ON UNSTEADY HYDROMAGNETIC TURBULENT SHEAR FLOW

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Unsteady hydromagnetic turbulent shear flow of viscous incompressible, electrically conducting fluid between two infinite uniform porous planes in the presence of an axial and a transverse magnetic field has been studied by the semi-empirical approach. The expressions for the mean distributions for the velocity and the magnetic field have been obtained for both cases. The solutions obtained for the axial magnetic field have been shown graphically for turbulent and laminar flows.

I. INTRODUCTION

Pai [1, 2] studied the turbulent shear flow of an incompressible viscous fluid between parallel planes and through a circular pipe by the semi-empirical method of Kampe de Ferriet [3]. These theoretical results agree with the experimental results of Laufer [4] and Nikurdse [5]. The hydromagnetic turbulent shear flow between two non-permeable parallel planes considered by Jain [6] is also in close conformity with the experimental results of Murgatroyd [7]. Mehta and Balasubramanyam [8] investigated the steady hydromagnetic turbulent shear flow through chanels with permeable walls by the semi empirical method. Sanyal and Roy Chowdhury [9] generalized this steady problem when the surfaces of the channel are moving with constant velocities and one of the surfaces is conducting.

An attempt has been made in the present paper to generalize the problem of Mehta and Belasubramanyam [8], taking into account the case when the mean flow is unsteady, i.e. the motion is a time-dependent phenomenon in contrast to earlier works. As in the case of [8], [9], two types of the magnetic field have been considered: (i) the axial and (ii) the transverse one. The results obtained here are applicable to all values of Reynold's number. Due to the

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non-availability of relevant experimental data, assumptions have been made regarding the numerical values of the constants and all the results have been calculated by using the Main Computer system and are shown graphically.

II. MAIN EQUATIONS FOR THE HYDROMAGNETIC TURBULENT SHEAR FLOW

We consider the unsteady hydromagnetic turbulent shear flow of an incompressible viscous, electricaly conducting fluid between two uniformly porous parallel planes at a distance. The axis of a x is in the direction of the flow parallel to the planes, the y axis is normal to the planes and z axis is transverse to both x and y. Let the lower plane be $y = 0$ and the hydromagnetic flow variables are functions of y alone. The planes of the channel are now $y = 0$, $y = a$.

Neglecting displacement currents, the hydromagnetic equations in SI units are [8]

$$\frac{\partial v_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial h_i}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = -\frac{1}{\varrho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 v_i}{\partial x_j x_j} + h_j \frac{\partial h_i}{\partial x_j} - \frac{1}{2}\frac{\partial h_j^2}{\partial x_i} \tag{3}$$

$$\frac{\partial h_i}{\partial t} + v\frac{\partial h_i}{\partial x_j} - h_j \frac{\partial v_i}{\partial x_j} = \nu_H \frac{\partial^2 h_i}{\partial x_j \partial x_j} \tag{4}$$

where $(i, j) = (1, 2, 3)$ and $(x_1, x_2, x_3) = (x, y, z)$, t is the time variable $v_i = (v_x, v_y, v_z)$ are velocity components, $h_i = \frac{\mu H_i}{\sqrt{\varrho}} = (h_x, h_y, h_z)$, μ is the magnetic permeability, H_i the magnetic field intensity vector, h_i has the dimension of velocity, ϱ desity, p pressure, ν kinematic viscosity, $\nu_H = 1/\mu\sigma$ magnetic diffusivity and σ is the electrical conductivity.

Let the flow be composed of a mean motion with superimposed random fluctuations and equations (1) to (4) are satisfied by the instantaneous flow variables. This may be expressed as

$$f = \bar{f} + f' \tag{5}$$

where $\bar{f}$ and f' denote the mean and fluctuating parts of the flow variables respectively.

