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CHIRAL SYMMETRY BREAKING AROUND QUARK MULTIPLETS

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Virtual quark-antiquark fluctuations around static quark multiplets are studied. The correlation function $\langle \boldsymbol{M}(0)\,\bar{\psi}\psi(r)\rangle$ between the multiplet system $\boldsymbol{M}(0)$ and the chiral condensate $\bar{\psi}\psi(r)$ is computed within lattice QCD. The behaviour of different quark multiplets seems to fit into a Casimir scaling law.

I. INTRODUCTION

The formulation of QCD on a space-time lattice led to new insights into the theory of strong interactions. One of the most important results was the demonstration of quark confinement: In the pure gluonic case the potential between a static quark and an antiquark rises linearly with distance r [1]. But in these first investigations the influence of dynamical fermions was not taken into account. The formulation of dynamical fermions on a space-time lattice was a highly non-trivial problem. According to the prescription of Wilson and Susskind and after several years of algorithm development also dynamical effects could be studied [2, 3]. Due to the dynamical fermions the potential between a static quark-antiquark source has no longer a linearly rising shape but becomes bounded. A screened potential between a static quark and antiquark is found both for the Susskind and Wilson definition of lattice fermions. This effect can be explained by the creation of virtual quark-antiquark pairs which leads to a gradual loss of the string tension [4].

To get a better idea of the basic physical mechanism and the influences of dynamical quarks to lattice QCD the polarisation cloud around a quark charge was investigated using the correlation $\langle L(0)\,\bar{\psi}\psi(r)\rangle$ between a static quark and the fermionic condensate as a measure for the quark vacuum occupation number density [5]. Extensive computer simulations showed that polarisation effects in the near surrounding of a quark are supressed. At the first glance this

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result was surprising because this is the opposite effect expected in QED with regard to fermionic vacuum polarisation.

The construction of static quark multiplets and the computation of the chiral order parameter on a space-time lattice are outlined in Sec. II. The results obtained in this way and some possible explanations are discussed in Sec. III. Finally, in Sec. IV we draw the conclusions and mention some interesting further investigations.

II. THEORY

In extension to former work [5] we intend to investigate the chiral symmetry breaking in the surroundings of static colour sources. The only property of static quarks $\psi^s(\boldsymbol{r}, t)$ is colour changing with time

$$\psi^s(\boldsymbol{r}, t) = T \exp\left\{-\mathrm{i}\int_0^t \mathrm{d}t'\, g\,\lambda_a A_a^0(\boldsymbol{r}, t')\right\}\psi^s(\boldsymbol{r}, 0). \tag{1}$$

The quark field $\psi^s(\boldsymbol{r}, t)$ obeys the static ($m \to \infty$) limit of the Dirac equation

$$\partial_t \psi^s(\boldsymbol{r}, t) = -\mathrm{i}\, g\,\lambda_a A_a^0(\boldsymbol{r}, t)\,\psi^s(\boldsymbol{r}, t). \tag{2}$$

T is the time-ordering operator, λ_a ($a = 1, \ldots, 8$) are the Gell-Mann matrices, A_a^0 is the time component of the gluon field A_a^μ and g denotes the coupling to the colour charge.

We want to study colour sources in various multiplet representations of SU(3). To construct the states $\Psi^s_{M,m}(\boldsymbol{r}, t)$ with the components m in the degeneration space of multiplicity M we have to couple the colour indices c of N quarks or antiquarks by the corresponding SU(3)-Clebsch-Gordan coefficients $(c_1, \ldots, c_N|Mm)$

$$\Psi^s_{M,m}(\boldsymbol{r}, t) = \sum_{c_1, \ldots, c_N} (c_1, \ldots, c_N|Mn)\,\psi^s_{c_1}(\boldsymbol{r}, t) \ldots \psi^s_{c_N}(\boldsymbol{r}, t). \tag{3}$$

To investigate the influence of these colour sources on the QCD vacuum we use the full QCD Langrangian $\mathscr{L}(A, \bar{\psi}, \psi) = \mathscr{L}_G(A) + \mathscr{L}_F(A, \bar{\psi}, \psi)$. The gauge field action S_G for 4-dimensional Euclidean lattice in the standard Wilson formulation reads

$$S_G = \int \mathscr{L}_G \mathrm{d}^4x = \beta\sum_{\square}\left[1 - \frac{1}{3}\mathrm{Re\,Tr}\,U_\square\right], \qquad \beta = \frac{6}{g^2}. \tag{4}$$

