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EFFECT OF CROSSED ELECTRIC AND MAGNETIC FIELDS ON THE DIFFUSIVITY-MOBILITY RATIO IN DEGENERATE SEMICONDUCTORS

MONDAL M.,[1] BANIK S. N.,[2] GHATAK K. P.[3] Calcutta

An attempt is made to study the Einstein relation for the diffusivity-mobility ratio of the carriers in degenerate semiconductors in the presence of crossed electric and magnetic fields by taking the electron spin and the broadening of the Landau levels into account. It is found, taking degenerate n-GaAs as an example, that the same ratio exhibits an oscillatory magnetic field dependence and the electric field enhances the amplitude of oscillations. In addition, the corresponding well-known result for the Einstein relation in bulk specimens of degenerate semiconductors having parabolic energy bands in the absence of any quatization has also been shown as a special case of our generalized expressions.

I. INTRODUCTION

Investigations of electrons in semiconductors in the presence of crossed electric and magnetic fields offer interesting physical possibilities, both experimental and theoretical [1]. The cross-field configuration is fundamental to classical and quantum transport in solids [1, 2]. Aronov [3] pointed out that electric field effects should be visible in interband magneto-optical transitions. It may be noted in this context that though considerable work has already been done in literature as regards this particular aspect, there still remain problems in connection with the investigations carried out, while the interest in further researches into other aspects of cross-fields in is becoming increasingly important. Incidentally, the Einstein relation for the diffusivity-mobility ratio (DMR) of the carriers in semiconductors is known to be a very useful one and is more accurate than any of the individual relations to the diffusivity or the mobility which are considered to be the two most widely used parameters of carrier transport in semiconductors. In recent years, the connection of the DMR with the velocity auto-correlation function [4], its modification due to non-linear charge transport [5], the relation of this ratio to the screening of the carriers in semiconductors [6] and the different modifications of the DMR in degenerate semiconductors under varying physical conditions have extensively been investigated [7—14]. Nevertheless, it appears from literature that the DMR in degenerate semiconductors under cross-fields configuration has yet to be investigated for the more interesting case which arises from the consideration of the electron spin and the broadening of the Landau levels, respectively. In what follows, this is done by taking degenerate n-GaAs as an example.

II. THEORETICAL BACKGROUND

The energy spectrum of the conduction electrons in degenerate semiconductors having parabolic energy bands can be written, in the presence of an electric field E_0 along the x axis and a quantizing magnetic field B along the z axis, as [15]

$$E = \left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{\hbar^2 k_z^2}{2m^*} - \frac{eE_0\hbar k_y}{m^*\omega_0} - \frac{e^2E_0^2}{2m^*\omega_0^2} \pm \frac{1}{2}g\mu_0 B \qquad (1a)$$

where E is the energy in the presence of a cross field configuration as measured from the edge of the conduction band in the absence of any quantization, n ($= 0, 1, 2, \ldots$) is the Landau quantum number, $\hbar$ is Dirac's constant, $\omega_0 \equiv eB/m^*$, e is the electron charge, m^* is the effective electron mass at the edge of the conduction band, g is the magnitude of the Lande g-factor and μ_0 is the Bohr magneton.

The electron concentration can be expressed, by including the electron spin and the broadening of the Landau levels, as

$$n_0 = (2L_x\pi^2)^{-1} \sum_{n=0}^{n_{max}} \int_{E'}^{\infty} I\left[-\frac{\partial f_0}{\partial E}\right] \mathrm{d}E, \qquad (1b)$$

where L_x is the sample length along the x axis, E' can be determined from the equation

$$E' = \left(n + \frac{1}{2}\right)\hbar\omega_0 - \frac{1}{2}\frac{e^2E_0^2}{m^*\omega_0^2} \pm \frac{1}{2}g\mu_0 B, \qquad (1c)$$

$I \equiv$ the real part of $\int_{x_1}^{x_h} \psi(E^*)\,\mathrm{d}k_y$, $E^* \equiv E + \mathrm{i}\Gamma$, $\mathrm{i} \equiv \sqrt{-1}$, Γ is the broadening parameter,

