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SPONTANEOUS DECAY AND SPONTANEOUS SQUEEZING

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It is shown that a two-level atom initially prepared in a coherent state and coupled to a near-continuum of cavity modes serves as a generator of squeezed light. The criterion is found for the squeezing of a particular field mode.

I. INTRODUCTION

In recent years a good deal of interest has been devoted to the theoretical description and the practical realization of the squeezed light [1—3] with less uncertainty in one quadrature of the field than the uncertainty associated with a coherent state. This reduction of fluctuation in one quadrature is accompanied by increasing fluctuations in the canonically — conjugate quadrature in such a way which preserves the restriction on the product of variances imposed by the Couchy—Schwarz inequality.

Quite recently it has been shown by Knight [4, 5] that squeezed light can be generated when a two-level atom is injected into a one mode vacuum state. Here the atom should be prepared in a coherent superposition of excited and ground states. In the original paper [4] the single field mode coupled to atwo-level atom through the dipole and rotting wave approximations (so-called Jaynes—Cummings model — JCM [6]) was studied and it was shown that the variance in one quadrature is squeezed below the vacuum value periodically — the period of the oscillations is just one half of the Rabi frequency.

In present survey we discuss the generation of the squeezed light in the process of a spontaneous decay of a two-level atom with atomic frequency ω_0 coupled to a near-continuum cavity modes of the frequency ω_k.

II. NEAR-CONTINUUM MODE JCM

We will consider the Hamiltonian in the electric-dipole and rotating-wave approximations:

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$\Gamma(\tilde{\varepsilon}) = \Gamma(\omega_0) \equiv \Gamma$ and $\Delta(\tilde{\varepsilon}) = \Delta(\omega_0)$. Further we shall absorb $\Delta(\omega_0)$ into ω_0, which we redefine as the excitation frequency of the excited state |2>.

Finally, for $C^{(2,0)}(t)$ and $C_k^{(1,1)}(t)$ we can write:

$$C^{(2,0)}(t) = C^{(2,0)} . \exp\left(-\mathrm{i}\frac{\omega_0}{2}t - \Gamma t\right) \tag{2.13a}$$

$$C^{(1,1)}(t) = \frac{\lambda_k C^{(2,0)}}{(\omega_0 - \omega_k - \mathrm{i}\Gamma)}\left[\exp\left(-\mathrm{i}\frac{\omega_0}{2}t - \Gamma t\right) - \exp . \left(+\mathrm{i}\frac{\omega_0}{2}t - \mathrm{i}\omega_k t\right)\right]. \tag{2.13b}$$

III. SPONTANEOUS SQUEEZING

Now we will show that the fluctuations in the field, after the interaction with a two-level atom, are squeezed. To do that, we define two hermitian quadrature operators $\hat{a}_k^{(1)}$ and $\hat{a}_k^{(2)}$, defined by

$$\hat{a}_k^{(1)} = \frac{1}{2}[\hat{a}_k + \hat{a}_k^+]$$
$$\hat{a}_k^{(2)} = \frac{1}{2\mathrm{i}}[\hat{a}_k - \hat{a}_k^+]. \tag{3.1}$$

The commutation relation for $a_k^{(i)}$ is $[\hat{a}_k^{(1)}, \hat{a}_q^{(2)}] = \frac{\mathrm{i}}{2}\delta_{k,q}$ and the variances

$$\langle(\Delta\hat{a}_k^{(i)})^2\rangle \equiv \langle(\hat{a}_k^{(i)})^2\rangle - \langle\hat{a}_k^{(i)}\rangle^2$$

satisfy the uncertainty relation $\langle(\Delta a_k^{(1)})^2\rangle\langle(\Delta a_k^{(2)})^2\rangle \geqslant 1/16$.

For the state vector $|\Psi(t)\rangle$ defined by (2.3) the variances are:

$$\langle(\Delta a_k^{\binom{(1)}{(2)}})^2\rangle = \frac{1}{4} + \frac{1}{2}|C_k^{(1,1)}(t)|^2 \mp [\overset{*}{C}{}^{(1,0)}(t)C_k^{(1,1)}(t) \pm C^{(1,0)}(t)\overset{*}{C}{}_k^{(1,1)}(t)]^2. \tag{3.2}$$

We will write here the explicit expressions for the variances in the limit $t \gg \Gamma^{-1}$ when almost all energy is in the field and the atom is in its ground state $(\lim_{t\to\infty}|C^{(2,0)}(t)|^2 = 0)$:

$$\langle(\Delta a_k^{(1)})^2\rangle = \frac{1}{4} + \frac{1}{2}\frac{\lambda_k^2\cos^2\Theta/2}{\xi_k^2 + \Gamma^2}\left[1 - \frac{2\sin^2\Theta/2}{\xi_k^2 + \Gamma^2}\cdot\right.$$
$$\left. \cdot(\xi_k\cos(\Phi + \omega_k t) - \Gamma\sin(\Phi + \omega_k t))^2\right] \tag{3.3a}$$

$$\langle(\Delta a_k^{(2)})^2\rangle = \frac{1}{4} + \frac{1}{2}\frac{\lambda_k^2\cos^2\Theta/2}{\xi_k^2 + \Gamma^2}\left[1 + \frac{2\sin^2\Theta/2}{\xi_k^2 + \Gamma^2}\cdot\right.$$
$$\left. \cdot(\xi_k\sin(\Phi + \omega_k t) + \Gamma\cos(\Phi + \omega_k t))^2\right], \tag{3.3b}$$

where $\xi_k = \omega_k - \omega_0$.

