acta phys. slov. 38 (1988), No. 1

SIZE EFFECT IN MULTILAYER METALIC FILM

KHATER, F.,[1] SEOUD, A.,[1] Tanta

On the basis of the Fuchs — Sondheimer theory, an analysis is given for some transport coefficients of a multilayer metallic film. This film has a large number of alternating layers of two different metals. Two different relaxation times, describing the bulk scattering of the conduction electrons in the layers, are taken as functions of energy. Beside the Fuchs specularity parameter P we introduce a parameter Q which corresponds to the fraction of the conduction electrons refracted at the interfaces. A numerical calculation was made for the electrical conductivity of the conduction electrons at the interface between the layers.

I. INTRODUCTION

The mean free path of the conduction electrons plays the dominant role in the transport properties of metals. When the thickness of a metal film is comparable to the bulk mean free path of the conduction electrons, the surface of the film contributes significantly to the scattering of the electrons. This leads to the size effects. The analysis of the size effects was carried out by Fuchs [1], Sondheimer [2] and many others by solving the Boltzmann transport equation with appropriate boundary conditions. An extension of their theory was made, e.g., by Lucas [3, 4], who introduced two specularity parameters P_1 and P_2 to characterize surface scattering at the outer surfaces of double-layer thin metallic films. Lucas assumed that the internal interface between the layers does not cause any reflection or additional scattering of the conduction electrons. The electrical conductivity of double — layer thin metallic films was calculated [5—9] where the scattering at the interface between the layers was taken into account. The authors used simple boundary conditions which were a generalization of the well-known Fuchs boundary condition.

In the last few years, transport properties of quasiparticles in multilayer structures were mostly studied experimentally. These experimental results were used for industrial applications [10—15] since thin film devices are generally multilayer structures of different materials. In the multilayer structures, scattering of quasiparticles at the interfaces has a strong influence on their transport properties if the thicknesses of the layers is comparable to their bulk mean free paths.

In this paper, a procedure is presented for computing some transport coefficients, in a multilayer metallic film, when internal reflections and refractions of conduction electrons at the interfaces are taken into account. The scattering of the conduction electrons at the interfaces is rather complex. For simplicity, we shall only consider the scattering due to some geometrical roughness of these interfaces. We restrict ourselves to the case of longitudinal transport, when the external thermodynamical forces are parallel to the film surface. The film is subjected to an electric field ε_x and a temperature gradient ∇T in the x-direction parallel to the film surface. The theoretical analysis in this paper utilizes two relaxation times with the energy dependence $\tau_1 \sim \epsilon^{\alpha}$ and $\tau_2 \sim \epsilon^{\beta}$ for the alternating layers of different materials, respectively. The vaues of α and β depend on the predominant scattering mechanisms ($\alpha, \beta = -0.5$ for lattice scattering, $\alpha, \beta = 1.5$ for ionized impurity scattering and $\alpha, \beta = 0$ for neutral defects scattering). In the numerical calculations, we shall only investigate for simplicity the case when $\alpha = \beta$.

II. EXPRESSIONS FOR TRANSPORT COEFFICIENTS IN A BULK METAL

For a bulk metal subjected to an external electric field ε_x and a temperature gradient ∇T in the x-direction, the current density J_x and the heat flux U_x are given, respectively, by the expressions:

$$J_x = -2e\left(\frac{m}{h}\right)^3 \iiint f v_x \mathrm{d}^3 v\,. \tag{1}$$

$$U_x = 2\left(\frac{m}{h}\right)^3 \iiint f(\epsilon - \zeta)\, v_x \mathrm{d}^3 v\,. \tag{2}$$

where e is the electronic charge, m the electronic mass, v_x the velocity of the conduction electrons in the x-direction, h the Planck's constant, f the distribution function of the conduction electrons obtained from the Boltzmann transport equation and f^0 the equilibrium Fermi-Dirac distribution function of the electrons:

$$f^0 = \frac{1}{\exp\{(\epsilon - \zeta)/K_B T\} + 1}\,. \tag{3}$$

(Here ϵ is the electronic energy, ζ the chemical potential which is a function of