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[1] Department of Physics, University College of Science and Technology, 92, Acharya Prafulla Chandra Road, CALCUTTA — 700009, India

[2] Belgachia Monohar Academy, 64/77 Belgachia Road, CALCUTTA — 700037, India

[3] Department of Electronics and Telecommunication Engineering, Faculty of Engineering and Technology, University of Jadavpur, CALCUTTA — 700032, India

$$\Psi(E^*) \equiv (\sqrt{2m^*}/\hbar)\Big[E^* - \left(n+\frac{1}{2}\right)\hbar\omega_0 + (eE_0\hbar k_y/m^*\omega_0) + $$

$$+ (e^2E_0^2/2m^*\omega_0^2) \pm \frac{1}{2}g\mu_0 B\Big]^{1/2}, \qquad x_1 \equiv [-m^*E_0/2B\hbar],$$

$x_h \equiv [eBL_x\hbar^{-1} + x_1]$ and f_0 is the Fermi-Dirac function.

The use of the generalized Sommerfeld lemma [16] together with some algebraic manipulations leads to the expression of the electron concentration as

$$n_0 = C_1 \sum_{n=0}^{n_{max}} [\alpha(n, E_F) + \beta(n, E_F)] \tag{2}$$

where

$$C_1 \equiv (B\sqrt{m^*}/3L_x\pi^2\hbar^2E_0),\ \alpha(n, E_F) \equiv [[a_1(n, E_F) + \{a_1^2(n, E_F) -$$

$$- a_2^2(n, E_F)\}^{1/2}]^{1/2} - [b_1(n, E_F) + \{b_1^2(n, E_F) - b_2^2(n, E_F)\}^{1/2}]^{1/2}],$$

$$a_1(n, E_F) \equiv A_1^3(n, E_F) - 3A_1(n, E_F)\Gamma^2,\ A_1(n, E_F) \equiv \left[E_F - A(n) + \frac{1}{2}g\mu_0 B\right],$$

$$A(n) \equiv \left[\left(n + \frac{1}{2}\right)\hbar\omega_0 + \frac{e^2E_0^2}{m^*\omega_0^2} - eE_0L_x\right],$$

$$a_2(n, E_F) \equiv [3\Gamma A_1^2(n, E_F) - \Gamma^3],\ b_1(n, E_F) \equiv [A_2^3(n, E_F) - 3\Gamma^2 A_2(n, E_F)],$$

$$A_2(n, E_F) \equiv \left[E_F - A_0(n) - \frac{1}{2}g\mu_0 B\right],\ A_0(n) \equiv A(n) + eE_0L_x,$$

$$b_2(n, E_F) \equiv [3\,\Gamma A_2^2(n, E_F) - \Gamma^3],$$

$$\beta(n, E_F) \equiv \sum_{\gamma=1}^{s} 2(k_BT)^{2\gamma}(1 - 2^{1-2\gamma})\,\zeta(2\gamma)\,\frac{d^{2x}}{dE_F^{2\gamma}}[\alpha(n, E_F)],$$

γ is the set of real numbers, k_B is the Boltzmann constant, T is the temperature. $\zeta(2\gamma)$ is the zeta function of order 2γ and E_F is the Fermi energy in the presence of the cross-field configuration as measured from the edge of the conduction band in the absence of any quantization. Since the DMR of the electrons in semiconductors under magnetic quantization can, in general, be expressed [8, 9, 11] as

$$\left(\frac{D}{\mu}\right)_B = \frac{1}{e}\,n_0\Big/\frac{dn_0}{dE_F}, \tag{3}$$

we can combine (2) and (3) as

$$\left(\frac{D}{\mu}\right)_B = \frac{1}{e}\left[\sum_{n=0}^{n_{max}}\{\alpha(n, E_F) + \beta(n, E_F)\}\right]\left[\sum_{n=0}^{n_{max}}\{\alpha'(n, E_F) + \beta'(n, E_F)\}\right]^{-1} \tag{4}$$

where / represent differentiation with respect to E_F.

