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A NEW ITERATIVE APPROACH TO THE RANDOM-FIELD ISING MODEL WITH A GAUSSIAN RANDOM-FIELD DISTRIBUTION

ŠAMAJ, L.,[1] Bratislava

The aim of the present paper is to apply an iterative method to the RFIM. We are concerned in RFIM correlation functions. The problem is reformulated by the use of the replica trick for the calculation of some correlations in a pure spin system S′. A special rewriting of the sum of δ-functions gives a new graphical representation of the thermodynamic quantities of the system S′. The lowering of the number of points on an arbitrarily chosen reference spin in this graphical representation leads to an iterative scheme. Quantities changing slowly and little during the iteration are found. Simple assumptions about their plots imply what we call model I and model II. Model I is based on the supposed independence of correlations in the surroundings of the reference spin from its presence or absence. It gives a satisfactory account of the critical behaviour above $d = 4$. More realistic forms of the correlations in model II enable us to make the calculations of model I more precise.

I. INTRODUCTION

The Ising model in a quenched random magnetic field (for a review see [1]) has been studied by means of various methods. In spite of this the main problems (the values of critical exponents, the determination of the lower critical dimensionality (*LCD*) below which there is no phase transition, etc.) have so far not been solved completely. Experimental materializations of the random-field Ising model (RFIM), i.e. dilute Ising antiferromagnets in a uniform magnetic field [2, 3], confirm that the theoretical treatment of the RFIM is not an academic problem.

The model was proposed originally by Imry and Ma [4] to examine the possibility of a ferromagnetic ground state. The stability of an assumed ferromagnetic state at zero temperature against its fragmenting into domains was investigated by simple energy arguments. The resulting $LCD = 2$ was shifted to $LCD = 3$ by the domain wall roughening calculations [5, 6]. The value of

[1] Inst. of Physics, EPRC, Slov. Acad. Sci., Dúbravská cesta 9, 842 28 BRATISLAVA, Czechoslovakia

$LCD = 2$ is in contradiction to the earlier renormalization-group analyses [7, 8, 9] in the $d = 6\text{-}\varepsilon$ dimensions yielding the critical exponents of the pure model in the $4\text{-}\varepsilon$ dimensions. Simultaneously, the supersymmetric formulation [10] predicts the dimensional reduction by two, and therefore $LCD = 3$. Recent treatments of the controversial specifying of the LCD, namely rigorous theoretical analyses [11, 12], renormalization-group arguments [13, 14], high-temperature series expansions [15], etc. all suggest $LCD = 2$.

This paper is based on the iterative method [16] applied originally to the pure spin system with arbitrary-range interactions. In order to study the RFIM correlation functions within the iterative method some introductory mathematical arrangements (following from the replica trick) are given in SEC. II. The next procedure is evident:

— an identity for the sum of δ-functions implies an elementary graphical representation of thermodynamic quantities we are interested in (SEC. III);

— the reduction of the point number on the reference lattice spin leads to an iterative scheme for the calculation of the RFIM correlation functions (Sec. IV);

— simple assumptions of the plots of quantities changing little during the iteration give the resulting equations for the RFIM correlations. They are solved and discussed in Sec. VI.

II. FORMALISM FOR THE CALCULATION OF CORRELATION FUNCTIONS

We are concerned with the RFIM on a regular d-dimensional lattice. On each site $u(u = 1, 2, \ldots N)$ there is a spin s_u (the corresponding spin variable s_u takes $+1$ or -1) affected by a magnetic field h_u. The Hamiltonian of this spin system S is given by

$$H = -\frac{1}{2}\sum_{u,v=1}^{N}{}' s_u J_{uv} s_v - \sum_{u=1}^{N} h_u s_u . \tag{2.1}$$

Here, J_{uv} is an arbitrary-range exchange coupling between spins on the sites u, v and the prime means the summation with exclusion of the terms $u = v$. Further we consider quenched random fields $\{h_u\}$ characterized by site-independent Gaussian distributions with a zero mean

$$P\{h_u\} = \prod_{u=1}^{N} p(h_u), \tag{2.2a}$$

$$p(h_u) = \frac{1}{\sqrt{2\pi\lambda^2}}\exp\left(-\frac{h_u^2}{2\lambda^2}\right), \tag{2.2b}$$

where λ is the standard deviation.

In order to obtain the thermodynamic behaviour of the spin system S at the temperature T we must calculate the thermodynamic quantities for a given configuration of random fields $\{h_u\}$ and then to average over the distribution $P\{h_u\}$. We are interested especially in the RFIM correlation functions defined by

$$g_{ij} = \int_{-\infty}^{\infty} \mathrm{d}^N h_u P\{h_u\} \frac{\sum_{\{s\}}^{N} (s_i - \langle s_i\rangle)(s_j - \langle s_j\rangle)\exp(-H/kT)}{\sum_{\{s\}}^{N}\exp(-H/kT)}, \tag{2.3}$$

where the sums run over all possible spin configurations. After a small modification, the expression for g_{ij} (2.3) can be written in a more suitable form

$$g_{ij} = \left[\frac{\partial^2}{\partial h_i' \partial h_j'}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} \mathrm{d}h_1 \ldots \mathrm{d}h_N p(h_1)\ldots p(h_N) \ln B(A, \{h_u\}, \{h_u'\})\right]_{\{h_u'\}=0}, \tag{2.4a}$$

where

$$B(A, \{h_u\}, \{h_u'\}) = \sum_{\{s\}} \exp\left(\frac{1}{2}\sum_{u,v=1}^{N} s_u A_{uv} s_v + \sum_{u=1}^{N}\frac{1}{kT}h_u s_u + \sum_{u=1}^{N} h_u' s_u\right), \tag{2.4b}$$

and $A_{uv} = J_{uv}/kT$.