[1] Department of Physics, Faculty of Science, Tanta University, TANTA, Egypt.

temperature and T the absolute temperature). The current density J_x and the heat flux U_x can be written in the form:

$$J_x = e^2 R_0 \epsilon_x + eR_1\left(-\frac{1}{T}\frac{\partial T}{\partial x}\right), \tag{4}$$

$$U_x = eR_1 \epsilon_x + R_2\left(-\frac{1}{T}\frac{\partial T}{\partial x}\right). \tag{5}$$

The coefficients R_0, R_1 and R_2 can be obtained from the equations for J_x and U_x. Thus using the basic definitions of the transport coefficients we get the results for the bulk metal as follows:

(i) The bulk electrical conductivity

$$\sigma_B = e^2 R_0. \tag{6}$$

(ii) The bulk thermal conductivity (electronic part)

$$K_B = \frac{R_2}{T} - \frac{R_1^2}{R_0 T}. \tag{7}$$

(iii) The bulk thermoelectric power

$$S_B = \frac{1}{eT}\left(\frac{R_1}{R_0}\right). \tag{8}$$

(iv) The bulk Peltier coefficients

$$\Pi_B = \frac{1}{e}\left(\frac{R_1}{R_0}\right). \tag{9}$$

III. RESULTS

In the case of a multilayer metallic film, we shall calculate the coefficients R_0, R_1 and R_2.

III.1. Expressions for Transport Coefficients in a Multilayer Metallic Films

Let us consider a multilayer metallic film, whose surfaces are parallel to the plane $z = 0$, with infinite dimensions in the x and y directions, subjected to an electric field ε_x and temperature gradient ∇T in the x-direction. The multilayer metallic film, under investigation, consists of a large number of periodically alternating layers (thin films) of two different metals. Since the period of the multilayer (i.e. the cell) is a doublelayer, our problem in to obtain the longitudinal transport coefficients of a doublelayer metallic film with periodic boundary conditions (see Figure 1).

The electron distribution function $f = f^0 + g(v, z)$ obeys the Boltzmann transport equation, where $g(v, z)$ is the small deviation from equilibrium caused by the thermodynamic forces, i.e. by ε_x and ∇T.

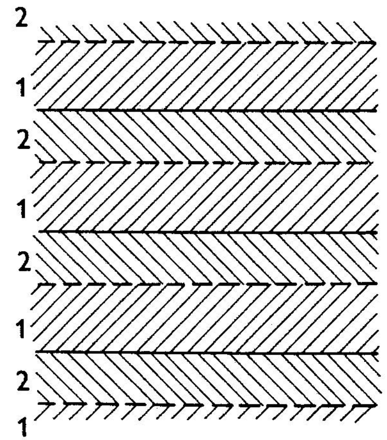

Fig. 1. Schematic diagram of multilayer metallic film which is composed of alternating layers of two different metals.

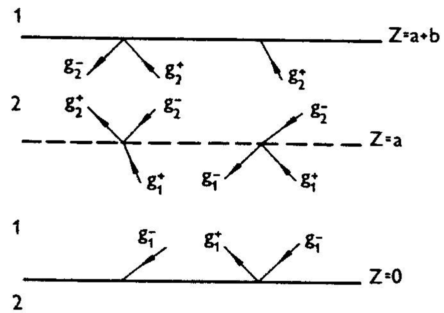

Fig. 2. Schematic illustration of the reflection (probability P) and refraction (probability Q) scattering of the conduction electrons on the interfaces between the layers. (For one cell of the periodic structure).

