^{1/2}}{\mu \cdot B^{3/2}} \quad (4)$$

and for the characteristic impedance

$$Z_0^+ = 6.73 \times 10^{11} \sqrt{\frac{B}{n}}, \quad (5)$$

where the imaginary part has been neglected. The measured voltage in Fig. 2 represents the absolute value of the difference of two complex quantities, the one corresponding to the output voltage of the transmission line and the other to the

voltage on the impedance  $R_0$ . Varying the impedance  $Z_B$  at a fixed value of the external magnetic field  $B$  it is possible to balance the microwave bridge, that means to zero the voltage  $U_g$ , which can be expressed as

$$U_{z2} = \frac{U_0 R_0}{Z_B + R_0} \quad (6)$$

$$U_1 = \frac{U_0 Z_{11}}{Z_{11} + Z_A} \quad (7)$$

$$Z_{11} = Z_0^+ \frac{Z_2 \cosh(\beta^+ + i\alpha^+)l + Z_0^+ \sinh(\beta^+ + i\alpha^+)l}{Z_2 \sinh(\beta^+ + i\alpha^+)l + Z_0^+ \cosh(\beta^+ + i\alpha^+)l} \quad (8)$$

$$U_2 = U_1 [\cosh(\beta^+ + i\alpha^+)l - \frac{Z_0^+}{Z_{11}} \sinh(\beta^+ + i\alpha^+)l] \quad (9)$$

$$U_g = |U_2 - U_{z2}|. \quad (10)$$

The relations (6) to (10) are well known from the transmission line theory. When one is interested only in relative changes of  $U_g$ , it is possible to put into them  $U_0 = 1$ .

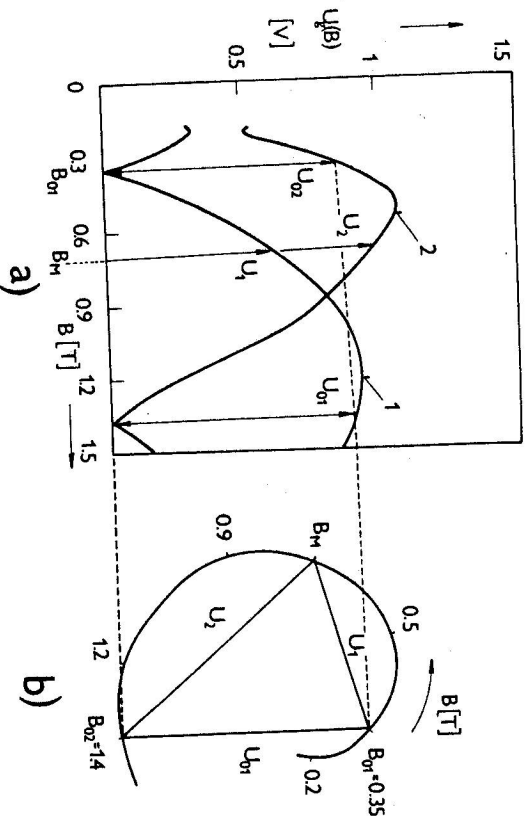


Fig. 3: a) Results of the computer simulation of the voltage dependence at the detector D.  $n = 1.7 \times 10^{21}$ ,  $\mu = 8.8$ ,  $l = 0.001$ ,  $R_0 = 50$ ,  $|Z_A| = 15$ ,  $\varphi_A = 34.4$ ,  $|Z_1| = 44$ ,  $\varphi_2 = 28.6$ ; all quantities in SI units, the angles in degrees. b) Construction of the curve in the complex plane from results in Fig. 3a.  $n = 9.54 \times 10^{20}$ ,  $\mu = 9.14$

For given semiconductor sample parameters the relations (3) to (10) enable the computer simulation of the conditions in the microwave bridge in Fig. 1. From the simulation it follows that the signal at the detector D depends on the value of the impedance  $Z_B$  of the equivalent network in Fig. 2. To zero the bridge means to find for a given value of the external magnetic field  $B = B_0$  the corresponding value of  $Z_B$ . Fig. 3a displays the calculated dependence of  $U_g(B)$  for two different values of the external magnetic field  $B_{01} = 0.35$  T and  $B_{02} = 1.4$  T (of course the value of  $Z_B$  is different). The sample parameters are in both cases the same.

### III. ANALYSIS OF SIMULATED RESULTS AND DISCUSSION

It is possible to interpret the curves in Fig. 3a through the complex quantity  $U_g(B)$  in the complex plane (Fig. 3b). The construction of the curve in Fig. 3b is indicated in Figs. 3ab for the value of  $B = B_M = 0.7$  T. As already mentioned the two curves in Fig. 3a represent the "as measured" voltage dependence  $U_g(B)$  as a function of the external magnetic field  $B$  at the detector D. The distances of the constructed point  $B_M$  from the two fixed points  $B_{01}$  and  $B_{02}$  are  $U_1$  and  $U_2$  (Fig. 3a) and form the triangle  $B_{01} B_{02} B_M$  in the complex plane. Repeating this procedure it is possible to construct the whole curve shown in Fig. 3b. The curves in Fig. 3a were obtained through computer simulation. The input values for the simulation are in the text to the figure. Unfortunately the inverse procedure of getting input data from the measured curve in the complex plane is not a simple task. We have therefore analysed the complex plane curves using different approaches for several simulated results. We have found that the best agreement between the present and the analysed values could be obtained under the assumption that the curve in the complex plane is described only by the forward propagating wave. In that case any point in the complex plane is described by the equation