The usual way to average the logarithm is the replica trick

$$\ln B(A, \{h_u\}, \{h_u'\} = \lim_{r\to 0}\frac{[B(A, \{h_u\}, \{h_u\}, \{h_u'\})]^r - 1}{r}. \tag{2.5}$$

We introduce r replicas $s^\eta (\eta = 1, \ldots r; s^\eta = \pm 1)$ to obtain $[B(A, \{h_u\}, \{h_u'\})]^r$. The integration over random-field variables $\{h_u\}$ in (2.4) then implies the following expression for the correlation function

$$g_{ij}(A, \Lambda) = \lim_{r\to 0}\frac{1}{r}\left[\frac{\partial^2}{\partial h_i'\partial h_j'}\sum_{\{s^\eta\}}\exp\left(\frac{1}{2}\sum_{u,v=1}^{N}{}'\sum_{\eta=1}^{r} s_u^\eta A_{uv} s_v^\eta + \frac{1}{2}\sum_{u=1}^{N}\sum_{\eta,v=1}^{N}{}' s_u^\eta \Lambda s_u^v + \sum_{u=1}^{N}\sum_{\eta=1}^{r} h_u' s_u^\eta\right)\right]_{\{h_u'\}=0}, \tag{2.6}$$

with $\Lambda = (\lambda/kT)^2$.

We define a new spin system S′. On each lattice site u ($u = 1, 2, \ldots N$) there is r spins u_η ($\eta = 1, \ldots r$; the corresponding spin variable $s_u = +1$ or -1). Arbitrary spins u_η and v_ν interact by an exchange integral (divided by kT)

$$A_{uv}^{\eta\nu} = A_{uv}\delta_{\eta\nu} + \Lambda\delta_{uv}(1 - \delta_{\eta\nu}), \tag{2.7}$$

i. e. spins u_η and v_η localized on different sites u and v are bounded by A_{uv}, while two spins u_η, u_ν $(\eta \neq \nu)$ interact within the same site u by Λ. Let us use the symbols:

$p_{uv}(A, \Lambda; r)$ for the correlation function of the spins u_η and v_η $(u \neq v)$;
$q_{uv}(A, \Lambda; r)$ for the correlation function of the spins u_η and v_ν $(u \neq v, \eta \neq \nu)$;
$q(A, \Lambda; r)$ for the correlation function of the spins u_η and u_ν $(\eta \neq \nu)$.

Finally, from (2.6) one obtains

$$g_{ij}(A, \Lambda) = \lim_{r \to 0} \frac{1}{r} \left[r p_{ij}(A, \Lambda; r) + r(r-1) q_{ij}(A, \Lambda; r) \right] \times$$
$$\times \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dh_1 \dots dh_N p(h_1) \dots p(h_N) \times$$
$$\times \left[\sum_{\{s\}} \exp\left(\frac{1}{2} \sum_{u,v=1}^{N}{}' s_u A_{uv} s_v + \sum_{u=1}^{N} \frac{1}{kT} s_u h_u \right) \right]^r =$$
$$= \lim_{r \to 0} [p_{ij}(A, \Lambda; r) - q_{ij}(A, \Lambda; r)]. \tag{2.8}$$

III. AN ELEMENTARY GRAPHICAL REPRESENTATION OF THERMODYNAMIC QUANTITIES OF THE SPIN SYSTEM S′

The correlation functions of the RFIM were expressed through the correlations of a new spin system S′ in (2.8). That is why we shall calculate the following thermodynamic quantities of the system S′:

i) the partition sum

$$Z(A, \Lambda; r) = \sum_{\{s^\eta_i\}} \exp\left(\frac{1}{2} \sum_{u,v=1}^{N}{}' \sum_{\eta=1}^{r} s^\eta_u A_{uv} s^\eta_v + \frac{1}{2} \sum_{u=1}^{N} \sum_{\eta,\nu=1}^{r} s^\eta_u \Lambda s^\nu_u \right), \tag{3.1}$$

ii) the correlation function of the spins i_α and j_α $(i \neq j)$

$$p_{ij}(A, \Lambda; r) = \frac{P_{ij}(A, \Lambda; r)}{Z(A, \Lambda; r)}, \tag{3.2a}$$

$$P_{ij}(A, \Lambda; r) = P^{\alpha\alpha}_{ij}(A, \Lambda; r) = \sum_{\langle s^\eta \rangle} s^\alpha_i s^\alpha_j \exp\left(\frac{1}{2} \sum_{u,v=1}^{N}{}' \sum_{\eta=1}^{r} s^\eta_u A_{uv} s^\eta_v + \frac{1}{2} \sum_{u=1}^{N} \sum_{\eta,\nu=1}^{r}{}' s^\eta_u \Lambda s^\nu_u \right), \tag{3.2b}$$

iii) the correlation function of the spins i_α and j_β $(i \neq j, \alpha \neq \beta)$

$$q_{ij}(A, \Lambda; r) = \frac{Q_{ij}(A, \Lambda; r)}{Z(A, \Lambda; r)}, \tag{3.3a}$$

$$Q_{ij}(A, \Lambda; r) = Q^{\alpha\beta}_{ij}(A, \Lambda; r) = \sum_{\{s^\eta\}} s^\alpha_i s^\beta_j \exp\left(\frac{1}{2} \sum_{u,v=1}^{N}{}' \sum_{\eta=1}^{r} s^\eta_u A_{uv} s^\eta_v + \frac{1}{2} \sum_{u=1}^{N} \sum_{\eta,\nu=1}^{r}{}' s^\eta_u \Lambda s^\nu_u \right), \tag{3.3b}$$

iv) the correlation function of the spins i_α and i_β $(\alpha \neq \beta)$

$$q(A, \Lambda; r) = \frac{Q(A, \Lambda; r)}{Z(A, \Lambda; r)}, \tag{3.4a}$$

$$Q(A, \Lambda; r) = Q^{\alpha\beta}_{ii}(A, \Lambda; r) = \sum_{\{s^\eta\}} s^\alpha_i s^\beta_i \exp\left(\frac{1}{2} \sum_{u,v=1}^{N}{}' \sum_{\eta=1}^{r} s^\eta_u A_{uv} s^\eta_v + \frac{1}{2} \sum_{u=1}^{N} \sum_{\eta,\nu=1}^{r} s^\eta_u \Lambda s^\nu_u \right). \tag{3.4b}$$

The summation over 2^{rN} spin configurations in (3.1)—(3.4) can be replaced by the integration over the continuous spin variables, where the sums of the corresponding δ-functions occur. The iterative method [16] is based on a special rewriting of the sum of the δ-functions

$$\delta(s^\eta_u + 1) + \delta(s^\eta_u - 1) = \lim_{m \to \infty} f_m(s^\eta_u), \tag{3.5a}$$

$$f_m(s^\eta_u) = \frac{2^{2m+1} m^m!}{(2m)!} \sqrt{\frac{m}{\pi}} (s^\eta_u)^{2m} \exp[-m(s^\eta_u)^2]. \tag{3.5b}$$

