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SCREENING IN AN EXPANDING QUARK-GLUON PLASMA*)

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Effects of expansion on the Debye length in quark-gluon plasma are calculated in an abelian, boost invariant model. It is found that for initial times the screening is significantly more efficient than that which follows from naive static considerations.

This research has been carried out with A. Białas and W. Czyż [1]. We analyse the effects of expansion of the quark-gluon plasma on the Debye length, λ_D, which is a measure of electric screening in an ionized gas [2]. Screening is a basic phenomenon in the plasma and it is essential to describe it as accurately as possible. So far the estimates for λ_D have assumed that the medium is at rest [3]. This is incorrect, since we know that the quark-gluon plasma, if created in relativistic heavy-ion collisions, will undergo a rapid expansion [4]. To incorporate this feature we modify the classical Debye-Hückel [5] theory of electric screening to the expanding case. As a result, we will find that the effects of expansion on λ_D are strong for early stages of the evolution.

To simplify the problem we make the following assumptions:

1) the expansion is boost invariant (Bjorken's expansion) [6],
2) the system reaches thermal equilibrium at some proper time τ_0,
3) ideal gas approximation works,
4) the abelian model of the quark-gluon plasma is used [7].

Let us begin by recalling the Debye-Hückel method [5]. One considers the Coulomb potential $\phi(\boldsymbol{x})$ in the vicinity of a specified charge of magnitude e placed at $\boldsymbol{x} = 0$. The presence of the potential causes the distribution of the background charges to be modified, $n(\boldsymbol{x}) = n_0 \exp(-e\phi(\boldsymbol{x})/kT)$, where n_0 is the average density. The Maxwell equation for ϕ becomes

$$\nabla^2\phi(\boldsymbol{x}) = e[n(\boldsymbol{x}) - n_0] + e\delta(\boldsymbol{x}) \approx -n_0 e\phi(\boldsymbol{x})/kT + e\delta(\boldsymbol{x}), \tag{1}$$

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where in the last equality condition $e\phi(\boldsymbol{x})/kT \ll 1$ was assumed. The solution of this equation has the form

$$\phi(\boldsymbol{x}) = -e/(4\pi r)\exp[-r/\lambda_D], \tag{2}$$

where $r = |\boldsymbol{x}|$, and the Debye screening length λ_D is

$$\lambda_D = [kT/(e^2 n_0)]^{1/2}. \tag{3}$$

After a highly relativistic heavy-ion collision the receding pancake-shaped fragments move at velocities close to the velocity of light. It is then fair to assume that for sufficiently short times after the reaction the expansion is one-dimensional and in the central rapidity region the system has approximate boost invariance. Making additional assumptions of the Landau hydrodynamic model (no viscosity nor thermal conductivity) Bjorken derived how various physical quantities depend on time [6]. Specifically, we will need the temperature law

$$T = T_0(\tau/\tau_0)^{-v_s^2}, \tag{4}$$

where τ is the proper time and v_s is the velocity of sound ($v_s^2 = 1/3$ for the case of an ideal gas). For simplicity we assume that the system has exact boost invariance in the whole rapidity range, which means that the receding ions are moving with the velocity $+c$ and $-c$ along the z-axis, and that the systems spacial extension in the $x - y$ direction is infinite.

In the abelian model of the quark-gluon plasma [7] only the charges associated with the diagonal 3 and 8 components of a colour octet are included. Thus, one has three quarks of charges ε_i^a, three antiquarks of charges $-\varepsilon_i^a$, and, additionally, six gluons of charges $\eta_{ij}^a = \varepsilon_i^a - \varepsilon_j^a (a = 3, 8, i, j = 1, 2, 3)$, where

$$\varepsilon_1^a = (1/2, 1/2\sqrt{3}),\ \varepsilon_2^a = (-1/2, 1/2\sqrt{3}),\ \varepsilon_3^a = (0, -1/2\sqrt{3}) \tag{5}$$

are vectors in the (3, 8) space. The interaction is mediated by two abelian fields, A_3^μ and A_8^μ. One can view this model as electrodynamic with two kinds of charges and twelve kinds of particles.

