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THE SYMMETRY PROBLEM OF THE ENERGY-MOMENTUM TENSOR AND THE CREATION OF A GRAVITATIONAL FIELD

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The symmetry problem of the energy-momentum tensor of an electromagnetic field in a medium is considered as a general problem of the Lagrange formalism with an external field. It is shown that the asymmetrical Minkowski tensor, which is the "*symmetrized*" canonical energy-momentum tensor, describes correctly the experiment in a homogeneous medium. But we need the symmetrical and the conserved source of a gravitationalfield, while the symmetrical Hilbert tensor, which should be used in this role, is not conserved. Hence a formal method of obtaining such a source is proposed.

1. INTRODUCTION

The symmetry of the energy-momentum tensor (EMT) is required for the conservation of the angular momentum of momentum (MM), the symmetry of its space-time components only for the invariance of this conservation with respect to the Lorentz transformations. With the development of field-theory it was shown that only the sum of the angular and the spin MM must be conserved and this conservation gives a general Belinfante method of obtaining the symmetrical EMT from the canonical one. However, when the Lagrange function depends on functions — let us call them external fields — which do not satisfy the Lagrange equations, the total MM need not to be conserved and the Belinfante "*symmetrization*" gives an asymmetrical EMT.

Just such a situation is considered in the present paper. It is shown that also the well-known Minkowski EMT of the electromagnetic field (EMF) in a continuous homogeneous medium is the result of such a "*symmetrization*". The problem of the EMT of the EMF in a medium has frequently been discussed in literature, it is considered rather in detail in [1]. However, the understanding of the physical meaning of this quantity has not been exact: the better the EMT the action of the EMF on the medium describes, the better the EMT of the EMF in a medium it has been regarded. However, we need the EMT describing the sum of a 4-momentum of the EMF and of the variation of the 4-momentum of medium caused by the field, only such a tensor will be called "*the* EMT *of the* EMF *in a medium*". And such an EMT must not describe the forces acting by the EMF on the medium, i.e. the momentum-exchange between the EMF and the medium, because the variation of both momenta has to be described by this EMT. Those forces are given exactly by the EMT of the EMF itself. In the present paper we shall see that in a homogeneous medium the sum of densities of the 4-momentum of the EMF and of the variation of the 4-momentum of the medium caused by the field is given by Minkowski EMT.

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The original aim of the present paper is to obtain the source of a weak gravitational field (GF) generated by a variable EMF in the medium. It should be given just by the above defined EMT of the EMF in the medium, however, it has to be conserving and symmetrical, because such has to be the source of the total GF and such is also the source of the GF in the absence of the EMF. But the Minkowski EMT is not symmetrical and the corresponding Hilbert tensor, which should be the source in the sense of the Einstein equations, is not conserved. Thus, the present paper is devoted to the solution of all these problems.

We use the natural units, i.e. $c = 1$.

2. CANONICAL FORMALISM WITH EXTERNAL FIELDS

We shall work in a Minkowski space with the metrics

$$\eta_{\mu\nu} = \mathrm{diag}\,(1,\ -1,\ -1,\ -1) \qquad (2.1)$$

and consider only the Lagrange function of a non-gravitational matter

$$L(\varphi_a,\ g_{\mu\nu},\ \varepsilon_a)\,, \qquad (2.2a)$$

in which we put

$$g_{\mu\nu} = \eta_{\mu\nu} \qquad (2.2b)$$

(but this cannot be done before obtaining the Hilbert EMT, see eq. (2.8)). Here φ_a are all the considered physical fields satisfying the Lagrange equation

$$\delta L/\delta\varphi_a\!: = \partial L/\partial\varphi_a - \partial_\mu \partial L/\partial\varphi_{a,\mu} = 0 \qquad (2.3)$$

and ε_a are the "external fields", not satisfying this equation. The indices $a, b \ldots$ denote various fields and their tensor indices.