The summation runs over all plaquettes of the lattice with N_t links in time and N_x links in the space direction and the lattice constant a. $U_{x\mu} \in \mathrm{SU}(3)$ is the gauge field matrix associated to each link beginning at the site x in the direction $\hat{\mu}$. For the fermion action S_F we choose the Kogut-Susskind discretisation

$$S_F = \int \mathscr{L}_F \mathrm{d}^4x = \frac{n_f}{4} a^3 \left\{ \sum_{x,\mu} \frac{1}{2} \Gamma_{x\mu} (\bar{\psi}_x U^+_{x\mu} \psi_{x+\hat{\mu}} - \bar{\psi}_{x+\hat{\mu}} U_{x\mu} \psi_x) + m \sum_x \bar{\psi}_x \psi_x \right\} = \tag{5}$$

$$= \sum_{xx'} \bar{\psi}_x (D(U) + m)_{xx'} \psi_{x'}.$$

Here n_f denotes the number of flavours and m is the bare mass of the quark field ψ. With the factor 1/4 fermion doubling is taken into account and the $\Gamma_{x\mu}$ play the rôle of Dirac matrices.

The colour sources partially restore the chiral symmetry which is broken in the QCD vacuum. To show this we have to measure the expectation value of the chiral condensate $\bar{\psi}\psi(\boldsymbol{r})$ in the vicinity of a static quark multiplet at $\boldsymbol{r} = 0$

$$\langle \boldsymbol{M}(0) \bar{\psi}\psi(\boldsymbol{r}) \rangle := \frac{1}{M} \sum_m \left\langle \Psi^s_{M,m}\left(\boldsymbol{r}=0,\ t=\frac{\mathrm{i}}{2T}\right) | \bar{\psi}\psi(\boldsymbol{r},\ t=0) | \Psi^s_{M,m}\left(\boldsymbol{r}=0,\ t=-\frac{\mathrm{i}}{2T}\right) \right\rangle. \tag{6}$$

According to eq. (1) the propagator $U(\boldsymbol{r})$ of the static quarks ψ^s at position $\boldsymbol{r}$ over the time period $t = \mathrm{i}/T$ is described by the product of SU(3) matrices in the time direction

$$U(\boldsymbol{r}) = \prod_{k=N_t}^{1} U^0(\boldsymbol{r} + ka\hat{O}). \tag{7}$$

We evaluate the expectation value (6) by a path integral

$$\langle \boldsymbol{M}(0)\, \bar{\psi}\psi(\boldsymbol{r}) \rangle = \int \mathscr{D}[U, \bar{\psi}, \psi] \frac{1}{M} \operatorname{Tr} \boldsymbol{M}(0)\, \bar{\psi}\psi(\boldsymbol{r}) \exp\{-S_G[U] - S_F[U, \bar{\psi}, \psi]\}, \tag{8}$$

where we introduced the trace of the colour multiplet $\boldsymbol{M}$

$$\operatorname{Tr} \boldsymbol{M} := \sum_m \sum_{\substack{c_1, \ldots, c_N \\ c_{1'}, \ldots, c_{N'}}} (c_{1'}, \ldots, c_{N'} | Mn)\, (c_1, \ldots, c_N | Mn)\, U_{c_1 c_{1'}} \ldots U_{c_N c_{N'}} \tag{9}$$

with the quark propagator U of eq. (7).

We are now able to express the trace of any desired multiplet by the Polyakov loop L, i.e. the trace of the fundamental triplet

$$L = \operatorname{Tr} U = \operatorname{Tr} \boldsymbol{3}. \tag{10}$$

Using the orthogonality relation of the Clebsch-Gordan coefficients

$$\sum_{M,m} (c_{1'}, \ldots, c_{N'} | Mm)\, (Mm | c_1, \ldots, c_N) = \delta_{c_1 c_{1'}} \ldots \delta_{c_N c_{N'}}, \tag{11}$$

where the sum over M takes all multiplets of the N-quark system into account, we can express the sum of the multiplet traces of a N-quark system as the fundamental Polyakov loop to the power of N

$$\sum_M \operatorname{Tr} \boldsymbol{M} = (\operatorname{Tr} U)^N = L^N \tag{12}$$

and similarly for the multiplets being built out of N_1 quarks and N_2 antiquarks

$$\sum_M \operatorname{Tr} \boldsymbol{M} = L^{N_1} L^{*N_2}. \tag{13}$$

From the decomposition laws for the SU(3) multiplets there follows the important result that all multiplet Polyakov loops can be represented as polynomials of the fundamental Polyakov loop

$$\begin{aligned} \operatorname{Tr} \boldsymbol{3} &= L \\ \operatorname{Tr} \bar{\boldsymbol{3}} &= L^* \\ \operatorname{Tr} \boldsymbol{6} &= L^2 - L^* \\ \operatorname{Tr} \boldsymbol{8} &= LL^* - 1 \\ \operatorname{Tr} \boldsymbol{10} &= L^3 - 2(LL^* - 1) - 1 = L^3 - 2LL^* + 1. \end{aligned} \tag{14}$$

