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RAY DEFLECTIONS IN NON-HOMOGENEOUS PLASMA CLUSTERS[1)]

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Space deflection of light rays in non-homogeneous plasma is discussed in the paper submitted. A system of ordinary differential equations of the first order describing the passage of the ray through the plasma with a given space distribution of concentration have been derived. This system of equations is advantageous for its simple numerical integration and a possibility of finding both the space and the time behaviour of the ray.

I. INTRODUCTION

A light ray passing through an non-homogeneous plasma cluster is deflected on the trasverse gradients of the plasma concentration. This is caused by the ray velocity and the refractive index dependence on the plasma concentration. According to Fig. 1 the points A, B of the plane wave front will be shifted to the points A′, B′ during the time interval Δt. If the refractive index in the points A, B is different, the deviation of the wave form (and the ray) will occur. This phenomenon is utilized in a great number of diagnostic methods, e.g. in studies of plasma non-homogeneities, in estimates of the concentration and the other parameaters of the plasma [1].

Let us denote by v the phase velocity of the electromagnetic wave with the frequency ω and the wave vector $\boldsymbol{k}$:

$$v = \frac{\omega}{k}. \tag{1}$$

The unit vector in the direction of the wave propagation is

$$\varkappa = \frac{\boldsymbol{k}}{k}. \tag{2}$$

The dispersion relation of the electromagnetic ordinary waves in plasma can be written in the form [2]

$$\omega^2 = \omega_P^2 + c^2 k^2, \tag{3}$$

where

$$\omega_P \equiv \sqrt{\frac{ne^2}{m_e \varepsilon_0}} \tag{4}$$

is the plasma frequency and n is the plasma concentration. With the aid of eq. (3) we can obtain for the refractive index $\mathcal{N} = ck/\omega$ a formula

$$\mathcal{N} = \sqrt{1 - \frac{\omega_P^2}{\omega^2}}. \tag{5}$$

As usual $\omega_P \ll \omega$, we can write

$$\mathcal{N} \approx 1 - \frac{\omega_P^2}{2\omega^2}. \tag{6}$$

The deflection angle of the ray α is in most cases calculated from the relation [1, 3]

$$\alpha = \int \frac{1}{\mathcal{N}} \frac{\partial \mathcal{N}}{\partial y} \, dl; \quad dl^2 = dx^2 + dy^2, \tag{7}$$

which is valid for small deflection angles from the x axis only and that is why approximately there takes place the relation

$$\alpha = \int \frac{1}{\mathcal{N}} \frac{\partial \mathcal{N}}{\partial y} \, dx. \tag{8}$$

This relation does not take into account the gradient of the refractive index in a general direction and so the ray deflection in all the three dimensions. Furthermore, eqs. (7), (8) are valid for small deflecations only and the limits of integration could in some cases be hardly determined. A quite different and sufficiently general method of description of this phenomenon via a system of ordinary differential equations of the first order will be treated in this paper.

II. EQUATIONS OF SPACE DEFLECTION

Deriving the equations of the space deflection of the ray, time will be taken as a parameter describing the ray trajectory. In the simple two-dimensional case in Fig. 1 it can be seen that for two sufficiently near points A, B and a sufficiently short time interval δt the deflection angle of the ray will be

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$$\delta\alpha = -\frac{\|B' - B\| - \|A' - A\|}{\|B - A\|} = -\frac{[v(B) - v(A)]\,\delta t}{\|B - A\|} = -\frac{\Delta v}{\Delta y}\delta t. \tag{9}$$

Permitting the limits $\Delta y \to 0$, $\delta t \to 0$, we get

$$\mathrm{d}\alpha = -\frac{\partial v}{\partial y}\,\mathrm{d}t. \tag{10}$$

This relation gives

$$\mathrm{d}\alpha = \frac{c}{\mathcal{N}^2}\frac{\partial \mathcal{N}}{\partial y}\,\mathrm{d}t, \tag{11}$$

where the fact that $v = c/\mathcal{N}$ has been used. If we used the length $l\,(t)$ as a parameter describing the ray trajectory, the eq. (11) gives

$$\mathrm{d}\alpha = \frac{1}{\mathcal{N}}\frac{\partial \mathcal{N}}{\partial y}\,\mathrm{d}l, \tag{12}$$

which is the differential shape of eq. (7).

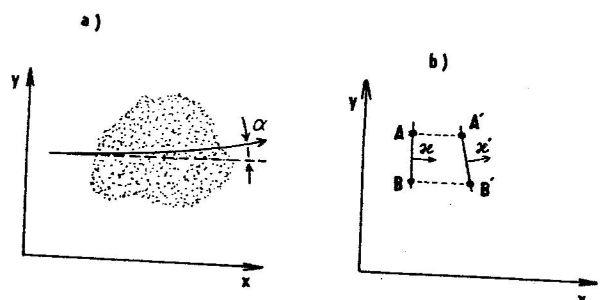

Fig. 1. The ray deflection in the plasma non-homogeneity in the two-dimensional case. a) a schematic figure b) the mechanism of deflection (A, B — two near points of the waveform)

In the three-dimensional case, the instantaneous axis of revolution (deflection) of the ray is perpendicular both to the phase velocity gradient and to the direction of the ray propagation, i.e.

$$\mathrm{d}\boldsymbol{\alpha} = (\nabla v \times \boldsymbol{\varkappa})\,\mathrm{d}t \tag{13}$$

or

$$\mathrm{d}\boldsymbol{\alpha} = -\frac{c}{\mathcal{N}^2}(\nabla\mathcal{N} \times \boldsymbol{\varkappa})\,\mathrm{d}t. \tag{14}$$

The vector $\boldsymbol{\alpha}$ is directed along the axis of deflection of the ray and its magnitude is equal to the deflection angle.

