EFFECT OF HALL CURRENTS IN THE HYDROMAGNETIC CONVECTIVE FLOW THROUGH A HORIZONTAL CHANNEL

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The object of the present paper is to study the effect of the Hall currents on the hydromagnetic free and forced convection flow through a horizontal channel with porous and moving conducting walls. Assuming the wall temperature to vary linearly with distance, closed form solutions are obtained for velocity, induced magnetic field and temperature. The behaviour of the velocity and the magnetic field is shown graphically.

I. INTRODUCTION

The free convective flow of a conducting fluid past a semi-infinite plate has been considered by Gupta [1] and others [2]. Gupta [3] also studied the free and forced convective flow of an electrically conducting liquid assuming the axial temperature variation along a wall. Majumdar et al. [4] and Rath et al. [2] extended the above study by considering the effects of the Hall currents on the flow. Such study finds application to the cooling of nuclear reactors.

This paper is concerned with the problem of the Hall effects in the free and the forced convection flow of a conducting fluid through a horizontal channel, the walls of which are porous, conducting and moving. Closed form solutions are obtained for the velocity, the induced magnetic field and the temperature. The behaviour of the velocity and the magnetic field are shown graphically. As the results are very complicated, we omitted the numerical results for the temperature.

II. FORMULATION OF THE PROBLEM

Let us suppose that an electrically conducting fluid flows between two horizontal porous walls at a distance of $2L$ apart. We take the x and y-axes along and perpendicular to the walls with the origin midway between them. A strong uniform magnetic field H_0 is applied parallel to the y-axis and we suppose that the axial pressure gradient $\frac{\partial p}{\partial x}$ is constant. Then for a fully developed steady laminar flow all physical quantities except temperture and pressure are functions of y alone [1]. We therefore take the velocity and the magnetic field as [2]

$$\boldsymbol{v} = (u, \vartheta_0, \omega), \qquad \boldsymbol{H} = (H_x, H_0, H_z),$$

where ϑ_0 is constant and represents the suction parameter.

The governing equations of motion are [2]

$$\varrho_0 \vartheta_0 \frac{du}{dy} = -\frac{\partial p}{\partial x} + \mu \frac{d^2 u}{dy^2} + \mu_e H_0 \frac{dH_x}{dy}, \tag{1a}$$

$$0 = -\frac{\partial p}{\partial y} - \varrho g - \frac{1}{2}\mu_e \frac{d}{dy}(H_x^2 + H_z^2), \tag{1b}$$

$$\varrho_0 \vartheta_0 \frac{d\omega}{dy} = \mu \frac{d^2\omega}{dy^2} + \mu_e H_0 \frac{dH_z}{dy}, \tag{1c}$$

where ϱ_0 is the value of the density ϱ at the reference temperature T_0, μ the coefficient of viscosity and μ_e is the magnetic permeability. $\frac{\partial p}{\partial z}$ is assumed to be zero, the motion being due to the eddy current velocity.

From Maxwell's velocity.

$$\nabla \times \boldsymbol{E} = 0, \qquad \nabla \times \boldsymbol{H} = \boldsymbol{J}$$

and a modified Ohm's law (including the Hall effects [5])

$$\boldsymbol{J} + \frac{\omega\tau}{H_0} \boldsymbol{J} \times \boldsymbol{H} = \sigma(\boldsymbol{E} + \mu_e \boldsymbol{v} \times \boldsymbol{H})$$

we get

$$\frac{d^2 H_z}{dy^2} + \omega\tau \frac{d^2 H_x}{dy^2} = \sigma\mu_e\left(\vartheta_0 \frac{dH_z}{dy} - H_0 \frac{d\omega}{dy}\right), \tag{2a}$$

$$-\frac{d^2 H_x}{dy^2} + \omega\tau \frac{dH_z}{dy^2} = \sigma\mu_e\left(H_0 \frac{du}{dy} - \vartheta_0 \frac{dH_x}{dy}\right), \tag{2b}$$

where $\boldsymbol{E}$ is the electric intensity, $\boldsymbol{J}$ the current density, ω the electron Larmor frequency, τ the mean interval between the successive collisions of an electron with ions and σ is the electrical conductivity. In the above equations we have

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neglected the slip effects between ions and neutrons and also the electron pressure gradient.