The functional integration in eq. (8) extends over all degrees of freedom of the gauge field U and of the fermion fields ψ, $\bar{\psi}$. After analytical integration over $\bar{\psi}_x$ and ψ_x only the path integration over $U_{x\mu}$ remains

$$\langle \boldsymbol{M}(0)\, \bar{\psi}\psi(r) \rangle = \frac{n_f}{4M} \int D[U] \operatorname{Tr} \boldsymbol{M}(0)\, [D(U) + m]^{-1}_{rr}\, \mathrm{e}^{-\left[S_G - \frac{n_f}{4} \operatorname{Tr} \ln (D+m)\right]}. \tag{15}$$

$\bar{\psi}\psi(r)$ is here replaced by the inverse fermionic matrix $(D + m)^{-1}_{rr}$. The fermionic determinant $\det(D + m)$ is evaluated using the pseudofermionic method [3].

The expressions $\langle \boldsymbol{M}(0)\, \bar{\psi}\psi(r) \rangle$ were calculated numerically. The system was simulated on a $8^3 \times 4$ lattice. As inverse gluonic coupling we took $\beta = 5.2$ lying in the confinement regime. Our investigations were done with a virtual quark mass $ma = 0.1$ and the flavour number n_f was set to 4. To equilibrate the pure gauge field we performed 1000 Metropolis Monte-Carlo iterations. Then we appended a few 100 MC iterations utilizing the pseudofermionic method with a heat bath algorithm with 50 steps to equilibrate the full QCD vacuum. Subsequently, data were taken over 1100 iterations.

For the chiral symmetry breaking around a colour multiplet we expect the same qualitative behaviour as for the chiral symmetry breaking around a single static quark charge presented in former works [5]. In figures 1—3 the corresponding correlation functions for the triplet, octet and sextet are normalised to the cluster value $\langle \boldsymbol{M} \rangle \langle \bar{\psi}\psi \rangle$ which describes the system at $r = \infty$. The error bars reflect the standard deviation assuming uncorrelated Monte-Carlo iterations. They are only indicated if they are greater than the symbols.

We find the remarkable result that even around the octet and the sextet as components of multiquark systems polarisation effects are suppressed, i.e. the correlation increases with increasing distance r. This behaviour can be explained in analogy to superconductivity: After inserting one or more quark sources into the empty QCD vacuum the system tries to become locally colourless by creation of virtual gluon strings. This is in analogy to force a hypothetical magnetic monopole into a superconducting medium. Then magnetic flux tubes will be produced which end at an antimonopole or leave the system at the boundary. The creation of massless gluons seems to be favoured in comparison to quark-antiquark pairs because of their finite mass. These gluons carry the colour charge away from the static multiquark system and having a charge they are themselves sources of further gluons amplifying the central charges. When the production of such gluon strings requires too much energy they end in virtual quark-antiquark pairs. The distance of maximum pair creation where the correlation reaches the cluster value is identical with the screening length found in hadronization studies of static quark-antiquark systems.

Having studied the qualitative behaviour of the virtual quark-antiquark pair creation around quark multiplets we compare the shape of the correlation function of the octet and the sextet to that of the single quark system. For this purpose we adjust the correlation function $\langle \boldsymbol{3}(0)\bar{\psi}\psi(r)\rangle/(\langle \boldsymbol{3}\rangle\langle\bar{\psi}\psi\rangle)$ for the triplet at integer distances $r/a = 0, 1, 2, 3$ and 4 by a three parameter exponential function $c - \alpha_3 \exp(-\mu r/a)$ which is drawn in figure 1. We find $c = 1$ within an accuracy of one percent, $\alpha_3 = 0.214 \pm 1$ and $\mu = 1.33 \pm 8$. A fit to the octet values gives almost the same screening mass μ as for the triplet and a value of the parameter α_8, which is in agreement with a Casimir scaling law whithin statistical errors

$$\alpha_M = \alpha_3 \frac{C_2(\boldsymbol{M})}{C_2(\boldsymbol{3})} \tag{16}$$

where $C_2(\boldsymbol{M})$ are the quadratic Casimir operators of SU(3) with $C_2(\boldsymbol{M}) = 0$, 4/3, 10/3 and 9/3 for the singlet, triplet, sextet and octet representations.

