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LYAPUNOV EXPONENTS OF THE GENERALIZED ONE-DIMENSIONAL ANDERSON MODEL

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We give the general formula for the particular sum of the Lyapunov exponents of an electron in the generalized one-dimensional Anderson model. It was used for the calculation of the weak disorder expansion of the Lyapunov exponents. The same expansion is derived from the *T*-matrix formulation of the problem.

1. INTRODUCTION

We define the generalized one-dimensional Anderson model (*GODAM*) by the Hamiltonian

$$H = H_0 + H_R, \tag{1}$$

where

$$H_0 = \sum_n \sum_{k=1}^{N} V_k \{|n+k\rangle\langle n| + |n\rangle\langle n+k|\}, \qquad V_N = 1, \tag{2}$$

and

$$H_R = \lambda \sum_n e_n |n\rangle\langle n| \tag{3}$$

represents the diagonal disorder (*DD*) with random independent energies e_n, $\langle e_n\rangle = 0$.

Model (1—3) has been studied in [1], where we used the supersymmetric representation of the Green function of the electron for the calculation of all Lyapunov exponents (*LE*) of system (1). The basic formula, from which our calculations in [1] started, however, holds only for the "smallest" *Le* γ_1. The main aim of this paper consists in the derivation of the true weak disorder expansion (*WDE*) of all *LE* γ_n of system (1). To do so, we choose two different methods: (i) We define the transfer-matrix which corresponds to the Hamiltonian (1), and formulate the problem as that of the calculation of *LE* of an infinite product of random matrices [2]. We proceed along these lines in §§ 3, 4. The form of the obtained *WDE* is very similar to that of the expansion of *LE*

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of quasi-one-dimensional systems [2, 3]. (ii) Using the properties of some antisymmetric combinations of Green functions, we derive in § 5 the general formula for the particular sums of LE. Then, by standard treatment developed in [1], we construct the graphical representation of the expansion and determine the rules of calculation of the contributions of diagrams. This method enables also the generalization of the obtained results for the case of systems with off-diagonal (ODD) disorder.

We show that although the starting formula in [1] does not hold for all LE, the discussion of the anomalies of WDE (which generalizes the study of the band-centre and the band-edge anomalies in [4—6]) remains valid; only a small correction of the coefficients of the $1/x$-expansion of $\gamma(x)$ for the case of the band-centre-anomaly is necessary.

2. THE GREEN FUNCTION

The Green function of the electron $G^0_{ab}(E+\mathrm{i}0)$ with the Hamiltonian (2) can be found in the same way as for the "classical" $1D$ Anderson model [7]

$$G^0_{ab}(E+\mathrm{i}0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{dk \exp[\mathrm{i}k|b-a|]}{E+\mathrm{i}0-E(k)} \tag{4}$$

where

$$E(k) = 2V_1\cdot\cos k + 2V_2\cdot\cos 2k + \ldots + 2\cos Nk. \tag{5}$$

The substitution $\exp \mathrm{i}k = w$ gives

$$G^0_{ab}(E+\mathrm{i}0) = \frac{-1}{2\pi\mathrm{i}}\oint \frac{dw w^{N-1} w^{|b-a|}}{(w-w_1)(w-w_2)\ldots(w-w_{2N})} \tag{6}$$

where the integration proceeds along the unit circle $|w| = 1$, and $w_1, w_2, \ldots, w_{2N}$ solve the equation

$$E(w) = E + \mathrm{i}0. \tag{7}$$

Owing to the infinitesimally small imaginary part $+\mathrm{i}0$ eq. (7) has no degenerate solutions; thus we can order all solutions of (7) as

$$|w_1| < |w_2| < \ldots < |w_{2N}|. \tag{8}$$

From (5) one easily finds

$$w_i = w^{-1}_{2N+1-i}, \qquad i = 1, 2, \ldots, N. \tag{9}$$

The integral (6) can be calculated using the Cauchy theorem and gives

$$G^0_{ab}(E+\mathrm{i}0) = \sum_{i=1}^{N} g_i(E+\mathrm{i}0) \tag{10}$$

$$g_i(E+\mathrm{i}0) = g_{q_i}(E+\mathrm{i}0) = g_{00}(E+\mathrm{i}0, q_i)\exp(\mathrm{i}q_i|b-a|)$$

with

$$w_i = \exp(\mathrm{i}q_i), \tag{11}$$

q_i, $i = 1, 2, \ldots, N$ represents the solutions of (5) with a positive imaginary part, and

$$g_{00}(E+\mathrm{i}0, q_i) = -\frac{w_i^{N-1}}{\prod_{k\neq i}(w_i - w_k)} \tag{12}$$

is the "density of states":

$$g_{00}(E+\mathrm{i}0, q_i) = -\mathrm{i}Q_i = \frac{\mathrm{i}}{\left.\dfrac{\partial E(k)}{\partial k}\right|_{k=q_i}} \tag{13}$$