Substituting (5) into equations (1) to (4) and taking averages we get

$$\frac{\partial \bar{v}_i}{\partial x_i} = 0, \tag{6}$$

$$\frac{\partial \bar{h}_i}{\partial x_i} = 0, \tag{7}$$

$$\frac{\partial \bar{v}_i}{\partial t} + \bar{v}_j \frac{\partial \bar{v}_i}{\partial x_j} - \bar{h}_j \frac{\partial \bar{h}_i}{\partial x_j} = -\frac{1}{\varrho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial b_{ij}}{\partial x_j} - \frac{1}{2}\left(\frac{\partial \bar{h}_j^2}{\partial x_i} + \frac{\partial \bar{h}_j^1}{\partial x_i}\right)^2 \tag{8}$$

$$\frac{\partial \bar{h}_i}{\partial t} + \bar{v}_j \frac{\partial \bar{h}_i}{\partial x_j} - \bar{h}_j \frac{\partial \bar{v}_i}{\partial x_j} = \nu_H \frac{\partial^2 \bar{h}_i}{\partial x_j x_j} + \frac{\partial a_{ij}}{\partial x_j} \tag{9}$$

where

$$a_{ij} = \langle v_i' h_j' \rangle - \langle v_j' h_i' \rangle \tag{10}$$

$$b_{ij} = \langle v_i' v_j' \rangle - \langle h_i' h_j' \rangle$$

and $\langle\ \rangle$ denote the average values.

III. FLOW I: AXIALLY ALIGNED MAGNETIC FIELD

As we have considered the mean flow to be unsteady, we assume $\bar{v}_i = \{\bar{v}_x(y), \bar{v}_y(y), 0\}\, e^{i\omega t}$, $\bar{h}_i = \{\bar{h}_x(y), 0, 0\}\, e^{i\omega t}$ and the components a_{ij}, b_{ij} are functions of y and t and there is a uniform external magnetic field h_0 applied in the direction of the x-axis. Equation (6) gives

$$\bar{v}_y = \text{constant} = v_0 \text{ (say)} \tag{11}$$

and equation (7) identically. We aso take $a_{ij} = a_{ij} e^{i\omega t}$, $b_{ij} = b_{ij} e^{i\omega t}$. Introducing the non dimensional quantities

$$\xi = \frac{x}{a}, \quad \eta = \frac{y}{a}, \quad v = \frac{\bar{v}_x}{u^*}, \quad H = \frac{\bar{h}_x}{u^*}, \quad A_{ij} = \frac{a_{ij}}{u^{*2}}, \quad B_{ij} = \frac{b_{ij}}{u^{*2}}$$

and Reynold's cross flow number $R = \frac{a v_0}{\nu}$,

$$R^* = \frac{a u^*}{\nu}, \quad R_m^* = \frac{a u^*}{\nu_H}, \quad \varepsilon = \frac{\nu}{\nu_H}, \quad \tilde{\omega} = \frac{p - p_0}{\varrho u^{*2}},$$

$$x = \frac{1}{u^{*2}} \langle hx'^2 + hy'^2 + hz'^2 \rangle$$

where $u^* = V(\tau/\varrho)$ is the reference velocity. τ being the shearing stress on the plane $y = a$ and p_0 is the reference pressure which may be taken as the mean pressure at $\xi = 0$, $\eta = 1$. Equations (8) and (9) will then give

$$\frac{d^2 r}{d\eta^2} - R\frac{dv}{d\eta} - \frac{i\omega a^2}{\nu} v = R^*\left(\frac{\partial \tilde{\omega}}{\partial \xi} + \frac{dB_{xy}}{d\eta}\right), \quad (12)$$

$$\frac{\partial \tilde{\omega}}{\partial \eta} + \frac{d}{d\tilde{\eta}}\left[B_{yy} + \frac{1}{2}(H^2 + x)\right] = 0 \quad (13)$$

$$\frac{dB_{yz}}{d\eta} = 0, \qquad \frac{dA_{yz}}{d\eta} = 0 \quad (14)$$

$$\frac{d^2 H}{d\eta^2} - R\varepsilon\frac{dH}{d\eta} - \frac{i\omega a^2}{\nu_H} H + R^*_m \frac{dA_{xy}}{d\eta} = 0. \quad (15)$$