According to Bezák and Krempaský [16], the distribution functions g_1^+, g_2^+, g_1^- and g_2^- (see Figure 2) satisfy the boundary conditions:

$$\left.\begin{aligned} g_1^+(v_{1z}, z=0) &= P_{12} g_1^-(-v_{1z}, z=0) + Q g_2^+(v_{2z}, z=a+b) \\ g_2^+(v_{2z}, z=a) &= P_{21} g_2^-(-v_{2z}, z=a) + Q g_1^+(v_{1z}, z=a) \end{aligned}\right\} \text{for } \begin{aligned} v_{1z} > 0 \\ v_{2z} > 0 \end{aligned}$$

$$\left.\begin{aligned} g_1^-(v_{1z}, z=a) &= P_{12} g_1^+(-v_{1z}, z=a) + Q g_2^-(v_{2z}, z=a) \\ g_2^-(v_{2z}, z=a+b) &= P_{21} g_2^+(-v_{2z}, z=a+b) + Q g_1^-(v_{1z}, z=0) \end{aligned}\right\} \text{for } \begin{aligned} v_{1z} < 0 \\ v_{2z} < 0 \end{aligned}, \tag{10}$$

where g_1^+, g_2^+ and g_1^-, g_2^- are the distribution functions of the conduction electrons with z-components of the positive and negative velocities, respectively. Here 1, 2 refers, respectively, to the layers whose thicknesses are a, b respectively.

The parameters P_{12} and P_{21} are called the Fuchs specularity parameters; they characterize the probability that an electron will be specularly reflected when scattered from the interface. On the other hand, the parameter Q characterizes the probability that an electron is refracted on the interface, according to the law of refraction. Because of the roughness of the surfaces of the interfaces, P_{12} need not be equal to P_{21}, but $Q_{12} = Q_{21} = Q$.

The boundary conditions, given by equation (10), respect the following identities:

$$g_2^+(v_{2z}, z = 0) = g_2^+(v_{2z}, z = a + b) \qquad \text{for } v_{2z} > 0$$

$$g_1^-(v_{1z}, z = 0) = g_1^-(v_{1z}, z = a + b) \qquad \text{for } v_{1z} < 0$$

Similarly for g_2^- and g_1^+.

The Boltzmann equation for the conduction electrons has the form

$$v_z = \frac{\partial g}{\partial z} + \frac{g}{\tau} = ev_x \frac{\partial f^0}{\partial \epsilon}\left(E_x + \frac{1}{e}\frac{\epsilon - \zeta}{T}\frac{\partial T}{\partial x}\right). \tag{11}$$

where $E_x = \varepsilon_x + \frac{1}{e}\frac{\partial \zeta}{\partial x}$ is the external electric field.

Solving the Boltzmann transport equations (four equations, i.e. two for each layer) for the conduction electrons with respect to the boundary conditions, we can obtain the distribution functions g_1^+, g_2^+, g_1^- and g_2^-.

The average current density $\bar{J}_x$ and average heat flux $\bar{U}_x$ for the multilayer metallic film are given by the relations:

$$\bar{J}_x = \frac{1}{a+b}(J_{1x} + J_{2x}). \tag{12}$$

$$\bar{U}_x = \frac{1}{a+b}(U_{1x} + U_{2x}). \tag{13}$$

where

$$J_{1x} = -2e\left(\frac{m_1}{h}\right)^3 \int d^3v_1 v_{1x} \int_0^a dz(g_1^+ + g_1^-). \tag{14}$$

$$J_{2x} = -2e\left(\frac{m_2}{h}\right)^3 \int d^3v_2 v_{2x} \int_0^{a+b} dz(g_2^+ + g_2^-). \tag{15}$$

$$U_{1x} = 2\left(\frac{m_1}{h}\right)^3 \int d^3v_1 v_{1x} \int_0^a dz(\epsilon_1 - \zeta_1)(g_1^+ + g_1^-). \tag{16}$$

$$U_{2x} = 2\left(\frac{m_2}{h}\right)^3 \int d^3v_2 v_{2x} \int_a^{a+b} dz(\epsilon_2 - \zeta_2)(g_2^+ + g_2^-). \tag{17}$$