We are now ready to begin the construction of Debye and Hückel. We consider the field generated by the charge ε_k^a ($k = 1, 2$ or 3) placed at rest at the origin, in the midpoint between the two receding nuclei. Of course, a general location between the two receding ions could be chosen, but this could not change our qualitative results. From now on we work in the units $c = k = \hbar = 1$. Maxwell equations are

$$\partial^\mu F_{\mu\nu}^a = g j_{n,\mathrm{cloud}}^a + g\varepsilon_k^a \delta(\boldsymbol{x})\delta_\nu^0, \tag{6}$$

where g denotes the strong coupling constant, and $j_{\nu,\mathrm{cloud}}^a$ is the current of the cloud:

$$j^a_{\nu,\mathrm{cloud}} = \int \frac{\mathrm{d}p_z \mathrm{d}^2 p_\perp}{(2\pi)^3 (p_z^2 + p_\perp^2)^{1/2}} p_\nu \left\{ \sum_i \varepsilon_i^a [\Delta Q_i^+ - \Delta Q_i^-] + \sum_{i,j} \eta_{ij}^a \Delta G_{ij} \right\}. \quad (7)$$

The quantities $\Delta Q_i^\pm$ and ΔG_{ij} denote the changes in the statistical distribution functions due to the presence of the potential. Introducing the Fermi and Bose statistical distribution functions

$$n_{F/B}(\xi) = [\exp(\xi) \pm 1]^{-1}, \quad (8)$$

we can write

$$\Delta Q_i^\pm = n_F(p_\mu u^\mu \mp g\varepsilon_i^a \phi^a/T) - n_F(p_\mu u^\mu) \approx \mp g\varepsilon_i^a \phi^a / T n_F'(p_\mu u^\mu),$$

$$\Delta G_{ij} = n_B(p_\mu u^\mu - g\eta_{ij}^a \phi/T) - n_B(p_\mu u^\mu) \approx -g\eta_{ij}^a \phi^a / T n_B'(p_\mu u^\mu). \quad (9)$$

The quantity $u^\mu = (t/\tau, 0, 0, z/\tau)$ is the four-velocity of the hydrodynamic flow of the plasma [6], and $n'_{F/B}$ are the derivatives of the distribution functions with respect to the argument.

We choose to work in the Coulomb gauge. Then the time component of eqn. (6) becomes

$$-\nabla^2 \phi^a(t, \boldsymbol{x}) = g j^a_{0,\mathrm{cloud}}(t, \boldsymbol{x}) + g\varepsilon_k^a \delta(\boldsymbol{x}). \quad (10)$$

The spacial components satisfy the equation

$$\Box A_i^a(t, \boldsymbol{x}) = -j^a_{i,\mathrm{cloud}}(t, \boldsymbol{x}) + \partial_i \partial_0 \phi^a(t, \boldsymbol{x}). \quad (11)$$

It can be shown [1] that the RHS of the above equation vanishes identically when eqn. (10) is satisfied, therefore the vector potential vanishes for our case, which simply means the absence of magnetic fields. Equation (10) is a differential equation in the variable $\boldsymbol{x}$, with the time treated as a parameter.

Using expressions (7) and (9), performing the integral over momentum, and using the identities

$$\sum_i \varepsilon_i^a \varepsilon_i^b = \delta^{ab}, \quad \sum_{i,j} \eta_{ij}^a \eta_{ij}^b = 3\delta^{ab}, \quad (12)$$

eqn. (10) assumes the form

$$[\nabla^2 - \varkappa^2(t, z)] \phi^a(t, \varrho, z) = -g\varepsilon_k^a \delta(\boldsymbol{x}), \quad (13)$$

with

$$\varkappa^2(t, z) = (4/3) g^2 T^2(t/\tau). \quad (14)$$

The system has cylindrical symmetry, hence we have introduced $\varrho = (x^2 + y^2)^{1/2}$. Note that T and τ in the above expression depend on z, therefore the screening mass $\varkappa$ is position dependent.