In the absence of an external field there can be obtained a conserving, canonical EMT [2]

$$a)\ T_{ca}^{\beta} = \sum_a (\partial L/\partial \varphi_{a,\beta})\, \varphi_{a,\alpha} - \delta_\alpha^\beta L; \tag{2.4}$$

$$b)\ T_{c,\beta}^{\beta} = 0$$

and the tensor of the spin-momentum

$$a)\ S^{\alpha\beta\mu} = \sum_{a,b} (\partial L/\partial \varphi_{a,\mu})\, \Omega_a^{b\alpha\beta} \varphi_b, \tag{2.5}$$

$$b)\ \partial_\mu [S^{\alpha\beta\mu} + M^{\alpha\beta\mu}] = 0,$$

$$c)\ M^{\alpha\beta\mu} = x^\alpha T_c^{\beta\mu} - x^\beta T_c^{\alpha\mu},$$

where $\Omega_a^{b\alpha\beta}$ is a rotation matrix. However, if the variation of the form of some field vanishes during the transformation, generating the considered conservation law, the field does not violate the conservation law, whereby we need not any additional restrictions of the kind (2.3). Therefore, the EMT conserves also in the presence of a constant external field

$$\varepsilon_a = \text{const.}, \tag{2.4c}$$

the form of the latter being invariant under translation. But the tensor of the total MM is not conserved, except for a trivial case of a scalar constant field.

The tensor (2.4a) can be asymmetrical. Then the Belinfante symmetrization procedure:

$$a)\ T_s^{\alpha\beta} = T_c^{\alpha\beta} + \frac{1}{2}(S^{\beta\mu\alpha} + S^{\alpha\mu\beta} - S^{\alpha\beta\mu})_{,\mu}; \tag{2.6}$$

$$b)\ \partial_\beta T_s^{\alpha\beta} \equiv \partial_\beta T_c^{\alpha\beta}$$

gives a symmetrical quantity T_s if the relations (2.5b, c) hold. So in the presence of a constant external field the EMT conserves, but the "*symmetrization*" (2.6) does not give a symmetrical tensor. Any EMT can always be symmetrized using the quantity $M^{\alpha\beta\mu}$ instead of $S^{\alpha\beta\mu}$ in the latter symmetrization:

$$a)\ T_T^{\alpha\beta} = 4T^{(\alpha\beta)} + x^\varrho \partial_\varrho T^{(\alpha\beta)} + \frac{1}{2}[x^\alpha T^{\varrho\beta} + x^\beta T^{\varrho\alpha}]; \tag{2.7}$$

$$b)\ T^{(\alpha\beta)}{:}= \frac{1}{2}(T^{\alpha\beta} + T^{\beta\alpha}).$$

But this EMT has, obviously, no physical meaning, depending on an arbitrary choice of the origin of space-time coordinates.

The Hilbert tensor

$$\sqrt{g}\, T_H^{\alpha\beta}{:}= 2\delta(\sqrt{g}\, L)/\delta g_{\alpha\beta}; \qquad g{:}= |\det g_{\alpha\beta}|, \tag{2.8}$$

is always symmetrical and according to ref. [1, 3] should satisfy the relation

$$T_H^{\alpha\beta} = T_s^{\alpha\beta}. \tag{2.9a}$$

However, the last relation is possible only providing

$$\varepsilon_a = 0, \tag{2.9b}$$

the same condition being required by the conservation law

$$\partial_\beta T_H^{\alpha\beta} = 0, \tag{2.9c}$$

since the latter is ensured only by the invariance of action under a variable displacement of coordinates.

3. THE ELECTROMAGNETIC FIELD IN A CONTINUOUS MEDIUM

First of all we have to remember that we need practically the EMT describing the sum of the 4-momentum of the field and of the variation of the 4-momentum of the medium caused by the field. Therefore, the EMT has to satisfy the condition

$$\partial_\beta T^{\alpha\beta} = -f^\alpha{:}= -F_\mu^{\ \alpha} j^\mu, \tag{3.1}$$

where j^μ describes only the conduction currents and

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{3.2}$$

is the tensor of the electromagnetic field. Such an EMT is known only in a homogeneous medium as a Minkowski tensor:

$$T_M^{\alpha\beta} = \frac{1}{4\pi}\left(-F^{\alpha\varrho} H^\beta_{\ \varrho} + \frac{1}{4} g^{\alpha\beta} F_{\varrho\sigma} H^{\varrho\sigma}\right), \tag{3.3}$$

here

$$a)\ H^{\alpha\beta} = \varepsilon^{\alpha\beta\varrho\sigma} F_{\varrho\sigma}, \tag{3.4}$$

$$b)\ \varepsilon^{\beta\alpha\varrho\sigma} = \varepsilon^{\alpha\beta\sigma\varrho} = -\varepsilon^{\alpha\beta\varrho\sigma} = -\varepsilon^{\varrho\sigma\alpha\beta}.$$

The last relation follows from the definition of the permitivity tensor $\varepsilon^{\alpha\beta\varrho\sigma}$ by means of its value in the rest frame of the medium (assuming its existence):

$$H^{ik} = \mu^{-1} F^{ik}, \qquad H^{ok} = \varepsilon F^{ok}, \tag{3.4c}$$

using the harmonic coordinates [4, 5, 6, 7]

$$\partial_\mu q^{\mu\nu} = 0; \qquad q^{\mu\nu}{:}= \sqrt{g}\, g^{\mu\nu}. \tag{3.5}$$

The field equations

$$\partial_\alpha H^{\alpha\beta} = 4\pi j\beta \tag{3.6}$$

are obtained from the lagrangian

$$L = -\frac{1}{16\pi} g_{\alpha\varrho} g_{\beta\sigma} F^{\alpha\beta} H^{\varrho\sigma} - g_{\alpha\beta} A^{\alpha} j^{\beta}. \tag{3.7a}$$

Further we put

$$j^{\beta} = 0. \tag{3.7b}$$

Then

$$\text{a)}\ T_{e\alpha}{}^{\beta} = \frac{1}{4\pi}\left[A_{\varrho,\alpha} H^{\varrho\beta} + \frac{1}{4}\delta^{\beta}_{\alpha} F_{\varrho\sigma} H^{\varrho\sigma}\right], \tag{3.8}$$

$$\text{b)}\ S^{\alpha\beta\mu} = \frac{1}{4\pi}[H^{\mu\beta}A^{\alpha} - H^{\mu\alpha}A^{\beta}],$$

and we can easily check that the Minkowski EMT (3.3) is in fact the result of the Belinfante "*symmetrization*" (2.6) of the canonical tensor (3.8a). In a homogeneous medium it conserves in the case (3.7b), but is not symmetrical, in accordance with the previous section. The Hilbert tensor (eq. (2.8)) corresponding to our lagrangian is not conserved and we are completely unable to obtain a symmetrical, conserved EMT from this lagrangian.

In literature also the EMT of Abraham is considered:

$$T_A^{i0}: = T_A^{0i}: = T_M^{0i}, \qquad T_A^{ik} = T_M^{ik}, \tag{3.9}$$

which is useless for us, being not conserved in a homogeneous, chargeless medium. There exists also the "*radiation tensor*", in the rest frame of the medium defined as

$$T_r^{i\alpha} = (\varepsilon\mu)^{-1} T_M^{i\alpha} = T_r^{\alpha i}, \tag{3.10}$$

giving by a $\varepsilon\mu$-times smaller force-density than the condition (3.1) requires.

Now we shall see that only the asymmetrical Minkowski EMT gives a correct relation of the energy- and momentum-densities, required by the 4-momentum conservation in the Tsherenkov radiation. Denoting the 4-momentum of the radiated photon as δk^{α} and the corresponding decrease of the 4-momentum of the radiating electron as δp^{α}, we have

$$\delta p^0 = \delta k^0, \qquad |\delta \boldsymbol{p}| = |\delta \boldsymbol{k}| \cos\varphi, \tag{3.11a}$$

where

$$\cos\varphi = (nv)^{-1}, \qquad \boldsymbol{n}: = \sqrt{\varepsilon\mu}, \qquad v = \boldsymbol{p}/p^0. \tag{3.11b}$$

