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ONE SOLUTION OF THE VAPORIZATION WAVE PHENOMENON IN THE EXPLODING WIRE TECHNIQUE[1]

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The vaporization wave phenomenon in exploding wire technique is considered in this paper. A convergent power series solution of the circuit equation was found. The coefficients of this series are given via a simple recurrent formula. Some numerical calculations are presented.

I. INTRODUCTION

The exploding wire technique offers a simple method of obtaining high temperature plasma. A condenser battery is usually used as an energy source. Formerly plasma has been gained in plasma accelerators using this technique [1]. The recent research showed the possibility of using the pressure wave generated by the explosion for moving various small bodies [4]. The study of both the phase transitions of the wire matter and the plasma parameters appears to be very important.

The complicated process of a wire explosion can be divided into several phases: wire heating in the solid state, melting, heating of the fluid metal, vaporization, current interruption and restrike. The complexity of this phenomenon may increase with the time overlapping of particular phases or with the fragmentation of the wire [5, 6]. These effects can be avoided under some experimental conditions [1] and the cylindrical symmetry of the wire can be preserved, too.

The wire vaporization begins at the temperature T_E considerably higher than the boiling point of the wire matter [2, 3]. The vaporization wave appears on the surface and moves to the centre separating the conductive and the nonconductive part of the wire. The temperature dependence of the wave speed was

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examined by Bennet [2]. It can be approximated with the rectangular relationship: $v = 0$ for $T < T_E$ and $v = v_0$ for $T \geq T_E$. As the wire radius decreases the wire resistance grows infinitely. The linear temperature dependence can therefore be neglected. The resistance then becomes

$$R(t) = R(t_0)\frac{\pi r_0^2}{\pi r^2(t)} = \frac{R(t_0)}{\left[1 - \frac{v_0}{r_0}(t - t_0)\right]^2}, \quad t \in \left\langle t_0,\ t_0 + \frac{r_0}{v_0}\right), \tag{1}$$

where r_0 is the wire radius and v_0 is the velocity of the vaporization wave, respectively.

II. SOLUTION OF THE CIRCUIT EQUATION

Let us consider the circuit equation in the form

$$L_0\frac{dI}{dt} + R(t)I + \frac{1}{C_0}\int_{t_0}^{t} I(t)\,dt = U(t_0), \quad t \in \left\langle t_0, t_0 + \frac{r_0}{v_0}\right), \tag{2}$$

where L_0, C_0 are parameters of the external circuit and the $U(t_0)$ is the initial voltage of the condenser battery, respectively. Having introduced the non-dimensional time and the current variables

$$\tau = \frac{v_0}{r_0}(t - t_0) \tag{3}$$

$$\iota = \frac{v_0}{r_0}\frac{L_0}{U(t_0)}I \tag{4}$$

and the coefficients

$$\Delta = \frac{R(t_0)}{2L_0}\frac{r_0}{v_0}, \tag{5}$$

$$\Omega = \frac{1}{(L_0C_0)^{1/2}}\frac{r_0}{v_0}, \tag{6}$$

the Eq. (2) becomes

$$\frac{d\iota}{d\tau} + 2\Delta\frac{\iota}{(1-\tau)^2} + \omega^2\int_0^{\tau}\iota(\tau)\,d\tau = 1, \quad \tau \in \langle 0, 1), \tag{7}$$

with the initial condition

$$\iota(0) = \frac{v_0}{r_0}L_0\frac{I(t_0)}{U(t_0)}. \tag{8}$$

In Eq. (7) we use the substitution

$$\iota(\tau) = (1 - \tau)^2 f(\tau). \tag{9}$$

The solution to $f(\tau)$ can be found in a power series form

$$f(\tau) = \sum_{k=0}^{\infty} a_k \tau^k. \tag{10}$$

After substituting relations (9) and (10) in Eq. (7) and comparing the τ^k coefficients we obtain the recurrent formula

$$\begin{aligned}
a_0 &= \iota(0)\\
a_1 &= 2(1-\Delta)\,a_0 + 1\\
a_2 &= (2-\Delta)\,a_1 - \left(1 + \frac{\Omega^2}{2}\right)a_0\\
a_3 &= \frac{1}{3}\left[2(3-\Delta)\,a_2 - \left(3 + \frac{\Omega^2}{2}\right)a_1 + \Omega^2 a_0\right)\\
a_{k+1} &= \frac{1}{k+1}\left[2(k+1-\Delta)\,a_k - \left(k+1+\frac{\Omega^2}{k}\right)a_{k-1} + \frac{2\Omega^2}{k}a_{k-2} - \right.\\
&\quad \left. - \frac{\Omega^2}{k}a_{k-3}\right], \qquad k \geq 3.
\end{aligned} \tag{11}$$

The power series converges for $\tau < 1$ due to d'Alambert's criterion. In addition we have $\iota(1) = 0$ for each finite sum thanks to the substitution (9).