Assuming a linear uniform temperature variation along the lower wall $y = -L$ in the form $T = T_0 + Nx$, where N is a constant, the temperature T inside the fluid can be taken as

$$T - T_0 = Nx + \Phi(y). \tag{3}$$

This, along with the equation of state

$$\varrho = \varrho_0[1 - \beta(T - T_0)], \tag{4}$$

where β is the coefficient of thermal expansion, gives from (1b) on integration

$$P = -\varrho_0 g y + \varrho_0 g \beta N x y + \varrho g \beta \int \Phi(y)\,\mathrm{d}y - \tfrac{1}{2}\mu_e (H_x^2 + H_z^2) + F(x). \tag{5}$$

Introducing the non-dimensional quantities

$$\eta = \frac{y}{L}, \quad \bar{u} = \frac{uL}{P_x}, \quad \bar{\omega} = \frac{\omega L}{P_x}, \quad \frac{\partial p}{\partial x} = P_x = -\frac{L^3}{\varrho_0 v^2}\frac{\mathrm{d}F}{\mathrm{d}x},$$

$$\bar{H}_x = \frac{H_x}{\sigma \mu_e H_0 v P_x}, \quad \bar{H}_y = \frac{H_y}{\sigma \mu_e H_0 v P_x}, \quad M^2 = \frac{\mu_e^2 H_0^2 L^2 \sigma}{\varrho_0 v}, \quad p_m = \sigma \mu_e v,$$

$$G = \frac{\beta g N L^4}{v P_x}, \quad R = \frac{\vartheta_0 L}{v},$$

$v = \mu/\varrho$ being the kinematic shear viscosity, we get from (1) and (2) (omitting the bars)

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\eta^2} - R\frac{\mathrm{d}u}{\mathrm{d}\eta} + M^2\frac{\mathrm{d}H_x}{\mathrm{d}\eta} - G\eta = -1, \tag{6a}$$

$$\frac{\mathrm{d}^2 \omega}{\mathrm{d}\eta^2} - R\frac{\mathrm{d}\omega}{\mathrm{d}\eta} + M^2\frac{\mathrm{d}H_z}{\mathrm{d}\eta} = 0 \tag{6b}$$

and

$$\frac{\mathrm{d}^2 H_z}{\mathrm{d}\eta^2} + \omega\tau\frac{\mathrm{d}^2 H_x}{\mathrm{d}\eta^2} = RP_m\frac{\mathrm{d}H_z}{\mathrm{d}\eta} - \frac{\mathrm{d}\omega}{\mathrm{d}\eta}, \tag{7a}$$

$$-\frac{\mathrm{d}^2 H_x}{\mathrm{d}\eta^2} + \omega\tau\frac{\mathrm{d}^2 H_z}{\mathrm{d}\eta^2} = \frac{\mathrm{d}u}{\mathrm{d}\eta} - RP_m\frac{\mathrm{d}H_x}{\mathrm{d}\eta}. \tag{7b}$$

The equations (6a), (6b) and (7a), (7b) together yield

$$\frac{\mathrm{d}^2 U}{\mathrm{d}\eta^2} - R\frac{\mathrm{d}U}{\mathrm{d}\eta} + M^2\frac{\mathrm{d}h}{\mathrm{d}\eta} - G\eta = -1, \tag{8a}$$

$$\frac{\mathrm{d}^2 h}{\mathrm{d}\eta^2} = \frac{RP_m}{1 + \mathrm{i}\omega\tau}\frac{\mathrm{d}h}{\mathrm{d}\eta} - \frac{1}{1 + \mathrm{i}\omega\tau}\frac{\mathrm{d}U}{\mathrm{d}\eta}, \tag{8b}$$

where $U = u + \mathrm{i}\omega$, $h = H_x + \mathrm{i}H_z$.
The boundary conditions are

$$U = U_1, \quad \Phi_1\frac{\mathrm{d}h}{\mathrm{d}\eta} + h = 0 \quad \text{at } \eta = 1,$$

$$U = U_2, \quad \Phi_2\frac{\mathrm{d}h}{\mathrm{d}\eta} - h = 0 \quad \text{at } \eta = -1, \tag{9}$$

where Φ_1, Φ_2 are the electric conductance ratios of the walls $\eta = 1$ and $\eta = -1$, respectively.

III. SOLUTIONS OF THE PROBLEM

Eliminating h between the equations (8) we get

$$\frac{\mathrm{d}^3 U}{\mathrm{d}\eta^3} - R\left[1 + \frac{P_m}{1 + \mathrm{i}\omega\tau}\right]\frac{\mathrm{d}^2 U}{\mathrm{d}\eta^2} + \left[R^2\frac{P_m}{1 + \mathrm{i}\omega\tau} - \frac{M^2}{1 + \mathrm{i}\omega\tau}\right]\frac{\mathrm{d}U}{\mathrm{d}\eta} +$$

$$+ \frac{GRP_m}{1 + \mathrm{i}\omega\tau}\eta - \frac{RP_m}{1 + \mathrm{i}\omega\tau} - G = 0,$$