The boundary conditions relevat to our problem are

$$A_{ij} = B_{ij} = 0, \quad \text{at} \quad \eta = 0{,}1, \quad (16)$$

$$V = 0, \quad \text{at} \quad \eta = 0{,}1, \quad (17)$$

$$H = H_0, \quad \text{at} \quad \eta = 0{,}1, \quad (18)$$

$$\tilde{\omega}(\xi, \eta) = 0, \quad \text{at} \quad \xi = 0, \ \eta = 1. \quad (19)$$

Intergating equations (13) and (14) and using the boundary conditions (16) and (19) we get

$$A_{yz} = B_{yz} = 0,$$

$$\tilde{\omega}(\xi, \eta) + B_{yy} + \frac{1}{2}(H^2 - H_0^2 + x) = A_0 \xi,$$

where $A_0 = \dfrac{\partial \tilde{\omega}}{\partial \xi}$ is the axial pressure gradient assumed to be given for the flow.

In the absence of turbulence $A_{ij} = B_{ij} = 0$ and the solutions of equations (12) and (15) satisfying the relevant boundary conditions are given by

$$V_l = \left\langle N\left[\exp\left\{\frac{R+S}{2}\eta\right\} - \exp\left\{\frac{R-S}{2}\eta\right\}\right] + M\left[1 - \exp\left\{\frac{R+S}{2}\eta\right\}\right]\right\rangle e^{i\omega t} \quad (20)$$

$$H_l = \left[\frac{1 - e^E}{e^D - e^E} e^{D\eta} + \frac{e^D - 1}{e^D - e^E} e^{E\eta}\right] H_0 e^{i\omega t} \quad (21)$$

where

$$S = \left(R^2 + \frac{i4\omega a^2}{\nu}\right)^{1/2}$$

$$M = \frac{iA_0 R^*_v}{a^2 \omega},$$

$$N = \frac{m\left(\exp\left\{\frac{R+S}{2}\right\} - 1\right)}{e^{R/2}(e^{S/2} - e^{-S/2})}, \qquad D = \frac{\varepsilon R + T}{2}, \qquad E = \frac{\varepsilon R - T}{2}$$

$$T = \left(\varepsilon^2 R^2 + \frac{i4\omega_a^2}{\nu_H}\right)^{1/2}.$$

IV. MEAN VELOCITY DISTRIBUTION FOR TURBULENT SHEAR FLOW

In the presence of turbulence A_{ij}, $B_{ij} \neq 0$ we may get the solution for the mean velocity distribution V_t for the turbulent shear flow compatible with the corresponding laminar flow with the same characteristic velocity. We assume V_t has the form

$$V_t = \left\langle N\left[\exp\left\{\frac{R+S}{2}\eta\right\} - (1 - A_{17}\eta)\exp\left\{\frac{R-S}{2}\eta\right\}\right] + M\left[1 - \exp\left\{\frac{R+S}{2}\eta\right\} + A_{20}\eta^n\right]\right\rangle e^{i\omega t}.$$

It may be noted that the choice of V_t in (22) satisfies the boundary condition $V_t = 0$ at $\eta = 0$. Introducing the empirical parameter

$$S_t = \frac{\tau_t}{\tau_l} = \left[\frac{dV_t}{d\eta}\right]_{\eta=1} \div \left[\frac{dV_l}{d\eta}\right]_{v=1}$$

and using the boundary condition V_t at $\eta = 0$ we find

$$A_{17} = \frac{Ne^x\langle x(S_t - 1) - n\rangle - Ne^Y\langle Y(s_t - 1) + n\rangle - M\langle Xe^x(S_t - 1) + n(e^x - 1)\rangle}{e^Y(n + NY - nN)}$$

$$A_{20} = \frac{1}{M}\left[Ne^Y - Ne^x + M(e^x - 1) - \frac{Ne^Y}{e^Y(N + NY - nN)}\left\{Ne^x\langle x(S_t - 1) - n\rangle - Ne^Y\langle YS_t - 1) + n\rangle - M\langle xe^x(S_t - 1) + n(e^x - 1)\rangle\right\}\right].$$

