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SYNERGETIC STRUCTURALISATION OF MATTER FROM THE GASEOUS STATE IN AN EXPANDING UNIVERSE

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The equation of evolution for the density of matter in an expanding universe is derived in this paper. The theory is based on the assumption that the formation of matter-structures (galaxies and stars) starts from a gas-like material, which is in a hydrodynamical motion due to Hubble's velocity. The influence of gravitation, rotation, diffusion and the scattering of particles due to thermal motion are taken into account.

It is shown that the equation of evolution has two bifurcation points: one of them corresponds to the formation of galaxies and the other to the formation of stars. The critical mass of galaxies and stars is determined by the formula which is practically identical with the well-known Jeans formula. Moreover our approach allows to calculate the critical time of the structuralisation of matter in an expanding universe, to explain the shape of galaxies and potentially also the mass spectrum of galaxies and stars.

1. INTRODUCTION

The problem of the structuralisation of matter in the universe has not been plausibly solved in general until recently. Practically all recent theories of the formation of galaxies are based on the Jeans theory [1] assuming the universe to be filled with non-relativistic fluid. The main result of this theory is the discovery that an instability in the uniform distribution of matter arises at a critical mass resulting in an exponential growth of the perturbation in time. Lifshitz [2] extrapolated this theory to the expanding universe and found that the perturbation in these circumstances grows not exponentially but with a given power of time. The discrepancy of the critical mass calculated by Jeans with the observation was partially removed by the assumption that the fluctuations corresponding to small values of the mass of the generated galaxies are damped [3—5].

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Today there exists a general relativistic theory of small perturbations [6, 7]. Problems of the fluctuations in the very early universe are considered by Carr [8], Rees et al. [9] and many others. The theory of the structuralisation of matter in the standard model of the universe was essentially developed by Zeldovich and his coworkers (see, e.g., [10—12]). Zeldovich himself is the author of the known theory of *"pancakes"*, which explains the dispersion of galaxies in the universe in a good agreement with observations.

Surprisingly, until recently the diffusion of gas molecules has not been taken into consideration, although it is evident that the matter of the universe in certain periods had an entirely fundamental behaviour of the (ideal) gas and, therefore, the diffusion could take place in the process of the formation of galaxies and stars.

It was understood in the last years (see, e.g., [13—15]) that it is the diffusion which can generate temporal and spatial structures under special circumstances in an originally uniform system. A new physical discipline (*"synergetics"*) is oriented to the problem of the origin of structures in uniform media, hence we can say in this sense that the problem of the formation of galaxies and stars is a synergetic one. We shall try to demonstrate this approach to the solution of the mentioned problem taking into account also processes of diffusion.

It follows from the Jeans theory why structuralisation was an unavoidable epoch in the evolution of a large cosmic cloud. Many problems, however, remain unsolved, e.g. the problem of the mass spectrum of galaxies (about 6 orders), the problem of the upper limit of the masses of galaxies (the Jeans theory gives in fact only the lower limit), the problem of the characteristic shapes of galaxies, the problem of the structuralisation of galaxies into stars and last but not least the problem of when the structuralisation taken place. We shall try to demonstrate that the theory based on the gaseous nature of a primary cosmic material gives (in given circumstances) the critical mass of galaxies identical with the Jeans result. Our theory offers, however, also non-trivial information related to the problems mentioned above.

2. THE EQUATION OF EVOLUTION

The principle of the synergetic method of the study of structures is the selection of the other parameter, the derivation of the equation of evolution and the mathematical analysis of its solution. Let us select the density of the matter (s) as an order parameter and suppose this material to be in the gaseous state[1]. For the sake of simplicity we shall consider only the atoms of one kind and

[1] To prove this assumption we discuss briefly the situation in the universe after the recombination of protons and the helium nucleus with free electrons at the end of the radiation-dominated era.

regarding the relatively low temperature we shall suppose that the neutral gas can be in the form of biatomic molecules.