the solution of which is

$$U = \frac{c_1}{k_1} + c_2 e^{P_1\eta} + c_3 e^{P_2\eta} + \frac{1}{k_2^2}[P_3 k_2^2 \eta^2 +$$

$$+ (2k_1 P_3 + k_2 P_4) k_2 \eta + 2(k_1^2 - k_2) P_3 + k_1 k_2 P_4], \tag{10}$$

where

$$P_1, P_2 = \tfrac{1}{2}(k_1 \pm \sqrt{k_1^2 - 4k_2}),$$

$$P_3 = -\tfrac{1}{2} GRM_2^2, \quad P_4 = RM_2^2 + G,$$

$$k_1 = R(1 + M_2^2), \quad k_2 = R^2 M_2^2 - M_1^2,$$

$$M_1^2 = \frac{M^2}{1 + \mathrm{i}\omega\tau}, \quad M_2^2 = \frac{P_m}{1 + \mathrm{i}\omega\tau}. \tag{11}$$

Then the solution for the magnetic field h is otained from the equations (8a) in the form

$$h = \frac{1}{M^2}\left[c_4 + c_2(R - P_1)e^{P_1\eta} + c_3(R - P_2)e^{P_2\eta} + \left(\frac{RP_3}{k_2} + \frac{1}{2}G\right)\eta^2 + \right.$$
$$\left. + \left(\frac{2Rk_1P_3}{k_2^2} + \frac{RP_4}{k_2} - \frac{2P_3}{k_2} - 1\right)\eta\right]. \tag{12}$$

The constants c_1, c_2, etc. are obtained from (10) and (11) by using the boundary conditions (9) in the following forms:

$$c_1 = k_2U_1 - k_2c_2e^{P_1} - k_2c_3e^{P_2} + \frac{1}{k_2^2}[(2k_1^2 + k_2^2 + 2k_1k_2 - 2k_3)P_3 + k_2(k_1 + k_2)P_4],$$

$$c_2 = \frac{1}{\Lambda}[\{4k_1P_3 + 2k_2P_4 - k_2^2(U_1 - U_2)\}\{(\Phi_1P_2 + 1)e^{P_2} +$$
$$+ (\Phi_2P_2 - 1)e^{-P_2}\}(R - P_2) - 2\{(2Rk_2P_3 + k_2^2G)(\Phi_1 - \Phi_2) +$$
$$+ (2Rk_1P_3 - k_2^2G)(\Phi_1 + \Phi_2 + 2)\}\sinh P_2],$$

$$c_3 = \frac{1}{\Lambda}[\{4k_1P_3 + 2k_2P_4 - k_2^2(U_1 - U_2)\}\{(\Phi_1P_1 + 1)e^{P_1} +$$
$$+ (\Phi_2P_1 - 1)e^{-P_1}\}(R - P_1) - 2\{(2Rk_2P_3 + k_2^2G)(\Phi_1 - \Phi_2) +$$
$$+ (2Rk_1P_3 - k_2^2G)(\Phi_1 + \Phi_2 + 2)\}\sinh P_1],$$

$$c_4 = c_2(R - P_1)(\Phi_2P_1 - 1)e^{-P_1} + c_3(R - P_2)(\Phi_2P_2 - 1)e^{-P_2} -$$
$$- \left(\frac{RP_3}{k_2} + \frac{1}{2}G\right)(2\Phi_2 + 1) + \left(\frac{2Rk_1P_3}{k_2^2} + \frac{RP_4}{k_2} - \frac{2P_3}{k_2} - 1\right)(\Phi_2 + 1),$$

$$\Lambda = 2k_2^2[\{(R - P_1)(\Phi_1P_1 + 1)e^{P_1} + (R - P_1)(\Phi_2P_1 - 1)e^{-P_1}\}\sinh P_2 -$$
$$- \{(R - P_2)(\Phi_1P_2 + 1)e^{P_2} + (R - P_2)(\Phi_2P_2 - 1)e^{-P_2}\}\sinh P_1. \tag{13}$$

Again, we can write [2]

$$\frac{dU}{d\eta} = F_1G + F_2, \tag{14}$$

where $F_j = F_j(\eta, M, P_m, R, \omega\tau)$ $(j = 1, 2)$. Putting $G = 0$ and in (14) successively we find that

$$F_1 = \left(\frac{dU}{d\eta}\right)_{G=1} - \left(\frac{dU}{d\eta}\right)_{G=0},$$
$$F_2 = \left(\frac{dU}{d\eta}\right)_{G=0}. \tag{15}$$

