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ON THE MODELLING OF HYSTERESIS IN MAGNETIC MATERIALS USING THE PREISACH DIAGRAM[1)]

О МОДЕЛИРОВАНИИ ГИСТЕРЕЗИСА В МАГНЕТИЧЕСКИХ МАТЕРИАЛАХ ПРИ ПОМОЩИ ДИАГРАММЫ ПРАЙЗАХА

HEJDA, P.,[2)] ZELINKA, T.,[2)] Praha

A new approach to the Preisach hysteresis model is presented. The analytical model of the Preisach function is proposed and its applicability is demonstrated on an α-Fe_2O_3 natural sample measured by a vibrating-sample magnetometer.

The term "*Preisach diagram*" is here used to denote a more general model of hysteresis than the original Preisach diagram [1], [2]. This generalization is based on the Krasnoselskii theory [3] of hysteresis. Mayergoyz [4], [5] applied this Krasnoselskii theory to develop a mathematical model of hysteresis which is closely related to the given physical process. Our model represents an alternative way tó the generalization proposed in [5].

Under the term "*hysteresis process*" we shall understand the multibranch non-linear dependence of M on H, where the transition between branches occurs in the local intensity extreme of H (so-called turning points), i.e.

$$M(H_n) = M\begin{pmatrix} & H_1 & & H_3 & & & H_n \\ 0 & & & & \cdots & & \\ & & H_2 & & & H_{n-1} & \end{pmatrix}.$$

We shall further suppose that the "*wiping-out property*" holds true [4], i.e. that each local maximum (minimum) removes all previous maxima (minima) which are below (above) them. Our basic model expresses the dependence of M on H in the form

$$M(H_n) = 1/2\,E(-H_1 - \gamma M(H_1), H_1 + \gamma M(H_1)) + \sum_{i=1}^{n-1} E(H_i + \gamma M(H_i), H_{i+1} + \gamma M(H_{i+1})), \tag{1}$$

where E (the Everett function) satisfies: $E(b, a) \geq 0$ for $a \geq b$, $E(a, a) = 0$, $E(a, b) = -E(b, a)$. The term $\gamma M(H)$ represents the mean interaction field. The inclusion of the mean interaction field improves substantially the stability of the Preisach diagram with respect to the various procedures of measurment, as shown in [6] and [7].

Although the Everett function can be approximated by the staircase function or charted by isolines, the analytical representation has undeniable advantages: The Everett function can be characterized by a few parameters and the Preisach density function of irreversible processes p as

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2) Geophysical Institute, Czechosl. Acad. Sci., Boční II, 141 31 PRAHA 4, Czechoslovakia

well as the density function of reversible processes q [8] can be obtained from the simple formulas

$$p(a, b) = -\partial^2 E(b, a)/\partial a\,\partial b, \quad q(a) = [\partial E(b, a)/\partial a]_{b=a}.$$

The model functions should be as simple as possible, but, at the same time, as general as possible to be able to cover a broad spectrum of applications. The following forms of E have proved to be useful:

I. $$E(b, a) = \alpha \cdot (a - b)$$

models the manifestation of para- or dia-magnetism.