The non-thermal velocity (v) of particles hat two components (we ignore at the beginning the rotation of the cloud): the Hubbles velocity (v_H) and the velocity due to gravitation (v_g), i.e.

$$v = v_H + v_g. \tag{2}$$

To find simple expressions for these terms we suppose the primary gaseous cloud to have approximately the shape of a sphere with the mass much greater than the critical Jeans mass. A particle in the point determined by a radius r (related to the centre of this sphere) has the Hubble velocity

$$v_H = Hr, \tag{3}$$

where H is the Hubble constant and the mean value of the gravitational part of the velocity can be determined by the relation

$$v_g \approx a\tau = E\tau, \tag{4}$$

where a is the gravitational acceleration (identical with the intensity of the gravitational field E) fulfilling the equation

$$\operatorname{div} E = -4\pi G s, \tag{5}$$

where G is the gravitational constant, τ is the relaxation time.

The flow corresponding to the hydrodynamical velocity v is $j_h = sv$ and the flow corresponding to the diffusion is $j_d = -D \operatorname{grad} s$, D being the coefficient of

The fundamental characteristics of the matter at the beginning of the matter-dominated era were probably the following ones:

composition: the radiation +92% H and 8% He,
density: $s \approx (10^{-19}$—$10^{-18})\ \mathrm{kg}^{-3}$,
concentration: $n \approx (10^{8}$—$10^{9})$ at. H/m^3,
temperature: $T \approx 4000$ K.

Using the well-known formulae from the kinetic theory of ideal gas it is easy to calculate the mean thermal velocity (v_T), the mean free length (L) an the mean free time (τ) characteristic for the free motion of hydrogen molecules: $v_T \approx 4 \,.\, 10^3$ m/s, $L \approx (10^{15}$—$10^{16})$ m and $\tau \approx (10^{11}$—$10^{12})$ s. To get the result in agreement with the observation we used the effective scattering cross-section of the hydrogen—hydrogen collision processes in the expression

$$L \approx \frac{1}{n\sigma} \tag{1}$$

the value $\sigma \approx 10^{-24}\ \mathrm{m}^2$. Regarding the dimensions of galaxies ($R \approx 10^{18}$—10^{20} m) and the time-scale characteristic for the evolution of the universe ($\sim 10^6$ years) we can assume that the hydrogen matter in the circumstances given above has a well-defined behaviour as an ideal gas.

the diffusion. For each closed area within the cloud the law of the conservation of the total number of particles must be fulfilled (we asume that no nuclear processes have taken place in the system), thus

$$\int (j_d + j_h) \,.\, \mathrm{d}S = -\frac{\partial}{\partial t}\int s \,\mathrm{d}V,$$

where $\mathrm{d}S$ is the plane a $\mathrm{d}V$ the volume element. Using the above relations and applying the Gauss theorem we get the relation

$$\frac{\partial s}{\partial t} = D\Delta s + \operatorname{grad} D \,.\, \operatorname{grad} s - (3H - A)s - Hr \,.\, \operatorname{grad} s, \tag{6}$$

where

$$A = \frac{\pi G m_0^{3/2}}{\sigma (5kT)^{1/2}}, \tag{7}$$

m_0 is the mass of molecules. Considering the fact that the coefficient of diffusion is in general a function of density, this equation is a complicated non-linear differential equation. Our aim is only to find a bifurcation point in which the structuralisation starts, therefore we can suppose that the density as well as the coefficient of diffusion are practically constants. In this case the equation (6) transforms into the form

$$\frac{\partial s}{\partial t} = D\Delta s - (3H - A)s - Hr \,.\, \operatorname{grad} s, \tag{8}$$

which will be the starting equation of evolution for the mathematical analysis.

In fact, parameters H and A in the equation (8) are not constants, too. The Hubble constant depends on the mean density of the universe, the parameter A depends on the temperature, which changes in the expanding universe. However, these changes are very slow in comparison with the time period in which qualitative transformations in the universe are realized, therefore we can solve the equation (8) postulating that the parameters H and A are also constant. On the basis of this assumption (identical with the assumption that the number density of particles changes very little in space and in time) all fundamental parameters are determined by the density s_0 of the unperturbed system.