III. RESULTS

The dependences $f(\tau)$, $\iota(\tau)$ anda $P(\tau) = (1 - \tau)^2 f^2(\tau)$ for different values of coefficients Δ and Ω are plotted in Figs. 1—6. The values were calculated for 100 members of the sum (10). Note that $f(\tau)$ means the non-dimensional wire voltage, $\iota(\tau)$ the non-dimensional current and $P(\tau)$ the non-dimensional power consumption in the wire. The amount of energy stored in the wire equals the area determined by the curve $P(\tau)$ and the τ axis.

The circuit equation with the vaporization wave was solved previously with the last member of the left-hand side of Eq. (2) negleated [3], in terms of our notation $\Omega = 0$. In our paper the solution for an arbitrary Ω is proposed in the form of a convergent power series with coefficients given by a simple recurrent formula.

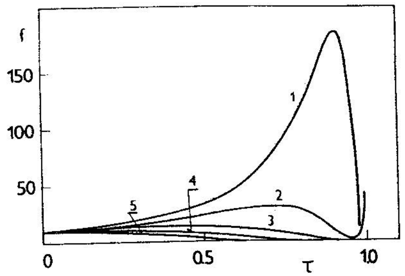

Fig. 1. The $f(\tau)$ dependence for $\iota(0) = 10$ and $\Omega = 0.015$. The curves 1—5 correspond respectively to the values $\Delta = 0.1, 0.3, 0.6, 1, 1.4$.

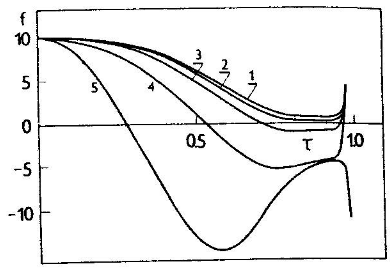

Fig. 2. The $f(\tau)$ dependence for $\iota(0) = 10$ and $\Delta = 1$. The curves 1—5 correspond respectively to the values $\Omega = 0, 0.5, 1, 2, 5$.

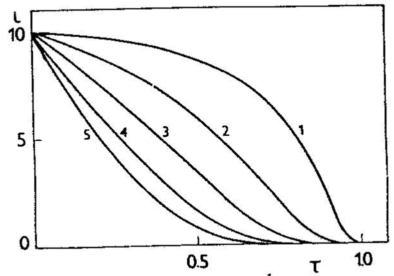

Fig. 3. The $\iota(\tau)$ dependence for $\iota(0) = 10$ and $\Omega = 0.015$. The curves 1—5 correspond respectively to the values $\Delta = 0.1, 0.3, 0.6, 1, 1.4$.

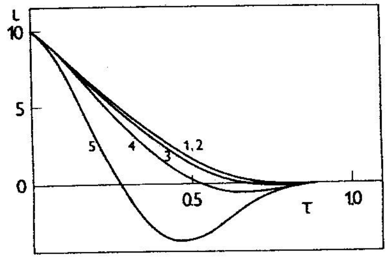

Fig. 4. The $\iota(\tau)$ dependence for $\iota(0) = 10$ and $\Delta = 1$. The curves 1—5 correspond respectively to the values $\Omega = 0, 0.5, 1, 2, 5$.

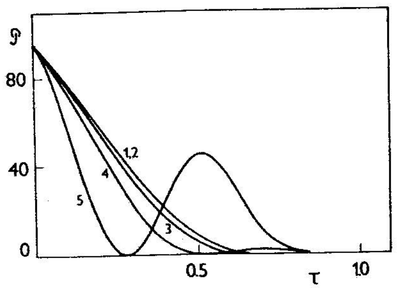

Fig. 5. The $P(\tau)$ dependence for $\iota(0) = 10$ and $\Omega = 0.015$. The curves 1—5 correspond respectively to the values $\Delta = 0.1, 0.3, 0.6, 1, 1.4$.

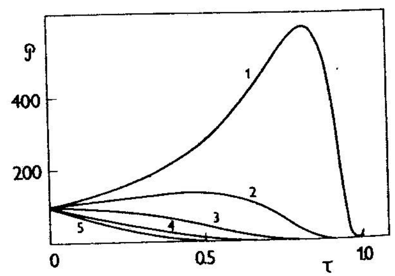

Fig. 6. The $P(\tau)$ dependence for $\iota(0) = 10$ and $\Delta = 1$. The curves 1—5 correspond respectively to the values $\Omega = 0, 0.5, 1, 2, 5$.

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ОДНО РЕШЕНИЕ ЯВЛЕНИЯ ИСПАРЯЮЩЕЙСЯ ВОЛНЫ В ТЕХНИКЕ ВЗРЫВАЮЩЕЙСЯ ПРОВОЛОКИ

В работе изучается явление испаряющейся волны в технике взрывающейся проволоки. Найдено решение для цепи в виде сходящегося степенного ряда. Коэффициенты этого ряда даны в виде простой реккурентной формулы. Приведены некоторые численные расчеты.