The coefficient of skin friction C_f for the turbulent shear flow is given by

$$\sqrt{\frac{2}{C_f}} = \int_0^1 V_t \, d\eta = \left\langle N\left[\frac{1}{x}(e^x - 1) - \frac{1}{Y}\{(1 - A_{17})e^Y - 1\} + \frac{A_{17}}{Y^2}(1 - e^Y)\right] + \right.$$
$$\left. + m\left[\frac{1}{x}(1 - e^x) + \frac{A_{20}}{n+1} + 1\right]\right\rangle e^{i\omega t}$$

where

$$x = \frac{R + S}{2}, \qquad Y = \frac{R - S}{2}.$$

The parameters S, and n are to be determined experimentally.

V. MEAN MAGNETIC FIELD DISTRIBUTION FOR TURBULENT SHEAR FLOW

A suitable choice for the magnetic field distribution in the turbulent case, satisfying the boundary condition $H_t = H_0 e^{i\omega t}$ at $\eta = 0$ is given by

$$H_t = \left[\frac{1 - e^E}{e^D - e^E} e^{D\eta} + \frac{e^D - 1}{e^D - e^E} e^{E\eta} + L_1\eta + L_2\eta^n\right] H_0 e^{i\omega t} \qquad (23)$$

Introducing the empirical parameter

$$L = (dH_t/d\eta)_{\eta=0}/(dH_t/d\eta)_{\eta=1}.$$

we get

$$L_1 = \frac{1}{l - 1 - ln}\left[\frac{D(1 - e^E)}{e^D - e^E}(l e^D - 1) + \frac{E(e^D - 1)}{e^D - e^E}(l e^E - 1)\right],$$

$$L_2 = \frac{1}{ln - l + 1}\left[\frac{D(1 - e^E)}{e^D - e^E}(l e^D - 1) + \frac{E(e^D - 1)}{e^D - e^E}(l e^E - 1)\right],$$

The parameter l is to be determined experimentally.

VI. NUMERICAL RESULTS

As the experimental results are not available, we assume for numerical discussion

$$A_0 = 2,\ R^* = 3,\ v = 1,\ a^2 = 2,\ \omega = 1,\ R = 2,\ n = 3,$$
$$S_1 = 2,\ \varepsilon = 1.25,\ l = 0.5,\ v_H = 2,\ t = 1,\ H_0 = 1.$$

We have plotted the mean magnetic field distribution for the turbulent shear flow (H_t) and the laminar shear flow (H_l) in Figure 1, and the mean velocity field distribution for the turbulent (V_t) and the laminar (V_l) shear flow in Figure 2. All the numerical results have been calculated by the Main Computer (EC — 1033) system.

It is observed in Fig. 1 that within $0 < \eta < 1$ the representations of H_t are almost the same, while H_t decreases with increasing value of η up to a certain stage and then increases again. In Fig. 2 V_l is increasing with η increasing and again decreasing to its minimum value. The same conclusion can be drawn for V_t. It is zero (minimum) for $\eta = 0$ and 1 and attains its highest value for $\eta = 0.6$.

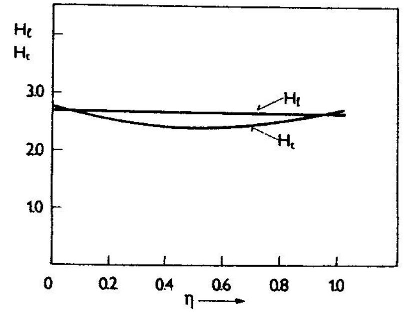

Fig. 1.