We define the matrix $\bar{A}(m)$ by

$$\bar{A}^{\eta\nu}_{uv}(m) = -A^{\eta\nu}_{uv} + 2m\delta_{uv}\delta_{\eta\nu}. \tag{3.6}$$

Its inverse matrix in the limit of $m \to \infty$ is obtained in the form

$$[\bar{A}^{\eta\nu}_{uv}]^{-1}(m) = \frac{1}{2m} \delta_{\eta\nu} + \frac{\Lambda}{(2m)^2}(1 - \delta_{\eta\nu}) \quad \text{for } u = v, \tag{3.7a}$$

$$= \frac{A_{uv}}{(2m)^2} \delta_{\eta\nu} \quad \text{for } u \neq v. \tag{3.7b}$$

We apply the Wick theorem to the Gaussian integrals arising from the substitution of the sums of the δ-functions in continuous versions of (3.1)—(3.4) by the identity (3.5). The graphical representation of the statistical sum (3.1), for example, is then found to be

$$Z(A, \Lambda; r) = \lim_{N, m \to \infty} [C(m)]^{rN} \cdot \sum_{\text{all diagrams}} \quad \text{(see Fig. 1.)} \tag{3.8a}$$

$$C(m) = \frac{2^{2m+1} m^m m!}{(2m)!} . \tag{3.8b}$$

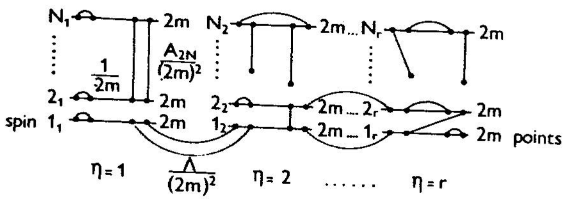

Fig. 1.

Here the sum proceeds over all the diagrams constructed as follows:

i) each spin is represented by $2m$ points;

ii) the diagram is created by connecting $2mrN$ points into mrN pairs by lines;

iii) the line between the points belonging to the spins u_η and v_ν represents the factor $[\bar{A}^{\eta\nu}_{uv}]^{-1}(m)$;

iv) the contribution of a given diagram is the product of all factors occurring in it.

If we rewrite the sum of the δ-functions by f_n in (3.5) (n is arbitrary) for an arbitrarily choosen spin, 1_1 for example, the diagrammatic formalism remains unchanged when:

— the number of points on the spin 1_1 is $2n$;

— the normalization factor is $C(n)[C(m)]^{rN-1}$;

— the inverse matrix $\bar{A}^{-1}(m)$ is modified little when compared to (3.7)

$$[\bar{A}^{1\nu}_{11}]^{-1}(m, n) = \frac{1}{2n}\,\delta_{1\nu} + \frac{\Lambda}{(2m)(2n)}\,(1 - \delta_{1\nu}), \tag{3.9a}$$

$$[\bar{A}^{1\nu}_{1v}]^{-1}(m, n) = \frac{A_{1v}}{(2m)(2n)}\,\delta_{1\nu} \qquad \text{for } v \neq 1 . \tag{3.9b}$$

By adding one point to the spins (i_α, j_α), (i_α, j_β), (i_α, i_β) $(i \neq j,\ \alpha \neq \beta)$ in the graphical representation of $Z(A, \Lambda; r)$ we obtain $P^{\alpha\alpha}_{ij}(A, \Lambda; r)$, $Q^{\alpha\beta}_{ij}(A, \Lambda; r)$ and $Q^{\alpha\beta}_{ii}(A, \Lambda; r)$, respectively.

IV. THE TRANSITION FROM THE GRAPHICAL REPRESENTATION TO THE ITERATIVE SCHEME FOR THE CALCULATION OF THERMODYNAMIC QUANTITIES

In this section we present the procedure of lowering the point number on an arbitrarily choosen reference spin, 1_1 for example, in our representation of the thermodynamic quantities of the spin system S′. In order to differentiate the reference spin 1_1 we use $f_n(s^1_1) f_n(s^1_1)$ $(n \to \infty,\ n \ll m)$ in (3.5) for the sum of the δ-functions $\delta(s^1_1 + 1) + \delta(s^1_1 - 1)$.

Let us define the new quantities $\bar{Z}(k)$, $\bar{P}^{\alpha\alpha}_{ij}(k)$, $\bar{Q}^{\alpha\beta}_{ij}(k)$, $\bar{Q}^{\alpha\beta}_{ii}(k)$ with $\alpha \neq \beta$, $i \neq j$. They are obtained from $Z(A, \Lambda; r)$, $P^{\alpha\alpha}_{ij}(A, \Lambda; r)$, $Q^{\alpha\beta}_{ij}(A, \Lambda; r)$, $Q^{\alpha\beta}_{ii}(A, \Lambda; r)$, respectively, by taking away $2(n - k)$ points of the spin 1_1. The normalization factor $C(n)[C(m)]^{rN-1}$ is necessarily unchanged. The relationships among these function are found by the exclusion of the last point on spin 1_1 in the graphical representation of $\bar{Z}(k)$,

$$\bar{Z}(k) = \lim_{\substack{N, m, n \to \infty \\ (n \ll m)}} C(n)[C(m)]^{rN-1} \cdot \sum_{\text{all diagrams}} \quad \text{(see Fig. 2)} \tag{4.1}$$

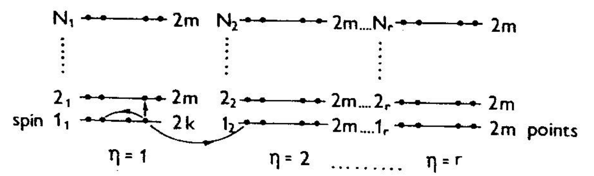

Fig. 2

for example. When we link the point we want to exclude from spin 1_1 with an arbitrary one of $(2k - 1)$ points on spin 1_1 by the line $[\bar{A}^{11}_{11}]^{-1}(m, n) = 1/2n$, the contribution of all remaining pairings is $\bar{Z}(k - 1)$. The connection of this point with an arbitrary one of $2m$ points belonging to the spin $j_1 \neq 1_1$ by the line $[\bar{A}^{11}_{1j}]^{-1}(m, n) = A_{1j}/(2m)(2n)$ leads to the respective contribution $\bar{P}^{11}_{1j}(k - 1)$ (the finite pair reduction of the point number on the spin j_1 does not change $\bar{P}_{1j}(k - 1)$ for $m \to \infty$). The linking of this point with an arbitrary one of $2m(r - 1)$ points on the spins $1_2, \ldots 1_r$ by the line $[\bar{A}^{12}_{11}]^{-1}(m, n) = \Lambda/(2m)(2n)$ implies the contribution $\bar{Q}^{12}_{11}(k - 1)$ (because of $[\bar{A}^{12}_{11}]^{-1}(m, n) = [\bar{A}^{13}_{11}]^{-1}(m, n) = \ldots = [\bar{A}^{1r}_{11}]^{-1}(m, n)$ and $\bar{Q}^{12}_{11}(k - 1) = \ldots = Q^{1r}_{11}(k - 1)$). Then