After substituting for g_1^+, g_2^+, g_1^- and g_2^- in these equations, using the theorem

$$\int_0^\infty F(\epsilon)\left(-\frac{\partial f^0}{\partial \epsilon}\right) d\epsilon = F(\zeta) + \frac{(\pi K_B T)^2}{6}\left[\frac{\partial^2 F}{\partial \epsilon^2}\right]_{\epsilon=\zeta},$$

introducing the spherical coordinates (v, Θ, Φ) in v-space with $v_z = v\cos\Theta$, averaging over thicknes and integrating over v, Φ, we obtain the following expressions for the average current density $\bar{J}_x$ and the average heat flux $\bar{U}_x$ for the multilayer metalic film:

$$\bar{J}_x = \frac{1}{a+b}\Bigg[\{a\sigma_{B1}F_1(K, P, Q) + b\sigma_{B2}F_2(K, P, Q)\}E_x - \left\{\frac{a\sigma_{B1}S_{B1}T}{\left(1+\frac{2a}{3}\right)}\Psi_1(K, P, Q) + \frac{b\sigma_{B2}S_{B2}T}{\left(1+\frac{2\beta}{3}\right)}\Psi_2(K, P, Q)\right\}\frac{1}{T}\frac{\partial T}{\partial X}\Bigg]. \tag{18}$$

$$\bar{U}_x = \frac{1}{a+b}\Bigg[\left\{\frac{a\sigma_{B1}S_{B1}T}{\left(1+\frac{2a}{3}\right)}\Psi_1(K, P, Q) + \frac{b\sigma_{B2}S_{B2}T}{\left(1+\frac{2\beta}{3}\right)}\Psi_2(K, P, Q)\right\}E_x - \{a_1K_{B1}TF_1(K, P, Q) + bK_{B2}TF_2(K, P, Q)\}\frac{1}{T}\frac{\partial T}{\partial X}\Bigg]. \tag{19}$$

We have taken into consideration that the layers may be of different metals, so we consider different effective masses m_1, m_2 Fermi velocities v_{F1}, v_{F2}, relaxation lengths L_1, L_2 (or bulk relaxation times τ_1, τ_2) and electronic densities n_1, n_2. Therefore there is a potential energy step at each interface f value $V_0 = \frac{1}{2}m_1 \times \times v_{F1}^2 - \frac{1}{2}m_2 v_{F2}^2$. The conduction electrons which pass through the potential step must obey the law of refraction $m_1v_1/m_2v_2 = \sin\Theta_2/\sin\Theta_1$. The Fermi surface for each layer is assumed to be spherical.

The funkctions $F_1(K, P, Q)$, $F_2(K, P, Q)$, $\Psi_1(K, P, Q)$ and $\Psi_2(K, P, Q)$ are defined as follows:

$$F_1(K, P, Q) = 1 - \frac{3}{2K_1}\int_0^1 dx_1(x_1 - x_1^3)(1 - A)D^{-1}(X_1 - CQ(1 - B)) \tag{20}$$

$$F_2(K, P, Q) = 1 - \frac{2}{2K_2}\int_0^1 dx_2(x_2 - x_2^3)(1-B)D^{-1}(X_2 - C^{-1}Q(1-A)). \quad (21)$$

$$\Psi_1(K, P, Q) = \left(1 + \frac{2\alpha}{3}\right) - \frac{2(1+\alpha)}{K_1}\int_0^1 dx_1(x_1(x_1 - x_1^3)(1-A)D^{-1}(X_1 - CQ(1-B)) - \frac{\zeta_1}{K_1}\int_0^1 dx_1(x_1 - x_1^3)\frac{\partial}{\partial\epsilon_1}(1-A)D^{-1} \times (X_1 - CQ(1-B))\}_{\epsilon=\zeta_1} \quad (22)$$

$$\Psi_2(K, P, Q) = \left(1 + \frac{2\beta}{3}\right) - \frac{2(1+\beta)}{K_2}\int_0^1 dx_2(x_2 - x_2^3)(1-B)D^{-1}(X_2 - C^{-1}Q \times (1-A)) - \frac{\zeta_2}{K_2}\int_0^1 dx_2(x_2 - x_2^3)\frac{\partial}{\partial\epsilon_2}(1-B)D^{-1}.(X - C^{-1}Q(1-A))\}_{\epsilon=\zeta_2} \quad (23)$$

Here K_1 and K_2 are the reduced thicknesses of the layers numbered by 1 and 2, respectively, i.e. $K_1 = a/L_1$ and $K_2 = b/L_2$.