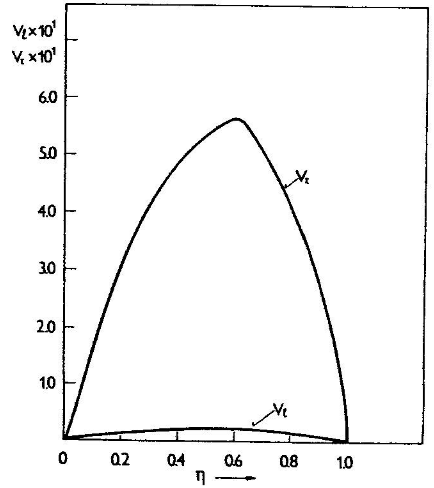

Fig. 2

VII. FLOW II. TRANSVERSE MAGNETIC FIELD

In this case we take $\bar{v}_i = \{\bar{v}_x(y), \bar{v}_y(y), 0\}e^{i\omega t}$, $\bar{h}_i = \{\bar{h}_x(y), \bar{h}_y(y), 0\}e^{i\omega t}$. The components a_{ij}, b_{ij} are functions of y, t and assume that there is a uniform transverse magnetic field h_0 normal to the main flow direction. Then equations (6) and (7) give

$$\bar{V}_y = \text{const.} = V_0, \qquad \bar{h}_y = \text{const.} = h_0 \qquad (24)$$

and Reynold's hydromagnetic equation in the non-dimensional form reduces to

$$\frac{d^2V}{d\eta^2} - R\frac{dV}{d\eta} - \frac{i\omega a^2}{v}V + \frac{R_M}{\varepsilon}\frac{dH}{d\eta} = R^*\left(\frac{\partial\omega}{\partial\xi} + \frac{dB_{xy}}{d\eta}\right), \qquad (25)$$

$$\frac{\partial\omega}{\partial\eta} + \frac{d}{d\eta}\left[B_{yy} + \frac{1}{2}(H^2 + x)\right] = 0, \qquad (26)$$

$$\frac{dB_{yt}}{d\eta} = 0 \text{ and } \frac{dA_{yz}}{d\eta} = 0, \tag{27}$$

$$\frac{d^2H}{d\eta^2} - \varepsilon R\frac{dH}{d\eta} - \frac{i\omega a^2}{\nu_H}H + R_M\frac{dV}{d\eta} + R_M^*\frac{dA_{xy}}{d\eta} = 0 \tag{28}$$

where $R_M = \dfrac{ah_0}{\nu_H}$ is Reynold's magnetic number characteristic of the uniform transverse magnetic fied h_0. The relevant boundary conditions are assumed to be

$$\begin{aligned} V &= 0, && \text{at} \quad \eta = 0, 1, \\ A_{ij} = B_{ij} &= 0, && \text{at} \quad \eta = 0, 1, \\ H &= 0, && \text{at} \quad \eta = 0, 1, \\ \tilde{\omega}(\xi, \eta) &= 0, && \text{at} \quad \xi = 0, \eta = 1. \end{aligned}$$

Equations (26) and (27) integrate to give

$$A_{yz} = B_{yz} = 0 \tag{29}$$

$$\tilde{\omega}(\xi, \eta) + B_{yy} + \frac{1}{2}(H^2 + x) = A_0\xi. \tag{30}$$

VIII. MEAN VELOCITY AND MAGNETIC FIELD DISTRIBUTION FOR TURBULENT SHEAR FLOW

Proceeding exactly along the same line as in flow I, we find that the solution for the velocity and the magnetic field distribution for the laminar flow are given by

$$V_l = A_{11}e^{A'\eta} + A_{12}e^{B'\eta} + A_{13}e^{C'\eta} + A_{14}e^{D'\eta} + \frac{i\omega a^2A_0R^*}{F\nu_H} \tag{31}$$

$$H_l = \frac{\varepsilon}{R_M}\Big[A_{11}A_1e^{A'\eta} + A_{12}B_1e^{B'\eta} + A_{13}E_1e^{C'\eta} + A_{14}D_1e^{D'\eta} + \left(A_0R^* - \frac{\omega^2a^4A_0R^*}{F\nu\nu_H}\right)\eta\Big] + E' \tag{32}$$