$$\bar{Z}(k) = (2k - 1)[\bar{A}^{11}_{11}]^{-1}(m, n)\,\bar{Z}(k - 1) + \sum_{j \neq 1} 2m[\bar{A}^{11}_{1j}]^{-1}(m, n)\,\bar{P}^{11}_{1j}(k - 1) + $$
$$+ 2m(r - 1)[\bar{A}^{12}_{11}]^{-1}(m, n)\,\bar{Q}^{12}_{11}(k - 1) . \tag{4.2a}$$

Analogous procedures performed in the graphical representations of $\bar{P}^{11}_{1i}(k)$, $\bar{Q}^{12}_{1i}(k)$, $\bar{Q}^{12}_{11}(k)$ $(i \neq 1)$ result in

$$\bar{P}^{11}_{1i}(k) = 2k[\bar{A}^{11}_{11}]^{-1}(m,n)\,\bar{P}^{11}_{1i}(k-1) + (2m+1)[\bar{A}^{11}_{1i}]^{-1}(m,n)\,\bar{Z}(k) + \sum_{j\neq 1,i} 2m[\bar{A}^{11}_{1j}]^{-1}(m,n)\,\bar{P}^{11}_{ji}(k) + 2m(r-1)[\bar{A}^{12}_{11}]^{-1}(m,n)\,\bar{Q}^{12}_{ii}(k),$$

$$\bar{Q}^{12}_{1i}(k) = 2k[\bar{A}^{11}_{11}]^{-1}(m,n)\,\bar{Q}^{12}_{1i}(k-1) + 2m[\bar{A}^{11}_{1i}]^{-1}(m,n)\,\bar{Q}^{12}_{ii}(k) + \sum_{j\neq 1,i} 2m[\bar{A}^{11}_{1j}]^{-1}(m,n)\,\bar{Q}^{12}_{ji}(k) + 2m[\bar{A}^{12}_{11}]^{-1}(m,n)\{(r-2)\,\bar{Q}^{23}_{ii}(k) + \bar{P}^{22}_{1i}(k)\}, \tag{4.2b, 4.2c}$$

$$\bar{Q}^{12}_{11}(k) = 2k[\bar{A}^{11}_{11}]^{-1}(m,n)\,\bar{Q}^{12}_{11}(k-1) + \sum_{j\neq 1} 2m[\bar{A}^{11}_{1j}]^{-1}(m,n)\,\bar{Q}^{12}_{j1}(k) + 2m[\bar{A}^{12}_{11}]^{-1}(m,n)\{(r-2)\,\bar{Q}^{23}_{11}(k) + \bar{Z}(k)\}. \tag{4.2d}$$

Let $Z((A^{1\nu}_{1v})' = \sqrt{k/n}\,A^{1\nu}_{1v})$, $P^{\alpha\alpha}_{ij}((A^{1\nu}_{1v})' = \sqrt{k/n}\,A^{1\nu}_{1v})$, $Q^{\alpha\beta}_{ij}(A^{1\nu}_{1v})' = \sqrt{k/n}\,A^{1\nu}_{1v})$, $Q^{\alpha\beta}_{ii}((A^{1\nu}_{1v})' = \sqrt{k/n}\,A^{1\nu}_{1v})$ be the statistical quantities of the considered spin system S' with modified interactions between the reference spin 1_1 and the others v_ν

$$(A^{1\nu}_{1v})' = \sqrt{\frac{k}{n}}\,A^{1\nu}_{1v} = \sqrt{\frac{k}{n}}\,[A_{1v}\delta_{1\nu} + \Lambda\delta_{1v}(1-\delta_{1\nu})]. \tag{4.3}$$

Using $f_k(s^1_1)$ for the sum of the δ-functions of the reference spin 1_1 in the thermodynamic quantities defined above with modified interactions one finds

$$\bar{Z}(k) \approx \frac{C(n)}{C(k)}\left(\frac{k}{n}\right)^k Z\left((A^{1\nu}_{1v})' = \sqrt{\frac{k}{n}}\,A^{1\nu}_{1v}\right), \tag{4.4a}$$

$$\bar{P}^{\alpha\alpha}_{ij}(k) \approx \frac{C(n)}{C(k)}\left(\frac{k}{n}\right)^k P^{\alpha\alpha}_{ij}\left((A^{1\nu}_{1v})' = \sqrt{\frac{k}{n}}\,A^{1\nu}_{1v}\right), \tag{4.4b}$$

$$\bar{Q}^{\alpha\beta}_{ij}(k) \approx \frac{C(n)}{C(k)}\left(\frac{k}{n}\right)^k Q^{\alpha\beta}_{ij}\left((A^{1\nu}_{1v})' = \sqrt{\frac{k}{n}}\,A^{1\nu}_{1v}\right), \tag{4.4c}$$

$$\bar{Q}^{\alpha\beta}_{ii}(k) \approx \frac{C(n)}{C(k)}\left(\frac{k}{n}\right)^k Q^{\alpha\beta}_{ii}\left((A^{1\nu}_{1v})' = \sqrt{\frac{k}{n}}\,A^{1\nu}_{1v}\right). \tag{4.4d}$$

Finally, the RFIM correlation function of the spins 1, i $g_{1i}(A, \Lambda)$ can be calculated from

$$g_{1i}(A, \Lambda) = \lim_{r\to 0}[p_{1i}(A, \Lambda; r) - q_{1i}(A, \Lambda; r)], \tag{4.5}$$

where

$$p_{1i}(A, \Lambda; r) = \lim_{n\to\infty}\frac{\bar{P}^{11}_{1i}(n)}{\bar{Z}(n)}, \tag{4.6a}$$

$$q_{1i}(A, \Lambda; r) = \lim_{n\to\infty}\frac{\bar{Q}^{12}_{1i}(n)}{\bar{Z}(n)}, \tag{4.6b}$$

$$q(A, \Lambda; r) = \lim_{n\to\infty}\frac{\bar{Q}^{12}_{11}(n)}{\bar{Z}(n)}. \tag{4.6c}$$