3. THE T-MATRIX

The system of equations for the wave function $\Psi_n(E)$ of the electron with energy E on the site n:

$$\Psi_{n+N} + V_{N-1}\Psi_{n+N-1} + \ldots + (\lambda e_n - E)\Psi_n + V_1\Psi_{n-1} + \ldots + \Psi_{n-N} = 0 \tag{14}$$

can be rewritten into the equivalent form

$$\begin{pmatrix}\Psi_{n+N}\\ \Psi_{n+N-1}\\ \vdots\\ \Psi_{n-N+1}\end{pmatrix} = (A+\lambda B_n)\begin{pmatrix}\Psi_{n+N-1}\\ \Psi_{n+N-2}\\ \vdots\\ \Psi_{n-N}\end{pmatrix} \tag{15}$$

where

$$A = \begin{pmatrix} -V_{N-1}, & -V_{N-2}, & \ldots, & E, & -V_1, & \ldots, & -V_{N-1}, & -1\\ 1 & & & & & & & \\ & 1 & & & & & & \\ & & & \ldots & & & & \\ & & & & & & 1 & 0\end{pmatrix} \tag{16}$$

is a $2N\times 2N$ matrix with a unit determinant:

$$\det A = 1, \tag{17}$$

and B_n is $2N\times 2N$ random matrix, which has only one non-zero term

$$(B_n)_{ij} = e_n\delta_{i1}\delta_{jN}. \tag{18}$$

Formula (15) enables us to find all LE γ of system (1) from LE $\tilde{\gamma}$ of the product of random matrices

$$\prod_{n=1}^{\infty} (A + \lambda B_n) \tag{19}$$

whith matrices A, B_n given by (16, 18).

4. LYAPUNOV EXPONENTS

In this section we calculate all LE $\tilde{\gamma}$ of the product (19). To do so, we use the formula of Derrida et al. [2] (see formula (A5) in the Appendix).

First we have to diagonalize the matrix A. We look for the matrices S, T such that

$$T \cdot A \cdot S = V, \tag{20}$$

V being a diagonal matrix with eigenvalues v_i:

$$|v_1| > |v_2| > \dots > |v_{2N}|. \tag{21}$$

One easy checks that

$$v_i = w_{2N+1-i} = w_i^{-1}. \tag{22}$$

Matrix S has the form

$$S_{ij} = v_j^{2N-i}. \tag{23}$$

In the new representation matrix B_n reads

$$(B_n)_{ij} = e_n T_{i1} S_{Nj}, \tag{24}$$

so that we need only the first column of matrix T. As

$$T \cdot S = S \cdot T = 1 \tag{25}$$

it is enough to solve the system of the linear equations

$$\sum_i S_{ki} T_{i1} = \delta_{k1},$$

which gives

$$T_{i1} = \frac{1}{\prod_{k \neq i} (v_i - v_k)} \tag{26}$$

Using (13, 22) we obtain

$$\begin{aligned} {}_{i1} &= -i \cdot Q_i v_i^{1-N} && i \leqslant N \\ &= +i \cdot Q_{2N+1-i} v_{2N+1-i}^{N-1} && i > N \\ S_{Nj} &= v_j^N && j \leqslant N \\ &= v_{2N+1-j}^{-N} && j > N \end{aligned} \tag{27}$$