II. $$E(b, a) = \begin{cases} \alpha f(a)\, f(-b), & a \geq 0,\ b \leq 0, \\ 0, & \text{otherwise,} \end{cases}$$

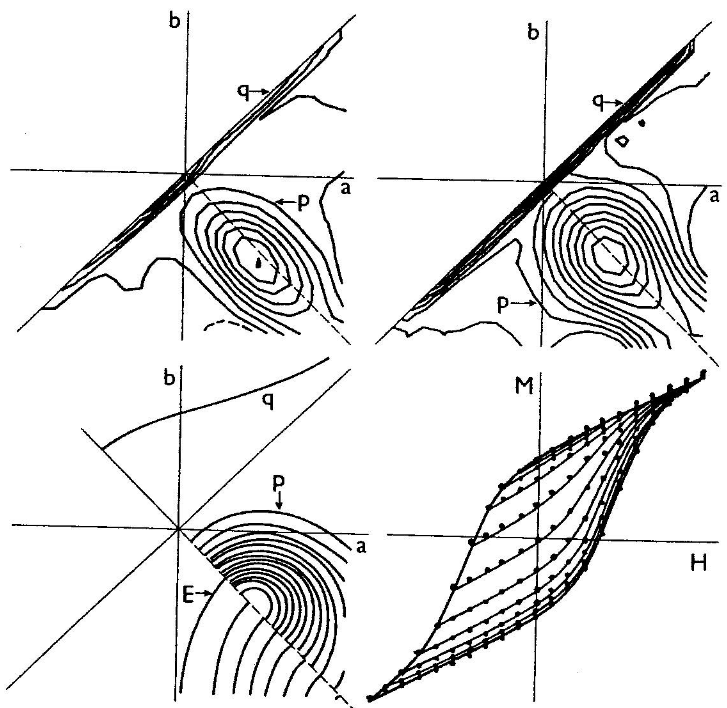

Fig. 1. Comparison of experimental data and the Preisach model for α-Fe_2O_3, $H_{max} = 500$ kA/m, M — arbitrary units. Upper part: isolines of density function derived directly from measured data (nonanalytical model); left: asymmetrical shape indicates the lack of stability in the case when the mean interaction field was not taken into consideration, right: improvement of symmetry when the mean interaction field was considered. Lower part: analytical model in the form $E(b, a) = c_1(a - b) + c_2(f_2(a) - f_2(b)) + c_3(f_3(a)f_3(-b))^{r_3} + c_4 g_4(f_4(a) - f_4(b))\, h_4(k_4(a) + k_4(b))$, where $f_i(x) = \tanh(a_i x)$, $k_4(x) = \tanh(\beta_4 x)$, $g_4(x) = \sinh(s_4 x) - s_4 x$, $h_4(x) = 1/\cosh(r_4 x)$. The constants were estimated using the nonlinear least-squares method; left: isolinies of p, q and E (only half of the symmetrical functions); right: experimental points and analytical curves.

where $f \in C^1(\langle 0, \infty))$, $f' \geq 0$, $f(0) = 0$, $f(x) \to 1$ for $x \to \infty$, is suitable for the representation of remanent characteristics.

III. $$E(b, a) = g(f(a) - f(b))\, h(k(a) + k(b)),$$

where $f \in C^2((-\infty, \infty))$ is an odd function, $f' \geq 0$, $f'' \leq 0$ on $\langle 0, \infty)$, $f(x) \to 1$ for $x \to \infty$; k fulfils the same conditions; $g \in C^2(\langle 0, 2\rangle)$, $g' \geq 0$, $g'' \geq 0$, $g(0) = 0$; $h \in C^2(\langle -2, 2\rangle)$ is an even function, $h \geq 0$, $h' \leq 0$ on $\langle 0, 2\rangle$, $h(0) = 1$ is the most general form and enables us to represent soft as well as hard magnetic materials. A great variability in the modelling of hysteresis processes can be attained by a selection of different parameters and by the sum of several functions.

The work on the model was motivated by the possibility of its testing on actual data measured by a vibrating-sample magnetometer of our construction [8], [9]. We demonstrate the application of the method to a sample formed by single-domain grains of α-Fe_2O_3 with a high concentration (see Fig. 1).

The work presents the essential generalization of the classical Preisach model. The generalization can be found in the following aspects:

1. The identification starts with the Everett function. The advantage of this procedure is considerable, since relation (1) and the "*wiping-out property*" enable us to develop a simple algorithm for the transition from the measured magnetization process to the Everett function at discrete points and for an inverse transition from the Everett function to the model. At the same time, no conditions are imposed on the measurement procedure.
2. The analytical forms of the Everett function given above enable to model hysteresis curves of various shapes.
3. The method allows the modelling of reversible as well irreversible magnetization changes.
4. The inclusion of the mean interaction field improves the stability of the Preisach diagram with respect to the various procedures of measurement.

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