and the corresponding solutions for the turbulent shear flow compatible with the above laminar flow are given by

$$V_t = \left[A_{11}e^{A'\eta} + A_{12}e^{B'\eta} + A_{13}e^{C'\eta} + A_{14}e^{D'\eta} + \frac{i\omega a^2A_0R^*}{F\nu_H}(1 - \eta A_{15}) + A_{16}\eta^n\right] \tag{33}$$

$$H_t = \frac{\varepsilon}{R_M}\Big[A_{11}A_1e^{A'\eta} + A_{12}B_1e^{B'\eta} + A_{13}E_1e^{C'\eta} + A_{14}D_1e^{D'\eta} + \left(A_0R^* - \frac{\omega^2a^4A_0R^*}{F\nu\nu_H}\right)\eta\Big] + E' + L_3\eta + L_4\eta^n\Big] \tag{34}$$

where

$$A' = \frac{1}{2}[p_1 + (p_1^2 - 4p_2)^{1/2}], \qquad B' = \frac{1}{2}[p_1 - (p_1^2 - 4p_2)^{1/2}],$$

$$C' = \frac{1}{2}[p_3 + (p_3^2 - 4p_4)^{1/2}], \qquad D' = \frac{1}{2}[p_3 - (p_3^2 - 4p_4)^{1/2}],$$

$$A_1 = R - A' + \frac{i\omega a^2}{\nu A'}, \qquad B_1 = R - B' + \frac{i\omega a^2}{\nu B'},$$

$$E_1 = R - C' + \frac{i\omega a^2}{\nu C'}, \qquad D_1 = R - D' + \frac{i\omega a^2}{\nu D'},$$

$$E' = -\frac{iA_0R^*\varepsilon R\nu_H}{\omega a^2}\left(\frac{\omega^2a^4}{F\nu\nu_H} - 1\right), \qquad A_2 = -\frac{i\omega a^2A_0R^*}{F\nu_H},$$

$$A_3 = -\frac{E'.R_M}{\varepsilon}, \qquad A_4 = -\left(A_0R^* + \frac{E'R_M}{\varepsilon} - \frac{\omega^2a^4A_0R^*}{F\nu\nu_H}\right)$$

$$A_5 = A_2(e^{A'} - 1), \; A_6 = A_1A_2 - A_3, \; A_7 = A_1A_2e^{A'} - A_4,$$

$$A_8 = A_5B_5 - A_6B_2, \; A_9 = A_5B_8 - B_2B_7, \; A_{10} = A_8B_{13} - A_9B_{11},$$

$$A_{11} = A_2 - \frac{A_5}{B_2} + \frac{B_3}{B_2B_{11}}\left(A_8 - \frac{A_{10}B_{12}}{B_{15}}\right) + \frac{A_{10}B_4}{B_{15}} - \frac{1}{B_{11}}\left(A_8 - \frac{A_{10}B_{12}}{B_{15}}\right) - \frac{A_{10}}{B_{15}},$$

$$A_{12} = \frac{A_5}{B_2} - \frac{B_3}{B_{11}B_2}\left(A_8 - \frac{A_{10}B_{12}}{B_{15}}\right) - \frac{A_{10}B_4}{B_2B_{15}},$$

$$A_{13} = \frac{1}{B_{11}}\left(A_8 - \frac{A_{10}}{B_{15}}B_{12}\right), \; A_{14} = \frac{A_{10}}{B_{15}},$$

$$B_2 = (e^{A'} - e^{B'}), \; B_3 = (e^{A'} - e^{C'}), \; B_4 = (e^{A'} - e^{D'}),$$

$$B_5 = A_1 - B_1, \; B_6 = A_1 - E_1, \; B_7 = A_1 - D_1,$$

$$B_8 = A_1 e^{A'} - B_1 e^{B'}, \quad B_9 = A_1 e^{A'} - E_1 e^{C'}, \quad B_{10} = A_1 e^{A'} - D_1 e^{D'},$$