In the absence of broadening, (2) and (4) get simplified as

$$n_0 = C_0 \sum_{n=0}^{n_{max}} [F_{1/2}(\eta_1) - F_{1/2}(\eta_2)] \tag{5}$$

and

$$\left(\frac{D}{\mu}\right)_B = \frac{k_BT}{e}\left[\sum_{n=0}^{n_{max}} F_{1/2}(\eta_1) - F_{1/2}(\eta_2)\right] \times$$

$$\times \left[\sum_{n=0}^{n_{max}} F_{-1/2}(\eta_1) - F_{-1/2}(\eta_2)\right]^{-1} \tag{6}$$

where

$$C_0 \equiv B\sqrt{2\pi m^*}\,(k_BT)^{3/2}(4L_x\pi^2\hbar^2E_0)^{-1},$$

$$\eta_1 \equiv (k_BT)^{-1}\left[E_F - A(n) + \frac{1}{2}g\mu_0 B\right],$$

$$\eta_2 \equiv (k_BT)^{-1}\left[E_F - A_0(n) - \frac{1}{2}g\mu_0 B\right]$$

and $F_j(\eta)$ is the Fermi-Dirac integral of order j as defined by Blakemore [17]. In the absence of the spin and the electric field (5) and (6) assume the well-known forms [17, 11] as

$$n_0 = N_C\Theta \sum_{n=0}^{n_{max}} F_{-1/2}(\eta) \tag{7}$$

and

$$\left(\frac{D}{\mu}\right)_B = \frac{k_BT}{e}\left[\sum_{n=0}^{n_{max}} F_{-1/2}(\eta)\right]\left[\sum_{n=0}^{n_{max}} F_{-3/2}(\eta)\right]^{-1} \tag{8}$$

where

$$N_C \equiv 2(2\pi m^* k_BT/\hbar^2)^{3/2},\ \ \eta \equiv (k_BT)^{-1}\left[E_F - \left(n + \frac{1}{2}\right)\hbar\omega_0\right]$$

and $\Theta \equiv \hbar\omega_0/k_BT$. For $B \to 0$ we get

$$n_0 = N_C F_{1/2}(\eta_0) \tag{9}$$

and

$$\left(\frac{D}{\mu}\right)_0 = \frac{k_BT}{e}[F_{1/2}(\eta)/F_{-1/2}(\eta_0)] \tag{10}$$

where $\eta_0 \equiv E_{F0}/k_B T$ and E_{F0} is the Fermi energy in the absence of any quantization. (9) and (10) were derived for thefirst time by Landsberg [8].

III. RESULTS AND DISCUSSION

Using (2), (4), (9) and (10) and taking the parameters [18] $m^* = 0.067 m_0$, $\Gamma = 0.4 \times 10^{-4}\,\mathrm{eV}$, $E_0 = 10^{-3}\,\mathrm{V/m}$, $L_x = L_y = L_z = 1\,\mathrm{m}$, $g = 2$, $n_0 = 2.28 \times 10^{22}\,\mathrm{m}^{-3}$ and $T = 4.2\,\mathrm{K}$, we have plotted $\left(\frac{D}{\mu}\right)_B \Big/ \left(\frac{D}{\mu}\right)_0$ versus $1/B$ as shown in Fig. 1, in which the same dependence for $E_0 = 0$ has also been shown for the purpose of comparison. It appears from the Fig. 1 that the DMR shows an oscillatory magnetic field dependence. The oscillatory dependence is due to the crossing over of the Fermi level by the subbands in steps resulting in a successive reduction in the number of the occupied Landau levels and it may be noted that the origin of the oscillations in DMR is the same as that of the SdH oscillations.

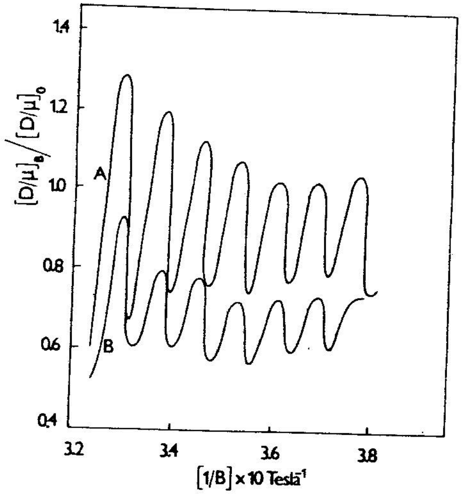

Fig. 1. The plot *A* exhibits the normalized dependence of the DMR as a function of $1/B$ in n-GaAs. The plot *B* corresponds to $E_0 = 0$.