$\bar{Z}(n)$, $\bar{P}^{11}_{1i}(n)$, $\bar{Q}^{12}_{1i}(n)$, $\bar{Q}^{12}_{11}(n)$ as functions of $\bar{Z}(0)$ are solutions of the following iterative scheme

$$\bar{Z}(k) = \frac{2k-1}{2n}\,\bar{Z}(k-1) + \frac{1}{2n}\left[\sum_{j\neq 1} A_{1j}\bar{P}^{11}_{1j}(k-1) + \Lambda(r-1)\,\bar{Q}^{12}_{11}(k-1)\right], \quad k = 1\ 2, \dots n \tag{4.7a}$$

$$\bar{P}^{11}_{1i}(k) = \frac{2k}{2n}\,\bar{P}^{11}_{1i}(k-1) + \frac{1}{2n}\left[A_{1i}\bar{Z}(k) + \sum_{j\neq 1,i} A_{1j}\bar{P}^{11}_{ji}(k) + \Lambda(r-1)\,\bar{Q}^{12}_{ii}(k)\right], \quad k = 0, 1, \dots n \tag{4.7b}$$

$$\bar{Q}^{12}_{1i}(k) = \frac{2k}{2n}\,\bar{Q}^{12}_{1i}(k-1) + \frac{1}{2n}\left\{A_{1i}\bar{Q}^{12}_{ii}(k) + \sum_{j\neq 1,i} A_{1j}\bar{Q}^{12}_{ji}(k) + \Lambda[(r-2)\,\bar{Q}^{23}_{ii}(k) + \bar{P}^{22}_{1i}(k)]\right\}, \quad k = 0, 1, \dots n \tag{4.7c}$$

$$\bar{Q}^{12}_{11}(k) = \frac{2k}{2n}\,\bar{Q}^{12}_{11}(k-1) + \frac{1}{2n}\left\{\sum_{j\neq 1} A_{1j}\bar{Q}^{12}_{j1}(k) + \Lambda[(r-2)\,\bar{Q}^{23}_{11}(k) + \bar{Z}(k)]\right\}, \quad k = 0, 1, \dots n. \tag{4.7d}$$

Here $\bar{Z}(k)$, $\bar{P}^{\alpha\alpha}_{ij}(k)$, $\bar{Q}^{\alpha\beta}_{ij}(k)$, $\bar{Q}^{\alpha\beta}_{ii}(k)$ are related to the corresponding thermodynamic quantities with modified interactions between the reference spin 1_1 and the others v_ν through the expressions (4.4a—d).

V. RESULTING EQUATIONS FOR THE RFIM CORRELATION FUNCTIONS MODELS I, II

All analyses of the RFIM correlation functions have so far been strictly within the replica trick. The iterative scheme (4.5)—(4.7), which is equivalent to the basic formulae for thermodynamic quantities (3.1)—(3.4), simplifies essentially the problem. We have found quantities changing slowly and little during the iteration:

— the value of $\bar{P}_{ij}^{11}(k)/\bar{Z}(k)$ $(i, j \neq 1)$ for $0 \leqq k \leqq n$ is from the range $\langle p_{ij}^{11}(1_1), p_{ij}\rangle$, where $p_{ij}^{11}(1_1)$ is the correlation of spins the i_1 and j_1 provided that the reference spin 1_1 is absent (the spin 1_1 is excluded from the Hamiltonian);

— the value of $\bar{Q}_{i1}^{12}(k)/\bar{Z}(k)$ $(i \neq 1)$ for $0 \leqq k \leqq n$ is from the range $\langle q_{i1}^{12}(1_1), q_{1i}\rangle$, where $q_{i1}^{12}(1_1)$ is the correlation of the spins i_1 and 1_2 in the absence of the spin 1_1. The values of $\bar{Q}_{ji}^{12}(k)/\bar{Z}(k)$, $\bar{Q}_{i1}^{23}(k)/\bar{Z}(k)$, $\bar{P}_{1i}^{22}(k)/\bar{Z}(k)$ and $\bar{Q}_{11}^{23}(k)/\bar{Z}(k)$ are limited analogously.

The exclusion of one spin from the spin system in high dimensions does not change the correlations in its surroundings essentially and so $p_{ij}^{11}(1_1) \approx p_{ij}$, $q_{i1}^{12}(1_1) \approx q_{i1}$, $q_{ji}^{12}(1_1) \approx q_{ji}$, $q_{i1}^{23}(1_1) \approx q_{i1}$, $p_{1i}^{22}(1_1) \approx p_{1i}$, $q_{11}^{23}(1_1) \approx q$. That is why it is reasonable to use, during the iteration, the relations

$$\bar{P}_{ij}^{11}(k) = p_{ij}\bar{Z}(k),\ \bar{Q}_{i1}^{12}(k) = q_{i1}\bar{Z}(k),\ \bar{Q}_{ji}^{12}(k) = q_{ji}\bar{Z}(k),$$
$$\bar{Q}_{i1}^{23}(k) = q_{i1}\bar{Z}(k),\ \bar{P}_{1i}^{22}(k) = p_{1i}\bar{Z}(k),\ \bar{Q}_{11}^{23}(k) = q\bar{Z}(k)$$

for arbitrary $k = 0, 1, \dots n$. Using these assumptions the iterative scheme (4.7) results in equations for the RFIM correlation functions of model I

$$g_{1i}(A, \Lambda) = \lim_{r\to 0}[p_{1i}(A, \Lambda; r) - q_{1i}(A, \Lambda; r)], \tag{5.1a}$$

$$p_{1i} = \bar{A}_{1i} + \sum_{j\neq 1,i} \bar{A}_{1j}p_{ji} + \bar{\Lambda}(r-1)\,q_{1i}, \tag{5.1b}$$

$$q_{1i} = q\bar{A}_{1i} + \sum_{j\neq 1,i} \bar{A}_{1j}q_{ji} + \bar{\Lambda}(r-2)\,q_{1i} + \bar{\Lambda}p_{1i}, \tag{5.1c}$$

$$q = \sum_{j\neq 1} \bar{A}_{1j}q_{1j} + \bar{\Lambda}(r-2)\,q + \bar{\Lambda}, \tag{5.1d}$$

where the notation

$$\bar{A}_{uv} = \frac{\tanh\alpha}{\alpha}A_{uv},\qquad \bar{\Lambda} = \frac{\tanh\alpha}{\alpha}\Lambda, \tag{5.1e}$$

$$(\tanh\alpha)^2 = \sum_{j\neq 1} \bar{A}_{1j}p_{1j} + \bar{\Lambda}(r-1)\,q \tag{5.1f}$$

is used for the renormalized parameters.