Relation $\delta p^2 = 0$ gives $p^0 \delta p^0 = |\boldsymbol{p}|\,|\delta\boldsymbol{p}|$ and so

$$|\delta\boldsymbol{k}/\delta k^0| = n. \tag{3.11c}$$

But just this is the ratio which gives the momentum-density

$$T_M^{k0} = \frac{1}{3\pi}(\boldsymbol{D} \times \boldsymbol{B})_k \tag{3.12}$$

with the energy-density T_M^{00} in a plane wave and, because the velocity of propagation of both quantities is the same, also the corresponding flux-densities. Similar conclusions were first obtained by Ginzburg [8].

Thus the correct EMT of the EMF in a continuous medium has to be asymmetrical. It gives the enlarged ratio (3.11c) with respect to the velocity of the field-propagation moving the "*photon*". The latter fact itself has no physical meaning because δk^0 describes only a part of the energy-density, but it implies the Lorentz non-invariance of conservation of the corresponding angular MM.

But althoug we do not consider a total energy-momentum of the EMF and the medium, we are looking for such an EMT, the variation of which, caused by the variation of the field, is equal to the variation of the total EMT. Just such a quantity would described the creation of the variable part of a weak GF (satisfying the principle of superposition) by the variable EMF in the medium. This requirement is mathematically described by the condition (3.1), because the EMT describing some system cannot give the force-density between the parts of this system.

4. THE SOURCE OF THE GF CREATED BY THE ELECTROMAGNETIC FIELD IN A MEDIUM

The resulting difficulties with the EMT are connected also with the approximation of the local relation (3.4a) between the tensors F and H. In the case of a homogeneous medium, we are able to introduce such a non-local term into the lagrangian, which influences neither the canonical EMT nor the dynamical equations of other (non external) fields, ensuring the Lagrange equations identically satisfied by the external field:

$$\text{a)}\ L_{\varepsilon} = L + A^{\alpha\beta\gamma\delta\varrho}\partial_{\varrho}\varepsilon_{\alpha\beta\gamma\delta}, \tag{4.1}$$

$$\text{b)}\ \partial_{\varrho} A^{\alpha\beta\gamma\delta\varrho}: = \delta L/\delta\varepsilon_{\alpha\beta\gamma\delta}.$$

Obviously, the canonical EMT, corresponding to both these lagrangians, do not differe when

$$\partial_{\varrho}\varepsilon_{\alpha\beta\gamma\delta} = 0. \tag{4.1c}$$

Let us obtain the part of the spin MM (2.5a), corresponding to the "*field*" of permitivity. Due to the symmetry-properties (3.4b) of the latter, it can be written as

$$\text{a)}\ S^{\alpha\beta\mu} = 4A^{\alpha\varrho\sigma\tau\mu}\varepsilon^{\beta}_{\varrho\sigma\tau} - 4A^{\beta\varrho\sigma\tau\mu}\varepsilon^{\alpha}_{\varrho\sigma\tau}, \tag{4.2}$$

$$\text{b)}\ \partial_{\mu} S^{\alpha\beta\mu} = T_M^{\alpha\beta} - T_M^{\beta\alpha},$$

$$\text{c)}\ T_M^{\alpha\beta} - T_M^{\beta\alpha} = -\frac{1}{4\pi}[F^{\alpha\varrho}H^{\beta}_{\varrho} - F^{\beta\varrho}H^{\alpha}_{\varrho}].$$

Of course, the determination of $S^{\alpha\beta\varrho}$ can start just by the relation (4.2b), symmetrizing any EMT. The previous consideration is useful to understand the physical sence of this symmetrization, which gives

$$T^{\alpha\beta} = T_H^{\alpha\beta} + \frac{1}{2}\,\partial_\varrho(S^{\alpha\varrho\beta} + S^{\beta\varrho\alpha}). \tag{4.3}$$