It may be noted in this context that the investigations of the physical properties of bulk semiconductors in the presence of crossed electric and quantizing magnetic fields began with the theoretical work of Hansel and Peter [2], who indicated that the influence of an electric field on the Landau levels should lead to observable effects in the cyclotron resonance phenomena of semiconductors. The Landau levels for parabolic energy bands in the presence and absence of an electric field are respectively given by

$$E_1(n, 0) = \left(n + \frac{1}{2}\right)\hbar\omega_0 \pm \frac{1}{2} g \mu_0 B \qquad (11a)$$

and

$$E_1(n, E_0) = E_1(n, 0) - \frac{1}{2}\frac{e^2 E_0^2}{m^* \omega_0^2} \qquad (11b)$$

Therefore it appears that the Landau energy in the presence of a quatizing magnetic field is greater than the same energy under a cross field configuration. Thus less (amount of) energy should be expended in changes of a rotational motion. Therefore, the ratio of contact of the Landau energy with the Fermi energy will by greater in the presence of the crossed electric and magnetic fields. Thus not only will the electric field enhance the amplitude of oscillations in the DMR under a cross field configuration but also in many magneto-oscillatory phenomena: magneto-optical absorption, magnetic permeability oscillations, magneto resistance oscillations, etc. Thus, all the electronic properties will exhibit a greater amplitude in the presence of crossed electric and oscillatory magnetic fields as compared with magnetic quantization even in the presence of the spin and the broadening effects due to an additional effect of the electric field on the Landau levels as stated above and this general conclusion is based on the work of Hansel and Peter.

It may be stated that the basic contents of the present paper could be of particular interest in view of the fact that the switching speed and the performance at the device terminals of the devices made of semiconductors having highly degenerate carrier concentrations can be related to the DMR [19, 20]. It is worth remarking that, since the available noise power is directly proportional to the DMR as discussed elsewhere [9], the experimental results of the thermal noise in degenerate semiconductors under a cross field configuration will provide an experimental check for the predictions concerning the above ratio and also a technique for investigating the band structures in degenerate materials. Though the effect of band-tails, electron-electron interactions, hot-electron effects and band non-parabolicity have been neglected in the work, this simplified analysis exhibits the basic qualitative features of the DMR in degenerate semiconductors under cross field configurations. Finally, it may be noted that the basic aim of the present work is not solely to demonstrate the effect of crossed electric and quatizing magnetic fields on the DMR, but also to formulate the carrier statistics by including the spin and the broadening effects, since the various transport phenomena and the derivation of the expressions of different electronic properties are based on the appropriate electron statistics in such materials.

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ВЛИЯНИЕ ПОПЕРЕЧНЫХ ЭЛЕКТРИЧЕСКОГО И МАГНИТНОГО ПОЛЕЙ НА ОТНОШЕНИЕ СПОСОБНОСТИ ДИФФУНДИРОВАТЬ К МОБИЛЬНОСТИ В ВЫСРОЖДЕННЫХ ПОЛУПРОВОДНИКАХ

В работе сделана попытка изучить соотношение Эйнштейна для отношения способности диффундирования к мобильности носителей в вырожденных полупроводниках в присутствии поперечных электрического и магнитного полей при учете спина электрона и расширении уровней Ландау. Найдено, в частности, для вырожденного n-GaAs, что одно и то же соотношение осциллирует в зависимости от магнитного поля и что электрическое поле усиливает амплитуду осцилляций. В добавлении показано, что соответствующий хорошо известный результат для выражения Эйнштейна в объемных образцах вырожденных полупроводников, имеющих параболические энергетические полосы в отсутствии квантования, является частным случаем нашего общего выражения.