It is interesting to see how the functions $F_1(K, P, Q)$, $F_2(K, P, Q)$, $\Psi_1(K, P, Q)$ and $\Psi_2(K, P, Q)$ behave in the limit of large values of K_1 and K_2 (asymptotic approximation):

$$F_1(K, P, Q) = 1 - \frac{3(1 - P_{12} - CQ)}{8K_1}. \quad (24)$$

$$F_2(K, P, Q) = 1 - \frac{3(1 - P_{21} - C^{-1}Q)}{8K_2}. \quad (25)$$

$$\Psi_1(K, P, Q) = \left(1 + \frac{2\alpha}{3}\right) - \frac{(1+\alpha)(1 - P_{12} - CQ)}{2K_1}. \quad (26)$$

$$\Psi_2(K, P, Q) = \left(1 + \frac{2\beta}{3}\right) - \frac{(1+\beta)(1 - P_{21} - C^{-1}Q)}{2K_2}. \quad (27)$$

Here we have used the notations:

$$X_1 = 1 - P_{12} - BP_{21} + B(P_{12}P_{21} - Q^2). \quad (28)$$

$$X_2 = 1 - P_{21} - AP_{12} + A(P_{21}P_{12} - Q^2). \quad (29)$$

$$D = 1 - (AP_{12} + BP_{21}) + AB(P_{12}P_{21} - Q^2). \quad (30)$$

The quantities A, B and C are

$$A = \exp\left(-\frac{a}{\tau_1|v_{1z}|_F}\right) = \exp\left(-\frac{a}{L_1\cos\Theta_1}\right) = \exp\left(-\frac{K_1}{x_1}\right). \quad (31)$$

$$B = \exp\left(-\frac{b}{\tau_2|v_{2z}|_F}\right) = \exp\left(-\frac{b}{L_2\cos\Theta_2}\right) = \exp\left(-\frac{K_2}{x_2}\right). \quad (32)$$

$$C = \frac{L_2 m_1 v_{1F}}{L_1 m_2 v_{2F}}. \quad (33)$$

We have introduced the angle Θ_i such that $v_{iz} = v_i\cos\Theta_i$, $i = 1, 2$. Comparing equations (4), (18) and (5), (19) we find that:

$$R_{0f} = \frac{a}{a+b}\frac{\sigma_{B1}}{e^2}F_1(K, P, Q) + \frac{b}{a+b}\frac{\sigma_{B2}}{e^2}F_2(K, P, Q). \quad (34)$$

$$R_{1f} = \frac{a}{a+b}\frac{\sigma_{B1}S_{B1}T}{\left(1 + \frac{2\alpha}{3}\right)e}\Psi_1(K, P, Q) + \frac{b}{a+b}\frac{\sigma_{B2}S_{B2}T}{\left(1 + \frac{2\beta}{3}\right)e}\Psi_2(K, P, Q). \quad (35)$$

$$R_{2f} = \frac{a}{a+b}K_{B1}TF_1(K, P, Q) + \frac{b}{a+b}K_{B2}TF_2(K, P, Q). \quad (36)$$

From the relations for R_{0f}, R_{1f} and R_{2f} (f, B represent film and bulk, respectively), we obtain the following transport coefficients for the case of the multilayer metallic film:

(1) Electrical conductivity

$$\sigma_f = \frac{a}{a+b}\sigma_{B1}F_1(K, P, Q) + \frac{b}{a+b}\sigma_{B2}F_2(K, P, Q). \quad (37)$$