Relations (27) together with (18) determine the matrix B_n. After substitution into (A5), we obtain the WDE of the LE of the product (19) in the form

$$\begin{aligned} \tilde{\Gamma}_p &= \tilde{\gamma}_1 + \tilde{\gamma}_2 + \dots + \tilde{\gamma}_p = -i(q_1 + \dots + q_p) + \\ &+ \frac{\lambda^2 \langle e^2 \rangle}{2} \sum_{ij=1}^{p} Q_i Q_j - \frac{\lambda^4 \langle e^4 \rangle}{4} \sum_{ijkm=1}^{p} Q_i Q_j Q_k Q_m - \\ &- \frac{\lambda^2 \langle e^2 \rangle^2}{2} \sum_{ij=1}^{p} \left\{ \sum_{km=p+1}^{N} \frac{v_k v_m}{v_i v_j - v_k v_m} Q_i Q_j Q_k Q_m - \right. \\ &- \sum_{k=p+1}^{N} \sum_{m=1}^{N} \frac{Q_i Q_j Q_k Q_m}{v_i v_j v_m / v_k - 1} + \left. \sum_{k,m=1}^{N} \frac{Q_i Q_j Q_k Q_m}{v_i v_j v_k v_m - 1} \right\} + \\ &+ \lambda^4 \langle e^2 \rangle^2 \sum_{ijk=1}^{p} \left\{ \sum_{m=p+1}^{N} \frac{Q_i Q_j Q_k Q_m}{v_i / v_m - 1} - \right. \\ &- \left. \sum_{m=1}^{N} \frac{Q_i Q_j Q_k Q_m}{v_j v_m - 1} \right\}. \end{aligned} \tag{28}$$

Expansion (28) represents the WDE of the first p LE of the product of random matrices, and so $\tilde{\gamma}_1, \dots, \tilde{\gamma}_p$ represent the LE with the p largest positive real part. Thus, each $\tilde{\gamma}_i$ corresponds with the LE of the Green function (10) γ_i as

$$\tilde{\gamma}_i = -\gamma_{N+1-i} \tag{29}$$

since $|\mathrm{Re}\,\gamma_1| < |\mathrm{Re}\,\gamma_2| < \dots$.

Expansion (28) is very similar to the WDE of the LE of the quasi-one-dimensional system [2, 3] (of course, some differences arise owing to the structure of the Hamiltonian). For the quasi-one-dimensional systems we have derived the WDE of the LE using a supersymmetric representation of the Green function [3], which provided the graphical representation of all terms of the expansion. The similarity of both expansions indicates that it is possible to derive (28) also from the graphical expansion, similar to that presented in [1, 3]. We find such an expansion in the next paragraphs.

5. GENERAL FORMULA FOR THE *LE*

In the previous sections we found the WDE of all LE of $GODAM$ to be similar with the WDE of the LE of quasi-one-dimensional systems, as derived in [2, 3]. This similarity enables us to suppose that we can easily generalize the supersymmetric treatment developed in [1] also for $GODAM$.

To do so, let us study the expression

$$S_2^0 = G^0_{a_1b_1}G^0_{a_2b_2} - G^0_{a_1b_2}G^0_{a_2b_1} \tag{30}$$

with $G_{x_1x_2}$ given by (10), and $a_i = a + \alpha_i$, $b_i = b + \beta_i$, $|\alpha_i|$, $|\beta_i| \ll |b-a|$. For $\alpha_1 = \beta_1 = 0$, $\alpha_2 = \beta_2 = \Delta$ one easily finds

$$S_2^0 \sim \exp\{\mathrm{i}(q_1+q_2)|b-a|\}\cdot Q_1\cdot Q_2\cdot 4\cdot\sin^2\left[\frac{q_1-q_2}{2}\Delta\right] + \ldots \tag{31}$$

where the dots supply the terms which, owing to (8, 11) decrease much more quickly in the limit $|b-a| \to \infty$. Thus,

$$\frac{1}{|b-a|}\log S_2^0 \xrightarrow[|b-a|\to\infty]{} \mathrm{i}(q_1+q_2). \tag{32}$$

We can generalize (32) as follows:
Let us construct

$$S_p^0 = \sum_\omega (-1)^{r_\omega} G^0_{a_1b_{k_1}}\cdot G^0_{a_2b_{k_2}}\cdot\ldots\cdot G^0_{a_pb_{k_p}} \tag{33}$$

where ω is the permutation

$$\omega: (1, 2, \ldots, p) \to (k_1, k_2, \ldots, k_p) \tag{34}$$

and r_ω is the number of changes of the positions of numbers (i, j), which ω defines. Then after a suitable choice of α_i, β_i one can prove that S_p^0 is linear in g_i (i.e. it does not contain terms with g_i^μ, $\mu > 1$). Then

$$\mathrm{i}(q_1+q_2+\ldots+q_p) = \lim_{|b-a|\to\infty}\frac{1}{|b-a|}\log S_p^0. \tag{35}$$