3. THE FORMATION OF GALAXIES

Equations of type (8) were very often analysed in physics. Kerner and Osipov proved in many papers (see, e.g., [16] and [17]) that a great variety of structures (e.g. solitons) can be produced in systems described by similar equations. We shall show now that a bifurcation point can be really expected in the system with the equation of evolution (8). In this point the density characterized by a constant or by a monotonous function rapidly changes into the state described by a spatial periodic function. We denote the solution of the equation (8) corresponding to the quasistationary state at some time as s^x. Let us suppose now that a perturbation $s' = s - s^x$ of the density arises in the system which enlarges with the velocity v_p in directions $+r$ or $-r$. Regarding the radial-symmetry of our system we can expect that this perturbation could be described by the function

$$s'(R, t) = \frac{1}{r} f(v_p t - r) = \frac{1}{r} f(w), \qquad w = v_p t - r. \tag{9}$$

For all $r < v_p/H$ the function f(w) is the solution of the equation

$$\frac{d^2 f}{dw^2} - \frac{v_p}{D}\frac{df}{dw} - \frac{2H - A}{D} f = 0. \tag{10}$$

It can be written in the form

$$f(w) = A_1 e^{\alpha_1 w} + A_2 e^{\alpha_2 w}, \tag{11}$$

where

$$\alpha_{1,2} = \frac{1}{2}\left\{\frac{v_p}{D} \pm \sqrt{\left(\frac{v_p}{D}\right)^2 + 4\frac{2H - A}{D}}\right\}.$$

We shall perform now a further although not necessary simplification. Let us suppose $(v_p/D)^2 \ll 4(2H - A)/D$. As we shall see later only the case in which $A \gg 2H$ will be interesting, hence it is necessary to fulfill the condition $v_p < 2\sqrt{AD}$. The velocity v_p can be identified with the velocity of the sound — in the situation defined above it is about 10^4 m/s, thus both inequalities ($rH < v_p$ and $v_p < 2\sqrt{AD}$) can be easily fulfilled. In this case

$$\alpha_{1,2} \approx \pm\left(\frac{2H - A}{D}\right)^{1/2}.$$

If $2H > A$, the solution (11) is real. If $2H < A$, the situation radically changes. In the latter case $\alpha_{1,2} \approx \pm ip$, where $p = \sqrt{(A - 2H)/D}$ and the function (11) gets the form

$$f(w) = A_1 e^{ipw} + A_2 e^{-ipw} = C\cos(pw + \varphi), \tag{12}$$

where C and φ are constants. This solution means that a new quality arises in the system. An originally homogeneous system with radical symmetry disintegrates into radial bands with the width λ determined by the condition $p\lambda \approx 2\pi$. These bands — except the first — will disintegrate in the course of the evolution

into globular clusters with the diameter $d \approx \lambda$. Of course, this concluston does not follow from our calculation because it has been deduced to small deviations from the constant density s_0 only. Before we try to determine the diameter d of the clusters, we must find if the bifurcation can really take place in the evolution of a cosmic cloud.

It is known that Hubble constant is approximately given by the reciprocal value of the time from Big Bang and therefore its value monotonously decreases with time. The constant A (7) increases with time due to the dependence $\sim T^{-1/2}$ on temperature. (The temperature of the expanding universe decreases with time.) It follows from these facts that there exists a critical time (the first is $-t_{cl}$)

$$t_{cl} \approx \frac{1}{H} \tag{13}$$

resulting from the condition $2H = A$ before which the solution (11) is monotonous, stationary and stable. After this critical time the solution (11) qualitatively changes and gets the form (12). According to the relation (7) this critical time is defined by the formula

$$t_{cl} \approx \frac{2\sqrt{5}\,\sigma\sqrt{kT}}{\pi G m_0^{3/2}}. \tag{14}$$

The numerical calculation needs the value of the temperature and of the scattering cross-section σ for molecules. Applying this result to the early universe (the epoch after the *"recombination"*) we get the value of $t_{cl} \approx 10^5$—10^6 years, which is not an unrealistic value.