Under an infinitesimal rotation through an angle $\delta\boldsymbol{\alpha}$ the unit vector $\boldsymbol{\varkappa}$ in the direction of the ray propagation transforms according to

$$\boldsymbol{\varkappa}(t + \delta t) = \boldsymbol{\varkappa}(t) + \delta\boldsymbol{\alpha} \times \boldsymbol{\varkappa}(t). \tag{15}$$

With the aid of the previously derived eq. (14) we can show that

$$\frac{\mathrm{d}\boldsymbol{\varkappa}}{\mathrm{d}t} = -\frac{c}{\mathcal{N}^2}(\nabla\mathcal{N} \times \boldsymbol{\varkappa}) \times \boldsymbol{\varkappa}, \tag{16}$$

i.e.

$$\frac{\mathrm{d}\boldsymbol{\varkappa}}{\mathrm{d}t} = \frac{c}{\mathcal{N}^2}[\nabla\mathcal{N} - (\boldsymbol{\varkappa}\,.\,\nabla\mathcal{N})\,\boldsymbol{\varkappa}]. \tag{17}$$

The points of the ray trajeactory satisfy the eq. $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{v}$ (time is the parameter describing the trajectory), i.e.

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \frac{c}{\mathcal{N}}\boldsymbol{\varkappa}. \tag{18}$$

Having assumed $\mathcal{N}(\mathbf{x})$ to be the known function given via space distribution of the plasma concentration in the inhomogeneity and a relation (5) or (6), then the system of equations (17), (18) is the system of the ordinary differential equations of the first order for the functions $\boldsymbol{\varkappa}(t)$, $\mathbf{x}(t)$.

III. SOME OTHER QUESTIONS CONCERNING SPACE DEFLECTIONS

From the initial configuration of the ray $\mathbf{x}(t_0)$, $\boldsymbol{\varkappa}(t_0)$ we can determine with the help of the system of eqs. (17), (18) the vectors $\mathbf{x}(t)$, $\boldsymbol{\varkappa}(t)$ for every time t. The total deflection angle at the time t can be obtained from the relation

$$\cos\alpha(t) = \boldsymbol{\varkappa}(t)\,.\,\boldsymbol{\varkappa}(t_0), \tag{19}$$

i.e.

$$\alpha(t) = \arccos(\boldsymbol{\varkappa}(t)\,.\,\boldsymbol{\varkappa}(t_0)). \tag{20}$$

Let us show now that for the situation from Fig. 1 ($\partial\mathcal{N}/\partial x = \partial\mathcal{N}/\partial z = \varkappa_z = 0$; $\alpha \ll 1$) the equations of the space deflection (17), (18) give the eq. (11) for the two-dimensional case. Under the assumptions mentioned above we have from (17)

$$\frac{\mathrm{d}\varkappa_x}{\mathrm{d}t} = -\frac{c}{\mathcal{N}^2}\frac{\partial\mathcal{N}}{\partial y}\varkappa_x\varkappa_y \tag{21}$$

$$\frac{\mathrm{d}\varkappa_y}{\mathrm{d}t} = \frac{c}{\mathcal{N}^2}\frac{\partial\mathcal{N}}{\partial y}(1 - \varkappa_y^2). \tag{22}$$

In this case

$$\varkappa = (\cos \alpha, \sin \alpha, 0) \tag{23}$$

and both eq. (21) and eq. (22) give the relation

$$\frac{d\alpha}{dt} = \frac{c}{\mathcal{N}^2} \frac{\partial \mathcal{N}}{\partial y} \cos \alpha, \tag{24}$$

which for the small angles α ($\cos \alpha \approx 1$) changes into eq. (11).

Let us remark that instead of the system of the six equations (17), (18) of the first order we could use a system of three equations of the second order. Substituting $\varkappa$ from (18) into (17), we get after simple manipulation

$$\frac{d^2\boldsymbol{x}}{dt^2} = \frac{c^2}{\mathcal{N}^3} \nabla \mathcal{N} - \frac{2}{\mathcal{N}} \left(\nabla \mathcal{N} \frac{d\boldsymbol{x}}{dt} \right) \frac{d\boldsymbol{x}}{dt}. \tag{25}$$

For the numerical integration the system of eqs. (17), (18) is more suitable. Furthermore, the total deflection angle can be better determined from it.

IV. CONCLUSION

The system of differential equations for space deflections derived in this paper can be transformed into nondimensional variables and easily numerically integrated. If the space distribution $n(\boldsymbol{x})$ of the plasma concentration in the nonhomogeneity (i.e. the refractive index function $\mathcal{N}(\boldsymbol{x})$) is known, the light ray trajectory can be determined from the system of eqs. (17), (18). The system of equations presented here holds for the deflections with an arbitrary magnitude and the concentration gradient can have a quite general direction.

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ОТКЛОНЕНИЕ ЛУЧА В НЕОДНОРОДНЫХ ПЛАЗМЕННЫХ КЛАСТЕРАХ

В настоящей работе рассмотрено пространственное отклонение световых лучей в неоднородной плазме. Была выведена система дифференциальных уравнений первого порядка, описывающая прохождение лучей через плазму с определенным пространственным распределением и концентрацией. Эта система уравнений привлекательна тем, что ее можно просто численно проинтегрировать. то позволяет найти как пространственное, так и временное поведение луча.