In order to understand the influence of the random fields on the thermodynamic properties of the spin system within the proposed approximation we consider the pure case $(\Lambda \equiv 0)$ first. As $q_{ij} = q = 0$ and $g_{ij} = p_{ij}$ we write immediately

$$g_{1i} = \bar{A}_{1i} + \sum_{j\neq 1,i} \bar{A}_{1j}g_{ji}, \tag{5.2a}$$

$$(\tanh\alpha)^2 = \sum_{j\neq 1} \bar{A}_{1j}g_{1j}. \tag{5.2b}$$

The Fourier transformation of the correlation function

$$G(\boldsymbol{k}) = \frac{\bar{A}(\boldsymbol{k}) - \sum_{j\neq 1}\bar{A}_{1j}g_{1j}}{1 - \bar{A}(\boldsymbol{k})}, \tag{5.3a}$$

$$\bar{A}(\boldsymbol{k}) = \sum_{j\neq 1}\bar{A}_{1j}\exp(\mathrm{i}\boldsymbol{k}\,.\,\boldsymbol{r}_{1j}) \tag{5.3b}$$

diverges at $\boldsymbol{k} = \boldsymbol{O}$ at the critical point $\bar{A}_c(\boldsymbol{O}) = 1$. As

$$(\tanh\alpha)^2 = 1 - \left\{\int_{BZ}\frac{\mathrm{d}^d k}{V_{BZ}}\left[\frac{1}{1-\bar{A}(\boldsymbol{k})}\right]\right\}^{-1}, \tag{5.4}$$

$T_c = 0$ $(\alpha_c \to \infty)$ for $d \leqq 2$ because of the divergency of Green's lattice function. For $d > 2$ model I is substantially more exact than a random-phase approximation, for example. The estimation of the critical points is satisfactory as well.

In spite of the incorrectness of $LCD = 2$ for the pure Ising systems, model I can explain the influence of the random fields on the Ising spins. Provided that $\Lambda \not\equiv 0$, we have from (5.1), in the limit $r \to 0$,

$$G(\boldsymbol{k}) = \frac{(1-q)\,\bar{A}(\boldsymbol{k}) - \sum_{j\neq 1}\bar{A}_{1j}g_{1j}}{1 + \bar{\Lambda} - \bar{A}(\boldsymbol{k})} \tag{5.5a}$$

$$q = \frac{\bar{\Lambda}I_2}{\bar{\Lambda}I_2 + I_1}, \tag{5.5b}$$

$$(\tanh\alpha)^2 = q + (1-q)\left(1 - \frac{1}{I_1}\right), \tag{5.5c}$$

with I_a defined by

$$I_a = \int_{BZ}\frac{\mathrm{d}^d k}{V_{BZ}}\frac{1}{[1 + \bar{\Lambda} - \bar{A}(\boldsymbol{k})]^a}\qquad \text{for } a = 1, 2\dots \tag{5.5d}$$

The divergency of I_2 at the critical point $\bar{A}_c(\boldsymbol{O}) = 1 + \bar{\Lambda}_c$ for $d \leqq 4$ implies $q_c = 1$ for arbitrary $\Lambda \not\equiv 0$. The resulting critical point $\alpha_c \to \infty$ $(T_c = 0)$ is not a physical one as $(g_{ij})_c = 0$ for arbitrary $i \neq j$. Therefore, the ground state cannot be ferromagnetic for $d \leqq 4$ in this approximation.

Model I, neglecting the change of correlations in the surroundings of the reference spin 1_1 caused by its absence, satisfactorily accounts for the critical behaviour above $d = 4$. As the approximation fails for $d \leqq 4$ (there is no critical point), we shall consider more realistic plots of $\bar{P}_{ij}^{\alpha\alpha}(k)$, $\bar{Q}_{ij}^{\alpha\beta}(k)$, $\bar{Q}_{ii}^{\alpha\beta}(k)$. By analogy with [16], we write

$$\bar{P}_{ij}^{\alpha\alpha}(k) = \left[\frac{k}{n} p_{ij} + \left(1 - \frac{k}{n}\right) p_{ij}^{\alpha\alpha}(1_1)\right] \bar{Z}(k), \tag{5.6a}$$

$$\bar{Q}_{ij}^{\alpha\beta}(k) = \left[\frac{k}{n} q_{ij} + \left(1 - \frac{k}{n}\right) q_{ij}^{\alpha\beta}(1_1)\right] \bar{Z}(k), \tag{5.6b}$$

$$\bar{Q}_{ii}^{\alpha\beta}(k) = \left[\frac{k}{n} q + \left(1 - \frac{k}{n}\right) q_{ii}^{\alpha\beta}(1_1)\right] \bar{Z}(k). \tag{5.6c}$$

Here the boundary conditions

$$\frac{\bar{P}_{ij}^{\alpha\alpha}(k)}{\bar{Z}(k)} = p_{ij} \qquad \text{for } k \to n,$$

$$= p_{ij}^{\alpha\alpha}(1_1) \qquad \text{for } k \to 0, \text{ etc.}$$

are fulfilled. Putting (5.6a—c) into the iterative scheme (4.5)—(4.7), we arrive at the equations for the correlation functions of model II

$$g_{1i}(A, \Lambda) = \lim_{r \to 0} [p_{1i}(A, \Lambda; r) - q_{1i}(A, \Lambda; r)], \tag{5.7a}$$

$$p_{1i} = \bar{A}_{1i} + \sum_{j \neq 1, i} \bar{A}_{1j} p_{ji} + \bar{\Lambda}(r-1) q_{1i} - - L(\alpha, \beta)\left[\sum_{j \neq 1, i} \bar{A}_{1j} \delta p_{ji}^{11} + \bar{\Lambda}(r-1)\, \delta q_{1i}^{21}\right], \tag{5.7b}$$