For each value of the colour index a eqn. (13) is a separable elliptic equation in two variables: ϱ and z. It is subject to the following boundary conditions:

$$\phi^a(\varrho, z = \pm t) = 0, \quad \phi^a(\varrho \to \infty, z) = 0. \quad (15)$$

The condition at $\varrho \to \infty$ is evident, while the condition at the light cone is a consequence of the singularity in $\varkappa$ at $z = \pm t$. Equations (14) with boundary conditions (15) are solved numerically [8]. For details of the method see ref. [1].

The solution for the parameters

$$T_0 = 2\,\mathrm{fm}^{-1}, \quad \tau_0 = 0.2\,\mathrm{fm}, \quad v_s^2 = 1/3, \quad \alpha_s = g^2/(4\pi) = 0.2, \quad (16)$$

and three different values of time is shown in fig. (1). We plot contours of ϕ at decimal multiples of 1, 2 and 5 fm^{-1}. Since the test charge is placed in the middle between the receding ions, the system has a reflection symmetry $z \to -z$, and we show the solution for $z \geq 0$ only. For the earliest time, $t = \tau_0 = 0.2$, we note clear anisotropy: the contours are compressed along the z-axis. For later times the anisotropy weakens, and for $t \approx 1$ fm the solution is nearly spherical, except for the immediate vicinity of the light cone. There are two sources of this effect: one is the explicit dependence of $\varkappa$ on z, and the second is the enforcement of the boundary condition (15).

We need to have some convenient global measure of the screening length. We notice that for the solution (2) the Debye length is the distance at which $r\phi$ drops e times from its original value. We generalize this definition to our case. Due to

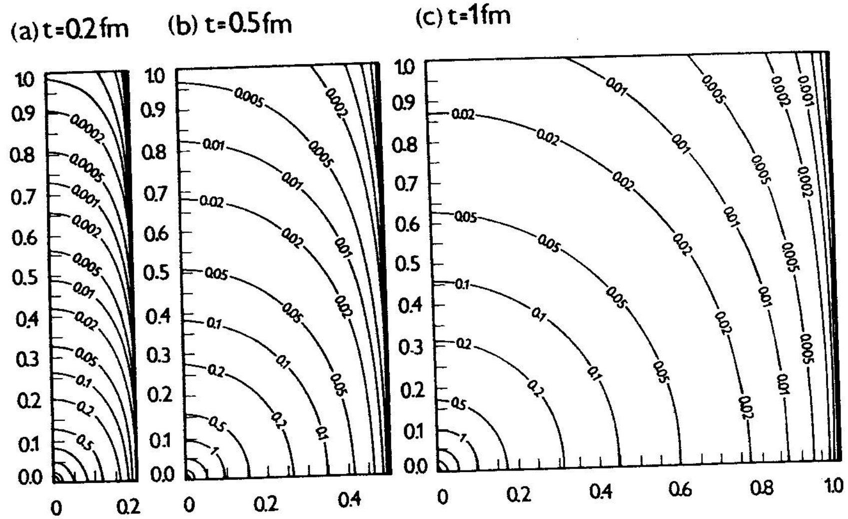

Fig. 1. Contour plots of the potential for $t = 0.2$ fm, (a), 0.5 fm (b) and 1 fm (c). Contour levels are at decimal multiples of 1, 2 and 5 fm^{-1}. The horizontal axis is z and the vertical axis is ϱ (in fm).

the mentioned asymmetry, we introduce two "global Debye lengths", Λ_z and Λ_ϱ, which are the distances along the z and ϱ axes at which $r\phi$ drops e times from the value at $x = 0$. Of course, Λ_z and Λ_ϱ depend on time. These global Debye lengths are displayed in Table 1. The first row of the table is the naive value of the screening length, which follows from a static calculation; in our notation it is just $1/\varkappa(t, 0)$. The next two rows are Λ_z and Λ_ϱ. We note that for all times the following inequalities hold:

$$1/\varkappa(t, 0) > \Lambda_\varrho > \Lambda_z. \tag{17}$$

The first inequality shows that our Debye lengths are shorter than the static estimate — the screening is more efficient due to the expansion — while the second inequality represents the above mentioned anisotropy of screening. Quantitatively, we find that for parameters (16) and for $t = 0.2$ fm the value of Λ_z is a factor of 2 smaller than $1/\varkappa(t = 0.2\,\text{fm}, 0)$. This is a surprisingly strong effect. Later the effect weakens, and around $t = 1$ fm practically disappears.