If the disorder is taken into account, we can formally express the Green function in the form

$$G_{ab} = \sum_n \exp\{\mathrm{i}\cdot q_n(b-a)\}\cdot\sum_m T_{nm} \tag{36}$$

with

$$T_{nm} = Q_nQ_mA_{nm}\exp\{\mathrm{i}(q_n-q_m)a\}. \tag{37}$$

Here A_{nm} contains the disorder together with all necessary summations. As T_{nm} does not contain $(b-a)$, we can generalize (35) as

$$\Gamma_p = \gamma_1+\gamma_2+\ldots+\gamma_p = \lim_{|b-a|\to\infty}\frac{1}{|b-a|}\langle\log S_p\rangle, \tag{38}$$

(note that there is, in agreement with (29), a difference in the numbering of γ_i, $\tilde{\gamma}_i$), where

$$S_p = \sum_\omega (-1)^{r_\omega}\prod_{i=1}^p G_{a_ib_{k_i}}. \tag{39}$$

The formulae (38, 39) provide the starting point for the construction of the *WDE*.

6. GENERALIZATION OF THE *WDE*

From (38) we can construct the *WDE* of the *LE* in a similar way as in [3]. First, we rewrite (38) as

$$\Gamma_p = \lim_{|b-a|\to\infty}\frac{1}{|b-a|}\frac{\mathrm{d}}{\mathrm{d}n}\langle S_p^n\rangle|_{n=0}, \tag{40}$$

which enables us to construct the graphical expansion of *WDE*: Each diagram consists of n groups of p lines; each line represents a Green function, and in each group the summation over all ω (34) should be performed, which secures that in the expansion terms with g_j^μ, $\mu > 1$ vanish in each group. Owing to (40), only contributions proportional to $n(b-a)$ are of interest, $\sim(b-a)^0$, n^2, etc. may be omitted. One can check that the terms $\sim(b-a)^2$ together make the contribution $\sim n^2$ and so vanish too.

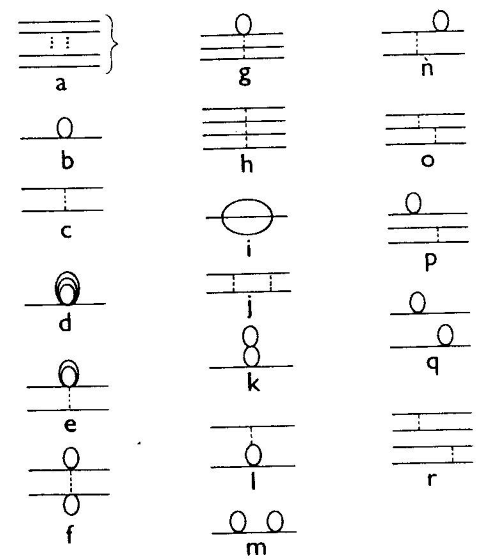

Fig. 1.

In Fig. 1 we present all diagrams of *WDE* up to the 4th order in disorder. For simplicity, we have described only the lines with "interactions". The construction of diagrams has been explained in [1] — here we only recall that the "two-particle interaction vertex" diagrams 1b, c represents $\lambda^2\langle e^2\rangle$. The "four-particle vertex" stands for $\lambda^4\langle e^4\rangle$, etc. Careful considerations lead, in analogy with the case of quasi-one-dimensional systems, to the following rules for the calculation of the contributions of diagrams:

1) the sign of the k-particle diagram is $(-1)^{k+1}$
2) the inner lines in diagrams should be represented by any g_m, $m = 1, 2, \ldots, N$
3) the external lines should be occupied by any g_m, $m \leqslant p$ in such a way that the contribution of the diagram is proportional to $(b-a)$. One has, however, to be careful and eliminate the terms $\sim n^2$. Thus, in diagram c one can occupy the four lines (from left to right and from top to bottom) by m_1, m_2, m_2, m_1, $m_1 \neq m_2$ (if both lines belong to the same group — the diagram is $\sim n$), or m_1, m_1, m_1, m_1 (the lines belong to different groups, and the contribution is proportional to $n(n-1)$), but not by m_1, m_1, m_2, m_2, $m_1 \neq m_2$, since now there is n^2 such a contribution
4) summation over all permutations ω, all positions of interaction points and all representations of lines should be performed.