$$q_{1i} = q\bar{A}_{1i} + \sum_{j \neq 1, i} \bar{A}_{1j} q_{ji} + \bar{\Lambda}(r-2) q_{1i} + \bar{\Lambda} p_{1i} - L(\alpha, \beta) \times \times \left[\bar{A}_{1i} \delta q_{ii}^{12} + \sum_{j \neq 1, i} \bar{A}_{1j} \delta q_{ji}^{12} + \bar{\Lambda}(r-2)\, \delta q_{1i}^{23} + \bar{\Lambda} \delta p_{1i}^{22}\right], \tag{5.7c}$$

$$q = \sum_{j \neq 1} \bar{A}_{1j} q_{1j} + \bar{\Lambda}(r-2) q + \bar{\Lambda} - - L(\alpha, \beta)\left[\sum_{j \neq 1} \bar{A}_{1j} \delta q_{j1}^{12} + \bar{\Lambda}(r-2)\, \delta q_{11}^{23}\right], \tag{5.7d}$$

where

$$L(\alpha, \beta) = \frac{2}{\beta^2}\left(1 - \frac{\tanh \beta}{\beta}\right) \frac{\alpha}{\tanh \alpha}, \tag{5.7e}$$

$$\beta^2 = \sum_{j \neq 1} A_{1j}^2 + \sum_{i,j \neq 1}' A_{1i} A_{1j} p_{ij}^{11}(1_1) + 2\Lambda(r-1) \sum_{j \neq 1} A_{1j} \times \times q_{1j}^{21}(1_1) + \Lambda^2(r-1)[(r-2) q_{11}^{23}(1_1) + 1], \tag{5.7f}$$

and $\delta p_{ij}^{\alpha\alpha} = p_{ij} - p_{ij}^{\alpha\alpha}(1_1)$, $\delta q_{ij}^{\alpha\beta} = q_{ij} - q_{ij}^{\alpha\beta}(1_1)$, $\delta q_{ii}^{\alpha\beta} = q - q_{ii}^{\alpha\beta}(1_1)$.

VI. DISCUSSION AND CONCLUDING REMARKS

The set of equations (5.7a—f) contains two types of the correlation functions — with and without the reference spin 1_1. In order to solve it the following procedure is proposed. First, we choose suitable equations, namely the resulting equations of model I written in a symmetric form

$$p_{ij}^{\alpha\alpha} = \bar{A}_{ij} + \frac{1}{2} \sum_{k \neq i, j} (\bar{A}_{ik} p_{kj}^{\alpha\alpha} + \bar{A}_{jk} p_{ki}^{\alpha\alpha}) + \frac{\bar{\Lambda}}{2} \sum_{\beta \neq \alpha} (q_{ij}^{\alpha\beta} + q_{ji}^{\alpha\beta}), \tag{6.1a}$$

$$q_{ij}^{\alpha\beta} = \frac{1}{2} \bar{A}_{ij}(q_{ii}^{\alpha\beta} + q_{jj}^{\alpha\beta}) + \frac{1}{2} \sum_{k \neq i, j} (\bar{A}_{ik} q_{kj}^{\alpha\beta} + \bar{A}_{jk} q_{ki}^{\beta\alpha}) + + \frac{\bar{\Lambda}}{2} \sum_{\gamma \neq \alpha, \beta} (q_{ij}^{\alpha\gamma} + q_{ji}^{\beta\gamma}) + \frac{\bar{\Lambda}}{2} (p_{ij}^{\alpha\alpha} + p_{ij}^{\beta\beta}), \tag{6.1b}$$

$$q_{ii}^{\alpha\beta} = \frac{1}{2} \sum_{j \neq i} (\bar{A}_{ij} q_{ji}^{\alpha\beta} + \bar{A}_{ij} q_{ji}^{\beta\alpha}) + \frac{\bar{\Lambda}}{2} \sum_{\gamma \neq \alpha, \beta} (q_{ii}^{\alpha\gamma} + q_{ii}^{\beta\gamma}) + \bar{\Lambda} \tag{6.1c}$$

by use of which the correlations with the excluded reference spin 1_1 are generated. The exclusion of spin 1_1 from (6.1a—c) yields the following equations for small differences $\delta p_{ij}^{\alpha\alpha}$, $\delta q_{ij}^{\alpha\beta}$, $\delta q_{ii}^{\alpha\beta}$

$$2\delta p_{ij}^{11} = \bar{A}_{i1} p_{1j} + \bar{A}_{j1} p_{1i} + \sum_{k \neq 1, i, j} (\bar{A}_{ik} \delta p_{kj}^{11} + \bar{A}_{jk} \delta p_{ki}^{11}) + + \bar{\Lambda}(r-1)(\delta q_{ij}^{12} + \delta q_{ji}^{12}), \tag{6.2a}$$

$$2\delta q_{i1}^{12} = \bar{A}_{i1} q + \bar{\Lambda} p_{1i} + \sum_{j \neq 1, i} \bar{A}_{ij} \delta q_{j1}^{12} + \sum_{j \neq 1, i} \bar{A}_{1j} \delta q_{ji}^{21} + + \bar{A}_{i1} \delta q_{ii}^{12} + \bar{\Lambda} \delta p_{1i}^{22} + \bar{\Lambda}(r-2)(\delta q_{i1}^{12} + \delta q_{1i}^{23}), \tag{6.2b}$$

$$2\delta q_{ii}^{12} = \bar{A}_{i1} q_{1i} + \sum_{j \neq 1, i} \bar{A}_{ij} \delta q_{ji}^{12} + \sum_{j \neq 1, i} \bar{A}_{ij} \delta q_{ji}^{21} +$$

$$+ \bar{A}_{i1}\,\delta q_{1i}^{21} + \bar{\Lambda}(r-2)(\delta q_{ii}^{12} + \delta q_{ii}^{23}), \tag{6.2c}$$

$$2\delta q_{ji}^{12} = \bar{A}_{j1} q_{1i} + \sum_{k \neq 1,i,j} (\bar{A}_{jk}\,\delta q_{ki}^{12} + \bar{A}_{ik}\,\delta q_{kj}^{21}) + + \bar{A}_{ji}\,\delta q_{ii}^{12} + \bar{A}_{ij}\,\delta q_{jj}^{12} + \bar{A}_{i1}\,\delta q_{1j}^{21} + \bar{\Lambda}(\delta p_{ij}^{11} + \delta p_{ij}^{22}) + + \bar{\Lambda}(r-2)(\delta q_{ji}^{32} + \delta q_{ij}^{21}), \tag{6.2d}$$