The most interesting result following from the solution (12) is the fact that for $2H < A$ a spatial periodical distribution of density arises. The originally homogeneous systems transform into the system with spatial clusters of a constant radius R. If we use the same definition of this radius as Jeans used ($R \approx \lambda/2 = \pi/p$), we get the formula ($D = v_T L/3$)

$$R \approx \left(\frac{5\pi kT}{3Gm_0 s_0}\right)^{1/2}. \tag{15}$$

The critical mass related to this critical radius is approximately

$$M_{cl} \approx \frac{9}{2}\left(\frac{\pi}{3}\right)^{5/2}\left(\frac{5kT}{G}\right)^{3/2} n_0^{-1/2} m_0^{-2}, \tag{16}$$

which is practically identical with the Jeans formula. (The numerical coefficient in the Jeans formula is $4(\pi/3)^{5/2}$.)

It is possible to summarize this result in the way that each super-critical system undergoes in the expanding universe in the time evolution at the critical time a process of the structuralisation resulting in the generation of galaxies with the mass defined by the Jeans formula. The main causes of such a structuralisation are gravitation, Hubble expansion an the diffusion of gas molecules.

4. THE INFLUENCE OF ROTATION — THE THEORY OF *"PANCAKES"*

Rotation generates the Coriolis and the centrifugal forces in the system corresponding to the acceleration

$$a = -2\omega \times v - \omega \times (\omega \times r), \tag{17}$$

where ω is the angular frequency. It is necessary to incorporate this formula into the relation (4). The influence of the first term in (17) on the dynamics of particles was analysed in papers [18] and [19]. It was found that the spiral-like clusters can arise in this case in convenient circumstances. If the necessary conditions are not fulfilled the cluster remains a spherical or eliptical one. We think that the same situation is in the case of the rotation of a primary cosmic cloud.

It is not very difficult to show that the influence of the second term in (17) results in the localization of clusters in space having the shape of a pancake. In the neighbourhood of poles equation (8) does not change, but in the neighbourhood of the equator it transforms into

$$\frac{\partial s}{\partial t} = D\Delta s - (3H + \omega^2\tau - A)\, s - Hr \,.\, \mathrm{grad}\, s. \tag{18}$$

Using the formal substitution $3H + (\omega^2\tau) \to 3H'$ it is possible to reduce this problem to the problem without rotation. Then it follows from the analysis that the critical time of a bifurcation determined by the condition $2H' = A$ now is greater than the time t_{c1} defined by the relation (14), i., e. $t'_{c1} > t_{c1}$. The times t_{c1} and t'_{c1} represent the times in which the hydrodynamical velocities of particles related to the centre of the cloud become zero — the expansion totally compensates the contraction. An interesting result follows — the structuralisation takes place at first in the region near the poles and later in the surroundings of the equator, where the expansion continues. The form of a primary cloud prolongates in the equatorial plane and new galaxies occupy their places in the space which has the shape similar to a pancake. Respecting the fact that the parameters H and A in this process change we come to the conclusion that the masses of the produced galaxies need not be equal.

5. THE FORMATION OF STARS

It follows from equation (8) or (18) that a second bufurcation point in the time evolution of the cloud is not to be expected, because in the following time

period the inequality $2H < A$ or $2H' < A$ is always fulfilled. To make it possible it would be necessary to postulate a new physical action resulting in the reversion of these inequalities. Such an action really appeared in the time evolution of the galaxies — it was rotation. The rotation of galaxies must be much greater than the rotation of the primary cloud from which the galaxies were produced. If a (spherical) system of the radius R rotating with the angular frequency ω disintegrates into many (equal) spheres of the radius r, the mean angular frequency of these spheres must be — due to the law of the conservation of the angular momenta — $(R/r)^2$ — times greater. Therefore, it is clear that in the galaxies *"in statu nascendi"* the inequality

$$3H + \omega_0^2\tau > A \tag{19}$$

is fulfilled. A further evolution of each galaxy must be the same as the evolution of a primary cloud, therefore, the results derived in the previous case remain plausible also for the galaxies. It is necessary only to replace $\omega \to \omega_0$. There exists the second bifurcation point t_{c2} defined by the relation $2H' = A$ after which the galaxy disintegrates into smaller clusters identical with stars. It is evident that the shape of each galaxy is determined first of all by the angular frequency and can be spherical. eliptic or spiral.