$$U_g(B) = U_{g0} \cdot e^{-i\alpha^+ l} \cdot e^{-\beta^+ l}. \quad (11)$$

Equation (11) holds exactly only for the transmission line (Fig. 2) loaded by the characteristic impedance  $Z_0^+$ . For the real case equation (11) is better satisfied for the thicker samples, where the influence of the reflected wave due to its damping is smaller. A further problem is the position of the  $B \rightarrow 0$  limiting point in Fig. 3b. Our analysis has shown that in a good approximation one can take for the limit the smallest value of  $B$  used in the experiment. After such a simplification the complex quantity  $U_g$  can be estimated for any value of  $B$ . Taking two different quantities  $U_g(B_1)$  and  $U_g(B_2)$  from (11) one obtains

$$\ln \frac{U_g(B_1)}{U_g(B_2)} = \ln \frac{U_g(B_1)}{U_g(B_2)} + i[\varphi(B_1) - \varphi(B_2)]. \quad (12)$$

where  $\phi(B_1)$  and  $\phi(B_2)$  are phases of the constructed complex quantities and  $U_e(B_1)$  and  $U_e(B_2)$  their amplitudes. From the comparison of (11) and (12) it follows that from the change of the phase it is possible to estimate the value of the phase constant  $\alpha^+$  and from the change of the phase and the amplitude the damping constant  $\beta^+$ . From  $\alpha^+$  and  $\beta^+$  using (3) and (4) we obtain the desired semiconductor plasma parameters, the concentration  $n$  and the mobility  $\mu$ . The mean values of  $n$  and  $\mu$  inferred from the curve in Fig. 3b are shown in the same figure. The initial values are shown in Fig. 3a.

The approach in the paper presented is comparable with that used in [7, 8]. The relations from [7] give, using some elementary algebra, our relations (3—5) and (11—12). The main difference between both approaches is that we need for the estimation of  $n$  and  $\mu$  only two points on the curve in the complex plane, while for the one used in [7, 8] it is necessary to measure at least two interference extremes of the signal. To show the applicability of our equivalent network we have made the computer simulation for the same parameters from [7] in the same range of the external magnetic fields. The results are shown in Fig. 4a, where the experimental results from [7] are indicated by crosses and the computer simulation with the full line. The evaluation procedure from measurements in Fig. 4a is shown in Fig. 4b.

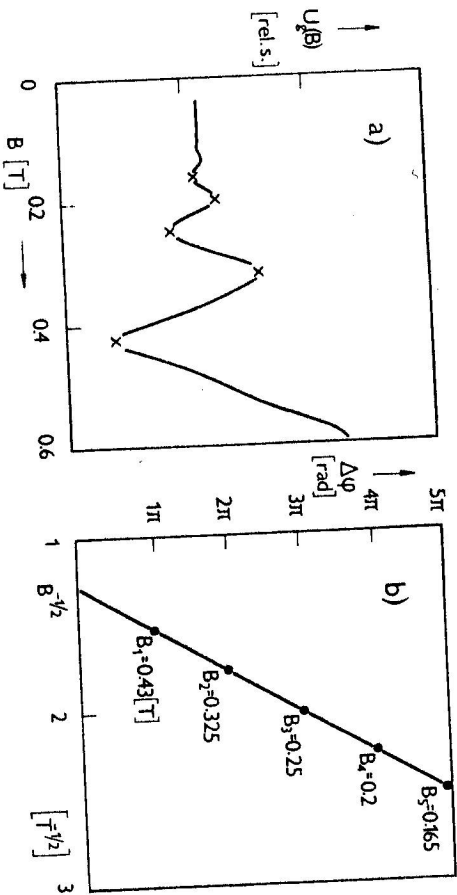


Fig. 4: a) Computer simulation of the results from [7]. Full line — computer simulation, crosses — some experimental values taken from [7]. Input parameters for the simulation are:  $n = 5.3 \times 10^{21}$ ,  $\mu = 34$ ,  $l = 8.5 \times 10^{-4}$ ,  $|Z_A| = 15$ ,  $\phi_A = -20$ ,  $|Z_B| = 68.7$ ,  $\phi_B = 2.4$ ,  $R_0 = 50$ ,  $Z_2 = 13$ ,  $f = 37$  GHz. The constants in (3) and (5) should be changed due to the change of frequency. b) Evaluation of  $n$  from the measurements shown in Fig. 4a. The concentration obtained from the slope of the line is  $n = 5.6 \times 10^{21}$ .

#### IV. CONCLUSION

We have presented in the paper an equivalent network model of the microwave bridge used for the measurement of semiconductor plasma parameters by means of excited helicon waves. The computer simulation based on the equivalent circuit gives results which are quite a good agreement with the experimental results and moreover it allows the optimization of the experimental procedure. Starting with the computer simulation a new approach to the estimation and its analysis has been introduced, which can be used for the estimation of the semiconductor plasma parameters from measurements in a much smaller range of the external magnetic fields than before. Concluding we would like to mention that the same network can be used for the simulation and measurement of, e.g., ferrite parameters. A study in that line is in progress and will be published.

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#### НОВЫЙ МЕТОД АНАЛИЗА ИЗМЕРЕНИЙ ПАРАМЕТРОВ ПОЛУПРОВОДНИКОВОЙ ПЛАЗМЫ

В работе излагаются методы симуляции и анализа микроволновых экспериментов, которые используются для ситуации геликонных волн. Отмечена эквивалентность экспериментальных результатов с результатами, полученными при помощи компьютерной симуляции. Эта схема позволяет определить параметры плазмы на основе изменения сигнала как функции внешнего магнитного поля даже в том случае, когда диапазон внешнего магнитного поля недостаточно велик для получения двух или более интерференционных картин измеряемого сигнала, которые были необходимы для анализа в предшествующих работах.