$$2\delta q_{1i}^{23} = \bar{\Lambda} q_{1i} + \sum_{j \neq 1,i} \bar{A}_{1j}\,\delta q_{ji}^{23} + \sum_{j \neq 1,i} \bar{A}_{ij}\,\delta q_{j1}^{32} + \bar{A}_{1i}\,\delta q_{ii}^{23} + + \bar{A}_{i1} q_{11}^{23} + 2\bar{\Lambda}\delta p_{1i}^{22} + 2\bar{\Lambda}(r-2)\,\delta q_{1i}^{23} + \bar{\Lambda}\delta q_{i1}, \tag{6.2e}$$

$$2\delta p_{1i}^{22} = \bar{\Lambda} q_{1i} + \sum_{j \neq 1,i} (\bar{A}_{1j}\,\delta p_{ji}^{22} + \bar{A}_{ij}\,\delta p_{j1}^{22}) + + 2\bar{\Lambda}(r-2)\,\delta q_{1i}^{23} + \bar{\Lambda}\delta q_{i1}^{12}, \tag{6.2f}$$

$$2\delta q_{11}^{23} = 2\bar{\Lambda} q + 2\sum_{j \neq 1} \bar{A}_{1j}\,\delta q_{j1}^{23} + 2\bar{\Lambda}(r-3)\,\delta q_{11}^{23}. \tag{6.2g}$$

The system of equations (6.2a—g) has to be solved iteratively. The leading terms in the lowest power of the parameters A_{ij}, $\bar{\Lambda}$

$$\delta p_{ij}^{11} \approx \frac{1}{2}(\bar{A}_{i1} p_{1j} + \bar{A}_{j1} p_{1i}), \qquad (i \neq j) \neq 1 \tag{6.3a}$$

$$\delta q_{i1}^{12} \approx \frac{1}{2}(\bar{A}_{i1} q + \bar{\Lambda} p_{1i}), \qquad i \neq 1 \tag{6.3b}$$

$$\delta q_{ii}^{12} \approx \frac{1}{2}\bar{A}_{i1} q_{1i}, \qquad i \neq 1 \tag{6.3c}$$

$$\delta q_{ji}^{12} \approx \frac{1}{2}\bar{A}_{j1} q_{1i}, \qquad (i \neq j) \neq 1 \tag{6.3d}$$

$$\delta q_{1i}^{23} \approx \frac{1}{2}\bar{\Lambda} q_{1i}, \qquad i \neq 1 \tag{6.3e}$$

$$\delta p_{1i}^{22} \approx \frac{1}{2}\bar{\Lambda} q_{1i}, \qquad i \neq 1 \tag{6.3f}$$

$$\delta q_{11}^{23} \approx \bar{\Lambda} q, \tag{6.3g}$$

after substituting into the right-hand sides of (6.2a—g) yield the expressions for $\delta p_{ij}^{\alpha\alpha}$, $\delta q_{ij}^{\alpha\beta}$, $\delta q_{ii}^{\alpha\beta}$ in the second power of A_{ij}, $\bar{\Lambda}$ and so on.

In the case of the vanishing random fields ($\Lambda \equiv 0$) the set (5.7a—f) in the lowest approximation (6.3a) implies the right $LCD = 1$. Having the right value of the LCD for the pure Ising models we expected model II for $\Lambda \neq 0$ to be applicable in the interesting region $d \leqq 4$. However, the expansion of $\delta p_{ij}^{\alpha\alpha}$, $\delta q_{ij}^{\alpha\beta}$, $\delta q_{ii}^{\alpha\beta}$ up to arbitrary finite powers of the renormalized parameteres $\bar{A}_{ij}$, $\bar{\Lambda}$ does not change the conclusions of model I, i.e. the equations (5.7a—f) do not give a critical point below $d = 4$ because of the I_2 divergency. This anomaly occurs in the RFIM with a bimodal random-field distribution [17] as well. The failure of the iterative method below $d = 4$ may be explained as follows:

i) the problem was investigated within the replica trick introduced in Sec. II. The questionable limit $r \to 0$ (corresponding to the zero spins on an arbitrary site) is used in the final calculation of the correlation functions. Therefore, the treatment of the replica-spin system S' as a uniform one would be incorrect;

ii) the consideration of all powers of $\bar{A}_{ij}$, $\bar{\Lambda}$ in (5.7a—f) can help elucidate the region $d \leqq 4$. In a nonperturbative approach the quantities p_{ij}, q_{ij}, q interchange the pair of the parameters $\bar{A}_{ij}$, $\bar{\Lambda}$ which are irrelevant from the point of view of the transition points. We suggest that for $d \leqq 2$ the divergency of I_1 implies $(g_{ij})_c = 0$ at $T_c = 0$, while the supposed absence of I_2 in the resulting equations for the correlation functions admits the existence of a critical point for $d > 2$.

If the $d > 4$ model II gives a satisfactory account of the critical behaviour and determines the transitions points more exactly when compared to model I.

For future investigations a nonperturbative approach to the set of equations (5.7a—f) is necessary.

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НОВЫЙ ИТЕРАЦИОННЫЙ ПОДХОД К МОДЕЛИ ИЗИНГА В СЛУЧАЙНОМ ПОЛЕ С ГАУССОВСКИМ РАСПРЕДЕЛЕНИЕМ СЛУЧАЙНЫХ ПОЛЕЙ

Цель данной работы состоит в применении одного итерационного метода к модели Изинга в случайном поле, для которой были рассмотрены корреляционные функции. Проблема переформулирована при помощи модельного приема для вычисления некоторых корреляций в чистой спиновой системе S'. Специальная запись суммы δ-функций дает новое графическое представление термодинамических величин в системе S'. Понижение числа точек на любом относительном спине в этом графическом представлении приводит к некоторой итерационной схеме. Найдены величины, которые в процессе итерации изменяются весьма мало и медленно. Простые предположения относительно их форм соответствуют тому, что мы называем моделью I и моделью II. Модель I основана на предположении о независимости корреляций в окрестности относительного спина от его присутствия или отсутствия. Это дает удовлетворительное описание критического поведения выше $d = 4$. Более реалистические формы корреляций в модели II позволяют сделать вычисления в модели I более точными.