$$B_{11} = B_3 B_5 - B_2 B_6, \quad B_{12} = B_4 B_5 - B_2 B_9, \quad B_{13} = B_3 B_8 - B_2 B_9,$$

$$B_{14} = B_4 B_8 - B_2 B_{10}, \quad B_{15} = B_{12} B_{13} - B_{11} B_{14},$$

$$A_{15} = \frac{1}{x'} \langle A_{11} e^{A'} + A_{12} e^{B'} + A_{13} e^{C'} + A_{14} e^{D'} + x' \rangle + \frac{A_{16}}{x'},$$

$$A_{16} = \frac{1}{n-1} \langle A_{11} e^{A'} \{A'(S_1 - 1) + 1\} + A_{12} e^{B'} \{B'(S_1 - 1) + 1\} + + A_{13} e^{C'} \{C'(S_1 - 1) + 1\} + A_{14} e^{D'} \{D'(S_1 - 1) + x' \rangle,$$

$$x' = \frac{i\omega a^2 A_0 R^*}{F \nu_H}$$

$$L_3 = -\frac{\varepsilon}{R_M(nl - l + 1)} \langle A_{11} A_1 \{e^{A'} nl + A'(1 - l e A')\} + A_{12} B_1 \{e^{B'} nl + + B'(1 - l e^{B'}) + A_{13} E_1 \{e^{C'} nl + C'(1 - l e^{C'})\} + A_{14} D_1 \{e^{D'} nl + + D'(1 - l e^{D'})\} + Y') nl + 1 - al) \rangle - \frac{E' nl}{nl - l + 1},$$

$$L_4 = -L_3 - \left[\frac{\varepsilon}{R_M} (A_{11} A_1 e^{A'} + A_{12} B_1 e^{B'} + A_{13} E_1 e^{C'} + A_{14} D_1 e^{D'} + Y') + E' \right],$$

$$Y' = \left(A_0 R^* - \frac{\omega^2 a^4 A_0 R^*}{F \nu \nu_H} \right) \eta,$$

and

$$p_1 + p_3 = R(1 + \varepsilon), \quad p_2 + p_1 p_3 + p_4 = -\left(\frac{i\omega a^2}{\nu_H} - \varepsilon R^2 + \frac{i\omega a^2}{\nu} + \frac{R_M^2}{\varepsilon} \right),$$

$$p_2 p_3 + p_1 p_4 = -\left(\frac{i\omega a^2 R}{\nu_H} + \frac{i\omega a^2 \varepsilon R}{\nu} \right), \quad p_2 p_4 = -\frac{\omega^2 a^4}{\nu \nu_H}.$$

In the above relations ε, l, n are the empirical parameters to be determined experimentally. It can be seen from the above relations that one cannot get the solutions for the laminar flow the corresponding solutions of the turbulent flow by any limiting conditions in contrast to the steady case.

The authors are thankful to the referee for some helpful and valuable comments upon the paper.

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Received March 24th, 1987
Revised received August 3rd, 1987.
Accepted for publication September 2nd, 1987.

К ВОПРОСУ О НЕУСТОЙЧИВОМ ГИДРОМАГНИТНОМ ТУРБУЛЕНТНОМ ТЕЧЕНИИ С ПОПОРЕЧНЫМ ГРАДИЕНТОМ СКОРОСТИ

В работе на основе полуэмпирического подхода изучается неустойчивое гидромагнитное турбулентное течение вязкой, несжимаемой, электрически проводящей жидкости с поперечным градиентом скорости, которая движется между двумя бесконечными одинаковыми пористыми плоскостями в присутствии аксиального и поперечного магнитных полей. Для обоих случаев получены выражения для средних распределений скорости и магнитного поля. Решения, полученные для аксиального магнитного поля, приведены в графическом виде для случая турбулентного и ламинарного течений.