(2) Thermal conductivity (electronic part)

$$K_f = \frac{a}{a+b}K_{B1}F_1(K, P, Q) + \frac{b}{a+b}K_{B2}F_2(K, P, Q). \quad (38)$$

(3) Thermoelectric power

$$S_F = \frac{S_{B1}\Psi_1(K, P, Q)/\left(1 + \frac{2\alpha}{3}\right) + \frac{n_2 b}{n_1 a}CS_{B2}\Psi_2(K, P, Q)/\left(1 + \frac{2\beta}{3}\right)}{F_1(K, P, Q) + \frac{n_2 b}{n_1 a}CF_2(K, P, Q)}$$

(4) Peltier coefficient

$$\Pi_f = \frac{\Pi_{B1}\Psi_1(K, P, Q)/\left(1+\frac{2\alpha}{3}\right) + \frac{n_2 b}{n_1 a} C\Pi_{B2}\Psi_2(K, P, Q)/\left(1+\frac{2\beta}{3}\right)}{F_1(K, P, Q) + \frac{n_2 b}{n_1 a} CF_2(K, P, Q)}$$

III.2 Numerical Results

For numerical analysis, we shall consider a simple model where the potential energy step at interfaces, is very small, and can be neglected, i.e. $\zeta_1 \simeq \zeta_2$. Therefore the conduction electrons at the interfaces can be assumed to be totally transmitted, i.e. both parameters P_{12} and P_{21} are equal to zero and the parameter Q is equal to unity. A multilayer of silver and gold can be viewed as a rough physical realization of the model proposed in this section.

For this model the functions $F_1(K, P, Q)$, $F_2(K, P, Q)$, $\Psi_1(K, P, Q)$ and $\Psi_2(K, P, Q)$ are defined as follows, where $x_1 = x_2 = x$,

$$F_1(K, P, Q) = 1 - \frac{3}{2K_1}\int_0^1 dx(x-x^3)(1-A)(1-B)(1-C)(1-AB)^{-1} \quad (41)$$

$$F_2(K, P, Q) = 1 - \frac{3}{2K_2}\int_0^1 dx(x-x^3)(1-A)(1-B)(1-C^{-1})(1-AB)^{-1} \quad (42)$$

$$\Psi_1(K, P, Q) = \left(1+\frac{2\alpha}{3}\right) - \frac{2(1+\alpha)}{K_1}\int_0^1 dx(x-x^3)(1-A)(1-B)\left(1-C\left(1+\right.\right.$$
$$\left.\left.+\frac{\beta-\alpha}{2(1+\alpha)}\right)\right)(1-AB)^{-1} + \frac{(1+2\alpha)}{2}\int_0^1 dx(1-x^2)A(1-B)^2(1-C)(1-$$
$$-AB)^{-2} - \frac{(1+2\beta)b}{2a}\int_0^1 dx(1-x^2B(1-A)^2(1-C^{-1})(1-AB)^{-2}. \quad (43)$$

$$\Psi_2(K, P, Q) = \left(1+\frac{2\beta}{3}\right) - \frac{2(1+\beta)}{K_2}\int_0^1 dx(x-x^3)(1-A)(1-B)\times$$
$$\times\left(1-C^{-1}\left(1+\frac{\alpha-\beta}{2(1+\beta)}\right)\right)(1-AB)^{-1}+$$
$$+\frac{(1+2\beta)}{2}\int_0^1 dx(1-x^2)B(1-A)^2(1-C^{-1})(1-AB)^2 -$$
$$-\frac{(1+2\alpha)a}{2b}\int_0^1 dx(1-x^2)A(1-B)^2(1-C)(1-AB)^{-2} \quad (44)$$

The dependences of the electrical conductivity ratio σ_f/σ_{B1} and the thermoelectric power ratio S_f/S_{B1} on the thickness ratio b/a for $L_1/L_2 = 10$, 50 and $K_1 = 10, 1, 0.5, 0.05$ were calculated using a computer. The results are shown in Figures (3—5). These dependences were calculated for the three scattering