In the considered case, the Hilbert EMT has here the form

$$T_H^{\alpha\beta} = -\frac{1}{8\pi}\left[F^{\alpha\varrho}H^{\beta}_{\varrho} + H^{\alpha\varrho}F^{\beta}{}_{\varrho} - \frac{1}{2}\,g^{\alpha\beta}F_{\varrho\sigma}H^{\varrho\sigma}\right]. \tag{4.4}$$

The addition to the Hilbert tensor, ensuring the conservation of the obtained quantity (4.3), is given by condition (4.2b) ambiguously, so we can put

$$S^{\alpha\beta\mu} = \partial^\mu\Phi^{\alpha\beta},\ \partial_\mu\partial^\mu\Phi^{\alpha\beta}{}_{;} = T_M^{\alpha\beta} - T_M^{\beta\alpha}; \tag{4.5a}$$

then

$$T^{\alpha\beta} = T_H^{\alpha\beta} + \frac{1}{2}\,\partial_\varrho(\partial^\beta\Phi^{\alpha\varrho} + \partial^\alpha\Phi^{\beta\varrho}). \tag{4.5b}$$

At first sight one could conclude that the given method gives an inadmissible appearance of the source of the GF outside the medium. However, one has to remember that this source serves just to ensure the conservation of the source inside the homogeneous medium; already on its boundary we do not have even a correct expression for $T_M^{\alpha\beta}$. But the contribution of the thin boundary layer to GF can be neglected, and outside the medium a usual Hilbert EMT in the role of a source of GF has to be used, rejecting the non-local additional term.

The creation of a weak GF can be best described by the equations [5]

$$\text{a)}\ q^{\varrho\sigma}\partial_\varrho\partial_\sigma q^{\alpha\beta} = q^{\alpha\varrho}_{,\sigma}q^{\beta\sigma}_{,\varrho} + 2\varkappa(-g)\,[t_L^{\alpha\beta} + T^{\alpha\beta}],$$
$$\text{b)}\ \partial_\alpha q^{\alpha\beta} = 0, \tag{4.6}$$

giving in the first order of $\varkappa$ simply

$$\partial_\varrho\partial^\varrho q^{\alpha\beta} = 2\varkappa T^{\alpha\beta}. \tag{4.6c}$$

Here the reader has to be warned of an incorrect statement occurring in literature [9], that in this approximation the source on the right-hand side includes the contribution of the GF. The latter statement originates in the following circumstance: when the source of the GF is a large, static bod, only the stresses caused by gravitational forces contribute to T^{ik}, so that the lowest order of contribution to q^{ik} is the second. In this order the source is in fact contributed, besides T, also by other terms of the right-hand side of eq. (4.6a). But in the first order only the EMT of the nongravitational matter has to be used

in the role of the source, taking no GF into account (the latter can, however, influence, e.g., the distribution of the mass).

The solution of a weak gravitational wave has the simple form

$$q^{\alpha\beta} = \frac{4k}{r}\int_{(ret)} \dot{T}^{\alpha\beta}\,\mathrm{d}V; \qquad \varkappa = 8\pi k. \tag{4.7}$$

Due to the conservation of $T^{\alpha\beta}$, the lowest non-vanishing order of expansion with respect to retarded time is a quadrupole one, i.e. the zero order for T^{ik}, the first-order for T^{0k} and the second order for T^{00} and the expression

$$\dot{q}^{ik} = \frac{k}{r}\int \dddot{T}^{00}x^i x^k\,\mathrm{d}V \tag{4.8}$$

can be obtained.

Here again one has to avoid the incorrect statement that the trace of space components $\dot{q}^{ik}$ and the components $\dot{q}^{0\alpha}$ can be removed by means of a transformation of the coorddinates conserving the condition (4.6b). Due to Fock's unique "*theorem*" for harmonic coordinates [4], for the case of a linear approximation correctly proved by Todorov [10] who used a more accurate mathematical formulation of the assumptions, no more non-linear transformations of coordinates can be performed without introducing an additional, non-physical ingoing gravitational wave (linear transformations are excluded by asymptotics).