When $\frac{dU}{d\eta} = 0$, we define a critical value of G for the reversal of the primary flow as

$$(G)_{critic} = -\frac{F_2}{F_1}. \tag{16}$$

IV. ENERGY EQUATION

The equation of energy is

$$u\frac{\partial T}{\partial t} + \vartheta_0\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\varrho c}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{d\omega}{dy}\right)^2\right] + \frac{1}{\sigma\varrho c}\left[\left(\frac{dH_x}{dy}\right)^2 + \left(\frac{dH_z}{dy}\right)^2\right],$$

where k is the thermal diffusivity, c the specific heat and T is the temperature of the liquid. Using the expression for T as stated earlier, we can write the above equation in a non-dimensional form as

$$\frac{d^2\Theta}{d\eta^2} - RP_r\frac{d\Theta}{d\eta} = P_ru - k_0\left[\frac{dU}{d\eta}\frac{d\bar{U}}{d\eta} + s^2\frac{dh}{d\eta}\frac{d\bar{h}}{d\eta}\right], \tag{17}$$

where P_r is the Prandtle number,

$$\Theta = \frac{\Phi}{NLP_x}, \qquad k_0 = \frac{v^3P_x}{ckNL^3}, \qquad S = M\left(\frac{\varrho}{\varrho_0}\right)^{-\frac{1}{2}}$$

and $\bar{U}$, $\bar{h}$ are the complex conjugates of U, h, respectively.

Since the temperature of the lower wall is $T_0 + Nx$, the boundary conditions for Θ are

$$\Theta(1) = \frac{\Phi(L)}{NLP_x} = N_1 \text{ (say)} \tag{18}$$

and

$$\Theta(-1) = 0.$$

The equation (17) can be solved by using the expressions for U and h given in (10) and (12) and the boundary conditions (18). Want of space prevents us from describing the calculation of the solution for the temperature.

V. NUMERICAL RESULTS

Figures 1 and 2 represent the natures of the velocity components u and ω and the figures 3 and 4 represent those of the magnetic field components H_x, H_z for different values of G and R. The continuous curves show the natures of different

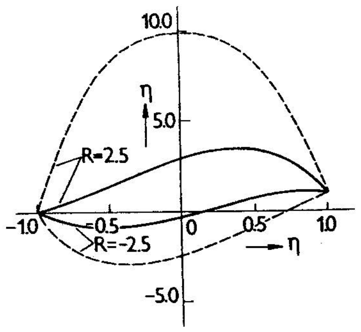

Fig. 1.

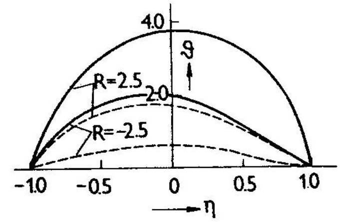

Fig. 2.

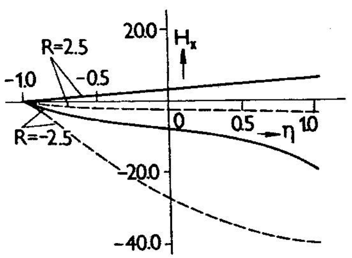

Fig. 3.

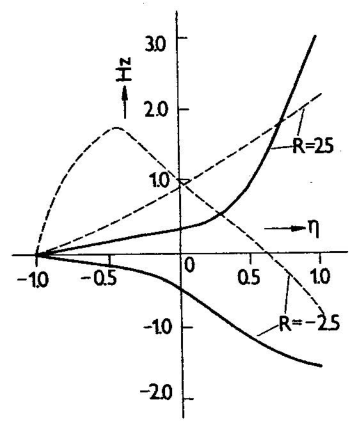

Fig. 4

entities for $G = 5$ and the dotted curves give those when $G = -5$. It is obvious that a negative G and injection induce a flow and a magnetic field reversal while a possitive G and suction prevent the same. Suction has a significant role in the velocity and the magnetic field. It prevents the reversal of the flow and seems to pull the induced magnetic field towards the upper wall, a role which is played by a positive G.

For numerical calculations, we assumed that $\omega\tau = 1$, $U_1 = \Phi_1 = 1$, $U_2 = \Phi_2 = 0$, $M = P_m = 1$.

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Received May 2nd, 1988
Accepted for publication May 23rd, 1988

ВЛИЯНИЕ ТОКОВ ХОЛЛА НА ГИДРОМАГНИТНОЕ КОНВЕКТИВНОЕ ТЕЧЕНИЕ СКВОЗЬ ГОРИЗОНТАЛЬНЫЙ КАНАЛ

В настояшей работе изучено влияние токов Холла на свободное и вынужденное гидромагнитное конвективное течение сквозь горизонтальный канал с движушимися пористыми проводящими стенками. В работе, при предположении, что температура стенок меняется линейно в зависимости от расстояний, получены в замкнутой форме решения для скорости, индуцированного магнитного поля и температуры. Поведение температуры и магнитного поля изображено графически.