In this way we can construct the *WDE* of the sum Γ_p of the first p *LE* with the smallest negative real parts (they coreespond to the $\mathrm{i}q_1, \ldots, \mathrm{i}q_p$ from (11)). As the resulting form of the expansion is rather complicated, we do not present it here, since, owing to (29), we have

$$\Gamma_p = -\tilde{\Gamma}_N + \tilde{\Gamma}_{N-p}. \tag{41}$$

According to rule (2) any diagram in Fig. 1 contributes to Γ_p if the lines which start from a_i are represented by $g_1, \ldots, g_p$. If one calculates Γ_{p-1}, then the external lines should be represented only by $g_1, \ldots, g_{p-1}$. As $\gamma_p = \Gamma_p - \Gamma_{p-1}$, we find that one can construct from the diagrams in Fig. 1 also the *WDE* of any γ_p if the rule (2) is modified as follows:

2a) the external lines should be occupied by any g_m, $m \leqslant p$, but at least one external line should be represented by g_p.

7. APPLICATION

First we present the *WDE* of γ_p of model (1) with *DD*. In the 2nd order approximation we need only the diagrams b, c and obtain

$$\gamma_p = \mathrm{i}q_p - \frac{\lambda^2\langle e^2\rangle}{2} Q_p\left\{2\cdot\sum_{i=p+1}^{N} Q_i + Q_p\right\}. \tag{42}$$

As pointed out in [1, 3], the graphical expansion can be used also for systems with any off-diagonal disorder. Thus replacing H_R in (3) by

$$H_R' = \Lambda\sum_n v_n\{|n+1\rangle\langle n| + |n\rangle\langle n+1|\} \tag{3'}$$

we find

$$\gamma_{p,\,ODD} = \mathrm{i}q_p - 2\cdot\Lambda^2\langle v^2\rangle\left\{\sum_{i=1}^{N}(1+\exp(\mathrm{i}q_i)\cdot\cos q_p)\cdot Q_iQ_p - 2\cdot\sum_{i=1}^{p}\cos^2\left(\frac{q_i+q_p}{2}\right)\cdot Q_iQ_p + \cos^2 q_p Q_p^2\right\}. \tag{43}$$

Details of the calculation of diagrams with the *ODD* can be found in [1].

Now let us verify the discussion of the anomalies of the *WDE* presented in [1].

1) Anomalies caused by the divergency of Q_i in (13). They arise in the extremal points q_{ex} of the dispersion relation (5), where

$$\left.\frac{\partial E(k)}{\partial k}\right|_{k=q_{ex}} = 0 \tag{44}$$

($q = 0$, $= \pi$, for instance). Now the most divergent diagrams, which contribute to the leading term of γ_p, are those with all lines occupied by $g_{q_{ex}}$ (10), as supposed in [1]. Thus, the analysis of these anomalies, as given in [1], does not need any correction.

2) Anomaly $q = \pi/2$. Now diagrams i, j, which make a contribution proportional to $\lambda^4\langle e^2\rangle^2/[\exp(-\mathrm{i}4q)-1]$ diverge. One can construct the $1/x$ expansion of the function $f(x)$:

$$\gamma(q=\pi/2, x) = \mathrm{i}q - \lambda^2\langle e^2\rangle Q_q^2 f(x), \tag{45}$$

$$x = (E - E(\pi/2))/(\lambda^2\langle e^2\rangle Q_q) \tag{46}$$

using the same diagrams as in [1]. The only difference consists in the occupation of the inner lines of the six- and higher-order diagrams; thus, instead of the expression (30) in [1] one should write

$$\gamma(q=\pi/2) = \mathrm{i}q - \lambda^2\langle e^2\rangle Q_q^2\left[-A + 1/2 - \mathrm{i}x - \mathrm{i}/8x + \frac{1}{32x^2}(8 + A + 3B)\right] \tag{47}$$

where $A = \sum_{i\geqslant p}^{N} Q_i/Q_q$, $B = \sum_{i=1}^{N} Q_i/Q_q$.

8. CONCLUSION

Let us briefly summarize the obtained results:
In the first part of the paper we have transformed the problem of the calculation of the Lyapunov exponents of the generalized one-dimensional Anderson model into the same problem for the infinite product of random matrices. Then, using the formulae of Derrida et al. [2] we have derived the weak disorder expansion of particular sums of the *LE* (formula (28)) up to the 4th order in disorder.