Therefore, it seems more reasonable to draw in the figures 2 and 3 the following curves

$$f(r) = 1 - \alpha_3 \frac{C_2(\boldsymbol{M})}{C_2(\boldsymbol{3})} \exp(-\mu r/a) \tag{17}$$

instead of fits. Because of the nice agreement of these curves with the correla-

TRIPLET

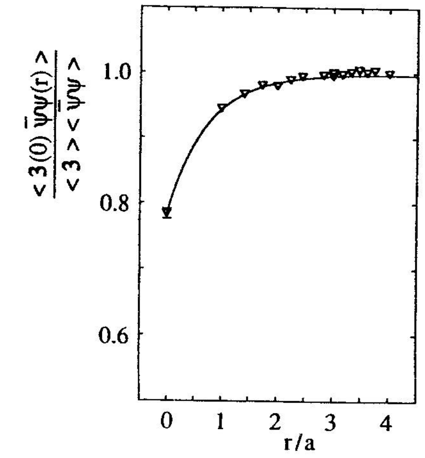

Fig. 1: The normalised correlation function $\frac{\langle \boldsymbol{3}(0)\,\bar{\psi}\psi(r)\rangle}{\langle \boldsymbol{3}\rangle\langle\bar{\psi}\psi\rangle}$ between a colour triplet and the chiral condensate for $m = 0.1$ and $\beta = 5.2$.

OCTET

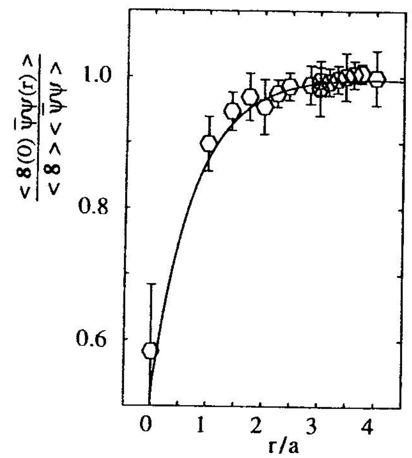

Fig. 2: The normalised correlation function $\frac{\langle \boldsymbol{8}(0)\,\bar{\psi}\psi(r)\rangle}{\langle \boldsymbol{8}\rangle\langle\bar{\psi}\psi\rangle}$ between a colour octet and the chiral condensate for $m = 0.1$ and $\beta = 5.2$.

SEXTET

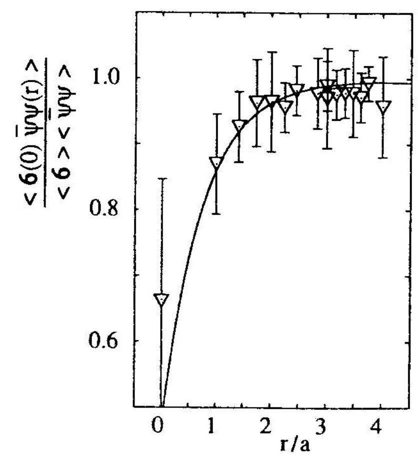

Fig. 3: The normalised correlation function $\frac{\langle \boldsymbol{6}(0)\,\bar{\psi}\psi(r)\rangle}{\langle \boldsymbol{6}\rangle\langle\bar{\psi}\psi\rangle}$ between a colour sextet and the chiral condensate for $m = 0.1$ and $\beta = 5.2$.

tions we believe that the local restoration $\bar{\psi}\psi(r)$ of chiral symmetry is proportional to the Casimir operators $C_2(\boldsymbol{M})$ at least for small deviations of chiral symmetry breaking from the QCD vacuum value.

IV. CONCLUSIONS

In the previous sections we have shown that the vacuum fluctuations are suppressed not only in the vicinity of an external single static quark but also close to static multiquark systems. This mechanism confirms the basic ideas of the MIT — bag model where the "true" QCD vacuum is partially destroyed inside the bag volume by the presence of coloured quarks. The state inside the bag is often called "perturbative" vacuum because it can be treated by the methods of the perturbation theory.

A comparison of the chiral condensate around a static quark with the behaviour around higher quark multiplets indicates a universality between the corresponding correlation function which should be investigated and proved in forthcoming works. Another topic of interest is the distribution of the charge density $\psi^+ \psi(r) = \bar{\psi} \gamma_0 \psi(r)$ around multiplet sources. All the studies should be carried out additionally with Wilson fermions. Further insight is expected to be achieved by the computation of the chromo-electromagnetic field around quark multiplets.

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НАРУШЕНИЕ КИРАЛЬНОЙ СИММЕТРИИ В ОКРЕСНОСТИ КВАРКОВЫХ МУЛЬТИПЛЕТОВ

В работе изучаются флуктуации виртуальных кварк-антикварковых пар, окружающих статические кварковые мультиплеты. В решеточной КХД вычисляется корреляционная функция $\langle \boldsymbol{M}(0)\, \bar{\psi}\psi(r) \rangle$ между мультиплетной системой $\boldsymbol{M}(0)$ и киральным конденсатом $\bar{\psi}\psi(r)$. Поведение различных кварковых мультиплетов выглядит соответствующим скейлинговому закону в терминах операторов Казимира.