Table 1

The static estimate of the Debye length, $1/\varkappa(t, 0)$, and the calculated Debye lengths for the expanding case, at four different values of time.

t	(fm)	0.2	0.5	1	2
$1/\varkappa(t, 0)$	(fm)	0.273	0.371	0.467	0.588
Λ_ϱ	(fm)	0.179	0.327	0.449	0.581
Λ_z	(fm)	0.140	0.293	0.433	0.574

The enhanced screening should directly influence the analyses of the problem of the J/ψ suppression in relativistic heavy-ion collisions, where the Debye mechanism is used [9]. Recent works include the kinematic effects of expansion on this problem [10]. However, such models have many components and it is not clear whether our effect could manifest itself clearly in this case.

Of course, one might wonder if our method is credible, particularly for short times, since it seems rather unlikely that thermal equilibrium is really achieved so soon, if at all. Certainly, all the assumptions 1) to 4) are questionable. Nevertheless, we believe that the result we have found may be far more general. This is because it is basically caused by the boundary conditions and by the dependence of the screening mass on z. These circumstances occur as long as boost invariance holds. As a matter of fact, there is experimental evidence for approximate boost invariance from the central rapidity plateau in the multiplicity distributions [11]. Hence, one may expect enhanced screening even if the assumptions 2), 3) or 4) are dropped.

REFERENCES

[1] Białas, A., Broniowski, W., Czyż, W.: Phys. Rev. *D39* (1989), 329.

[2] Screening effects in the magnetic interactions are described in a recent work by Baym, G., Monien, H., Pethick, C. J.: *Proc. XVI International Workshop on Gross Properties of Nuclei and Nuclear Excitations*, Hirschegg, Austria, Jan. 18—22, 1988, p. 128.

[3] Satz, H.: Nucl. Phys. *A418* (1984), 447c; Kanaya, K., Satz, H.: Phys. Rev. *D34* (1986), 3193; De Grand, T. A., De Tar, C. E.: Phys. Rev. *D34* (1986), 2469.

[4] See, e.g., Białas, A., Czyż, W.: Z. Phys. *C38* (1988), 173.

[5] Debye, P., Hückel, F.: Phys. Z. *24* (1923), 185, for a reviews see for example Fetter, A. L., Walecka, J. D.: *Quantum Theory of Many-Particle Systems*. McGraw New York 1971.

[6] Bjorken, J. D.: Phys. Rev. *D27* (1983), 140.

[7] Białas, A., Czyż, W., Dyrek, A., Florkowski, W.: Nucl. Phys. *B296* (1988), 611. Białas, A., Czyż, W.: Ann. Phys. (N. Y.) *187* (1988), 97.

[8] Adams, J., Sarztrauber, P., Sweet, R.: computer code FISHPAK, ver. 3.1 (1980); Swarztrauber, P., Sweet, R.: *"Efficient Fortran Subprograms for the Solution of Elliptic Partial Differential Equations"*, NCAR-TN/IA-109, pp. 135—137, (1975).

[9] Matsui, T., Satz, H.: Phys. Lett. *B178* (1986), 416.

[10] Karsch, F., Petronzio, R.: Phys. Lett. *B193* (1987), 105; Blaizot, J. P., Ollitrault, J. Y.: Phys. Lett. *B199* (1987), 499; Chu, M.-C., Matsui, T.: Phys. Rev. *D37* (1988), 1851.

[11] UA1 collab., Arnison, G., et al.: Phys. Lett. *107B* (1981), 310. UA5 collab., Alpgård, K., et al.: Phys. Lett. *107B* (1981), 320.

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ЭКРАНИРОВКА В РАСШИРЯЮЩЕЙСЯ КВАРК-ГЛЮОННОЙ ПЛАЗМЕ

В работе в рамках абелевой буст-инвариантной модели исследованы эффекты расширения на дебаевской клине в кварк-глюонной плазме. Обнаружено, что экранировка на первоначальном этапе развития более эффективна, чем та, которую можно получить, исходя из наивных статистических предположений.