Due to the fact phase-sensitive detectors do not respond to the rapid oscillations at the field mode frequencies ω_k, the phase $\Phi + \omega_k t$ can be substituted just by the phase Φ[5].

Thus for the measured variances $\langle(\Delta a_k^{(1)})^2\rangle$ of the particular field mode with the frequency ω_k one can find the squeezing condition

$$\frac{(\xi_k^2 - \Gamma^2)\cos 2\Phi - 2\xi_k\Gamma\sin 2\Phi}{\xi_k^2 + \Gamma^2} > \mathrm{cotg}^2\,\Theta/2. \tag{3.4}$$

This means that if the atomat $t = 0$ is prepared in that particular coherent state, when phases Φ and Θ do obey condition (3.4), then the variance in the measured quadrature $\langle(\Delta a_k^{(1)})^2\rangle_{meas}$ with the frequency ω_k displays squeezing, which means that

$$\langle(a_k^{(1)})^2\rangle_{meas} < 1/4. \tag{1.5}$$

Particularly, the resonant mode ($\xi_k = 0$) displays squeezing for $\Theta = 2\pi/3$; $4\pi/3$ and $\Phi = \pi/2$:

$$\langle(\Delta a_{\omega_0}^1)^2\rangle = \frac{1}{4} - \frac{1}{16}\frac{\lambda^2\omega_0}{\Gamma^2} \tag{3.6a}$$

and

$$\langle(\Delta a_{\omega_0}^2)^2\rangle = \frac{1}{4} + \frac{1}{8}\frac{\lambda^2\omega_0}{\Gamma^2}. \tag{3.6b}$$

In Fig. 1. the evolutions of the resonant mode variances $\langle(\Delta a_{\omega_0}^{(i)})^2\rangle$

$$\langle(\Delta a_{\omega_0}^{(1)})^2\rangle = \frac{1}{4} - \frac{1}{16}\frac{\lambda^2\omega_0}{\Gamma^2}(1 - \mathrm{e}^{-\Gamma t})^2$$
$$\langle(\Delta a_{\omega_0}^{(2)})^2\rangle = \frac{1}{4} + \frac{1}{8}\frac{\lambda^2\omega_0}{\Gamma^2}(1 - \mathrm{e}^{-\Gamma t})^2$$

are plotted. The oscillating lines in Fig. 1. correspond to the variances of the quadrature operators $a_{\omega_0}^{(i)}$ in a single-resonant mode problem analysed by Knight [4, 5].

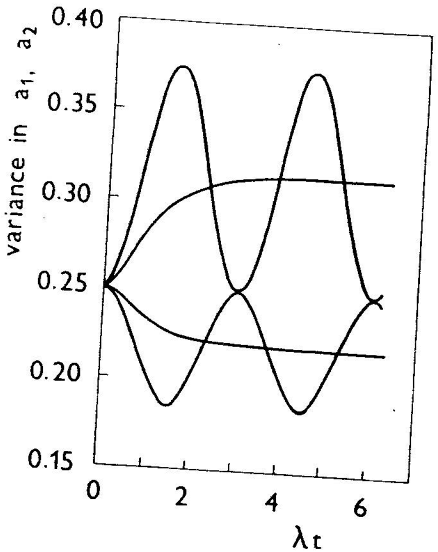

Fig. 1. Time dependence of variances of quadrature operators on the resonant mode. Variance $\langle(\Delta a^{(1)}_{\omega_0})^2\rangle$ is squeezed below its vacuum value. The resonant coupling constant λ_{ω_0} is chosen tobe $\Gamma/\sqrt{2}$. Oscillating lines correspond to a single-resonant mode problem analysed by Knight [4, 5].

IV. CONCLUSIONS

In the paper presented it was shown that the quantum fluctuations of the near-continuum mode field coupled to a two-level atom, which is initially prepared in a coherent superposition of the ground and the excited states, display squeezing. The main difference compared to a single-mode problem consists in the fact that the variances of the quadrature operators do not oscillate with the Rabi frequency — they reach at $t \gg \Gamma^{-1}$ some constant value. This is analogous with the atomic inversion, which in a single-mode case oscillates, but in the near-continuum mode case tends to -1 (atom in the ground state).

So, when the atom is initially prepared in a particular coherent state (with defined phases Φ and Θ), then at $t > 0$ the atom passes to its ground state and some of the field modes (for which the squeezing condition (3.4) is fulfilled) do exhibit squeezing.

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СПОНТАННЫЙ РАСПАД И СПОНТАННОЕ СЖАТИЕ

В работе показано, что двухуровневый атом, находящийся в начальный момент времени в когерентном состоянии и взаимодействующий с континуумом мод, служит источником сжатых состояний света. Найден критерий сжатия для определенных мод поля излучения.