In the second part, we have derived the exact formulae for the particular sum of the *LE* for any Hamiltonian, the Green function of which is given as a sum of exponentials (10). The antisymmetric formulation (33) enables us to eliminate the product $g_1 \dots g_p$ instead of the undersirable terms g_1^p, $g_1^{p-1} g_2$, etc. Then, using the standard techniques, presented in [1, 3], we have constructed the graphical expansion of the *LE*, which is equivalent to that found by the *T*-matrix method (28).

The possibility of the construction of a graphical expansion has two important consequences: (i) it can be used also for the off-diagonal disorder, and (ii) the higher order terms of the expansion can be easily constructed. As an application of the graphical expansion, we derive the leading term of γ_p for systems with the *DD* and the *ODD*, and analyse the anomalies of expansion, first discussed in [1].

The results presented in this paper modify some ideas presented in [1]; namely, as it can easily be shown in (38), the formula (7b) in [1], proposed for the calculation of any *LE*, works only for the smallest *LE* γ_1. Nevertheless, the analysis of the anomalies of the *WDE*, as given in [1], needs only small, non-essential corrections.

We believe that the antisymmetric combination of the Green functions, proposed in § 5, can find further applications to different physical problems.

APPENDIX

LYAPUNOV EXPONENTS OF THE PRODUCT OF RANDOM MATRICES

Let us consider the product of the random matrices

$$p = \prod_{n=1}^{\infty} M_n \qquad \text{(A1)}$$

where

$$M_n = A + \lambda B_n \qquad \text{(A2)}$$

A is a diagonal matrix with nondegenerate eigenvalues $v_1, v_2, \dots, v_M$:

$$|v_1| > |v_2| > \dots > |v_M| \qquad \text{(A3)}$$

and B_n is a random matrix,

$$\langle (B_n)_{ij} \rangle = 0. \qquad \text{(A4)}$$

Derrida et al. [2] derived for the particular sum of the *LE* of the product (A1) the following *WDE*:

$$\tilde{\Gamma}_p = \tilde{\gamma}_1 + \tilde{\gamma}_2 + \dots + \tilde{\gamma}_p = \sum_{i=1}^{p} \log v_i - \frac{\lambda^2}{2} \sum_{ij=1}^{p} \frac{\langle B_{ij} B_{ji} \rangle}{v_i v_j} + \frac{\lambda^3}{3} \sum_{ijk=1}^{p} \frac{\langle B_{ij} B_{jk} B_{ki} \rangle}{v_i v_j v_k} - \frac{\lambda^4}{4} \sum_{ijkm=1}^{p} \frac{\langle B_{ij} B_{jk} B_{km} B_{mi} \rangle}{v_i v_j v_k v_m} - \frac{\lambda^4}{2} \sum_{ij=1}^{p} \sum_{km=p+1}^{M} \frac{\langle B_{ik} B_{jm} \rangle \langle B_{kj} B_{mi} \rangle}{v_i v_j (v_i v_j - v_k v_m)} + \lambda^4 \sum_{ijk=1}^{p} \sum_{m=p+1}^{M} \frac{\langle B_{im} B_{ki} \rangle \langle B_{mj} B_{jk} \rangle}{v_i v_j v_k (v_k - v_m)}. \qquad \text{(A5)}$$

REFERENCES

[1] Markoš, P.: J. Phys. C *21* (1988), 2647.
[2] Derrida, B., Mecheri, K., Pichard, L.: J. Physique *48* (1987), 733.
[3] Markoš, P.: J. Phys.: Condensed Matter — will be published.
[4] Derrida, B., Gardner, E.: J. Physique *45* (1984), 1283.
[5] Kappus, M., Wegner, F.: Z. Phys. B *45* (1981), 15.
[6] Kirkman, P. D., Pendry, J. B.: J. Phys. C *17* (1984), 5707.
[7] Economou, E. N.: *Green's Functions in Quantum Physics*, 2nd ed. Springer Verlag, Berlin.

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ЭКСПОНЕНТЫ ЛЯПУНОВА В ОБЩЕЙ ОДНОМЕРНОЙ МОДЕЛИ АНАЕРСОНА

Получена общая формула для сумм экспонент Ляпунова (ЭЛ) электрона в общей одномерной модели Андерсона. Она используется для построения разложения ЭЛ в степенях беспорядка. Полученное разложение эквивалентно разложению, полученному из Т-матричной формулировки проблемы.