Nevertheless, the mentioned "*longitudinal*" components of the tensor density $q^{\alpha\beta}$ do not contribute to the EMT of the GF in a weak, flat gravitational wave, when they can be written as:

$$\text{a)}\ t^{\alpha\beta} = n^\alpha n^\beta t^{00}, \qquad t^{00} = \frac{1}{4\varkappa}\sum_{i,k}(\dot{q}^{ik}_{tt})^2; \tag{4.9}$$

$$\text{b)}\ \boldsymbol{n}^2 := 1,\ n^0 = 1; \qquad \dot{q}^{0\alpha} = n^k\dot{q}^{k\alpha} \Rightarrow \dot{q}^{00} = n^i n^k \dot{q}^{ik},$$

$$\text{c)}\ \dot{q}^{ik}_{tt} = \dot{q}^{ik} - \dot{q}^{0i}n^k - \dot{q}^{0k}n^i + \dot{q}^{00}n^i n^k + \frac{1}{2}(\delta_{ik} - n^i n^k)\,Sp\dot{q}^{\alpha\beta} =$$
$$= \dot{q}^{ik}_t - \dot{q}^i_t n^k - \dot{q}^k_t n^i + \frac{1}{2}(n^i n^k + \delta_{ik})\,\dot{q}_t,$$

$$q^{ik}_t = q^{ik} - \frac{1}{3}\,\delta_{ik}q^{ll}, \quad q^i_t = q^{ik}_t n^k, \quad q_t = q^i_t n^i. \tag{4.9d}$$

The relation in (4.9d) is the harmonic condition (4.6b) on the coordinates. In these coordinates the EMT of the GF is defined as Einstein's canonical quasitensor [5, 6], which has in the considered case the same value as the quasitensor t_L of Landau-Lifshic—Fock.

5. CONCLUSIONS

When external fields, not satisfying the Lagrange equations, are present in the lagrangian, the dynamical invariants generally cannot be obtained from a canonical formalism. And those equations cannot be satisfied by completing the lagrangian when the external fields describe the macroscopical properties of the system. Only in the special case of a homogeneous, constant external field we obtain a conserving, but not symmetrical canonical EMT and we do not obtain a conserving tensor of MM. The latter fact prevents us from symmetrizing the canonical EMT.

Considering the EMT of the EMF in a medium, one has to understand clearly its sense. We define this quantity as the EMT describing the sum of the energy-momentum of the EMF and of the variation of the energy-momentum of the medium caused by this field. In homogeneous medium, the corresponding densities of this sum are given by the components of the Minkowski EMT, this follows from the conservation of the energy-momentum in the Tsherenkov radiation. The Minkowski EMT has been obtained as a result of the Belinfante "*symmetrization*" of the corresponding canonical EMT. In an inhomogeneous medium we are not able to obtain the EMT as a local function of the EMF.

We need a symmetrical, conserving source of the GF generated by the variation of the EMF in a medium, while the Minkowski EMT is not symmetrical and the symmetrical Hilbert tensor is not conserved. Since a microscopical source can always satisfy both these requirements, the difficulties are connected with the macroscopical description. Even while the problem has not been solved completely, we need at least a formal solution (for the case of a homogeneous medium). The latter is suggested in the present paper, introducing into the lagrangian the non-local additional terms in a way which does not disturb either the equations of the EMF, nor the canonical EMT. Then the Belinfante symmetrization gives the original Hilbert EMT with a non-local addition ensuring the conservation of the source.

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ПРОБЛЕМА СИММЕТРИИ ТЕНЗОРА ЭНЕРГИИ-ИМПУЛЬСА И ГЕНЕРИРОВАНИЕ ГРАВИТАЦИОННОГО ПОЛЯ

В работе рассматривается проблема симметрии тензора энергии-импульса электромагнитного поля в среде как общая проблема в рамках лагранжевского формализма с внешнимполем. Показано, что асимметрический тензор Минского, представляющий «симметризированный» канонический тензор энергии-импульса, правильно описывает экспериментальные данные в однородной среде. Однако нам необходим симметрический и сохраняющийся источник гравитационного поля, в то время как симметрический тензор Гильберта, который следует использовать для этой цели, не сохраняется. Предложен формальный метод получения такого источника.