Considering the relations (9) and (12) one can expect that the biggest cluster arises in the centre of the galaxy (at $r \to 0$). It can be — due to a very high density — a black hole. This result is in a very good agreement with theories of the structures of our galaxy.

It is possible to calculate approximately the masses of stars. The value of Hubble constant in the time period of the stars formation was approximately $10^{-16}\,\mathrm{s}^{-1}$, the angular frequency ω_0 (deducing from today's approximate values related to the galaxies with the mass of $(10^5$—$10^6)\, M_s$, M_s being the mass of the Sun) could be $(10^{-11}$—$10^{-12})\,\mathrm{s}^{-1}$, so (at $\tau \approx 10^{11}$ s) there is always $H \ll \omega_0^2\tau$. Thus the problem of the formation of stars reduces into the problem of the formation of galaxies. The critical time t_{c2} is therefore defined by the condition

$$\omega_0^2\tau = A. \tag{20}$$

Using the relation (7) we find the critical density s'_{0c} related to the second bifurcation point

$$s'_{0c} \approx \frac{\omega_0^2}{4\pi G}. \tag{21}$$

The critical mass of the new clusters is defined by the relation (16), too. It is seen that this second critical mass is smaller than the first one by the coefficient

$$K \approx \left(\frac{T_{c1}}{T_{c2}}\right)^{3/2} \left(\frac{s'_{c0}}{s_0}\right)^{1/2}.$$

Supposing the temperature in the time period of the stars formation to be 10-times smaller than in the time of the galaxies formation we get the value $K \approx 10^4$. So, clusters with the critical mass equal approximately to $1 M_s$ are produced within the galaxy. These clusters are identical with stars.

6. CONCLUSION

It has been shown that the structuralisation of matter in the expanding universe can start from the gaseous state. The model based on the existence of a primary gaseous cloud gives more information than the theory based on the fluid-like primary matter. It is possible to expect even better results when one finds the solution of the derived equation of evolution without the approximations made in this paper.

Lately the solution of the fundamental equation of evolution (6) with respect to the dependence of the coefficient of diffusion on the density and without approximations performed in this paper has been found [20]. It can be stated that all conclusions given in this paper are correct, only the structuralisation does not start from a homogeneous state (with the density s_0), but from a nonhomogeneous one determined by the function $s \sim \exp[a/r]$. This is a very important result — the new system (e.g. a galaxy) must be characterized by a nonhomogeneous distribution of the matter from the centre to the boundaries according to the function $\exp[a/r]$. This is in very good agreement with the observation in our galaxy.

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СИНЕРГЕТИЧЕСКАЯ СТРУКТУРАЛИЗАЦИЯ МАТЕРИИ ИЗ ГАЗОВОГО СОСТОЯНИЯ В РАСШИРЯЮЩЕЙСЯ ВСЕЛЕННОЙ

В настоящей работе выведено уравнение эволюции для плотности материи в расширяющейся Вселенной. Приведенная в работе теория основана на предположении, что формирование материальных структур, таких, как галактики и звезды, начинается из газообразного состояния материи, которая находится в гидродинамическом движении, обусловленном законом Хаббла. Теория учитывает влияние гравитации, вращения диффузии и рассеяние частиц, связанное с тепловым движением.

Показано, что уравнение эволюции имеет две сингулярные точки: одна из них соответствует формированию галактик, вторая формированию звезд. Критическая масса галактик и звезд определяется соотношением, которое практически идентично хорошо известному соотношению Джинса. Более того, наш подход позволяет вычислить критическое время структурализации материи в расширяющейся Вселенной и объяснить форму галактик и потенциально также спектр масс галактик и звезд.