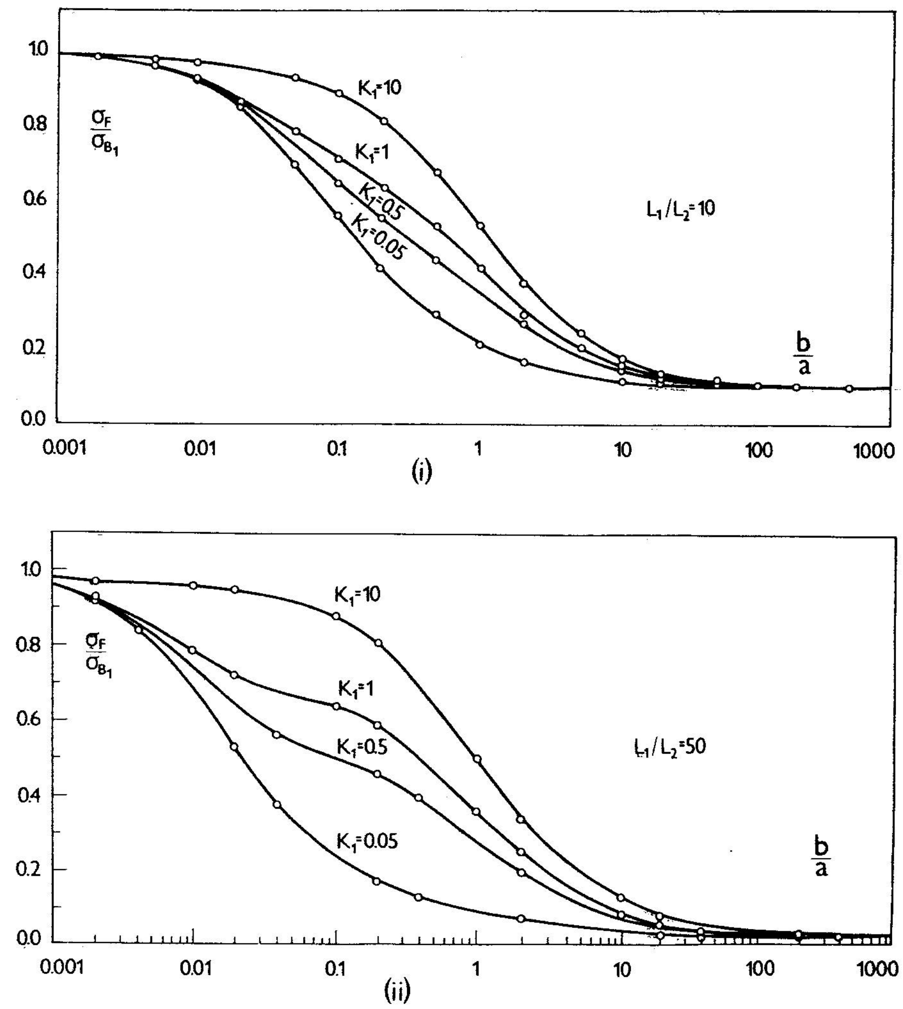

Fig. 3. Variation in σ_F/σ_{B1} with b/a for $K_1 = 10, 1, 0.5, 0.05$ and (i) $L_1/L_2 = 10$, (ii) $L_1/L_2 = 50$.

mechanisms: scattering of the conduction electrons by lattice phonons, ionized impurities and neutral defects, i.e. $\alpha = \beta = -0.5$, $\alpha = \beta = 1.5$ and $\alpha = \beta = 0$, respectively. The curves approach their asymptotic values as follows:

a) When $b/a \ll 1$, i.e. the thicknesses of the layers labelled by number 2 are very small, σ_f/σ_{B1} and S_f/S_{B1} tend to unity for all values of K_1, L_1/L_2 and α, β. Therefore the film behaves as a single metal (i.e. bulk).

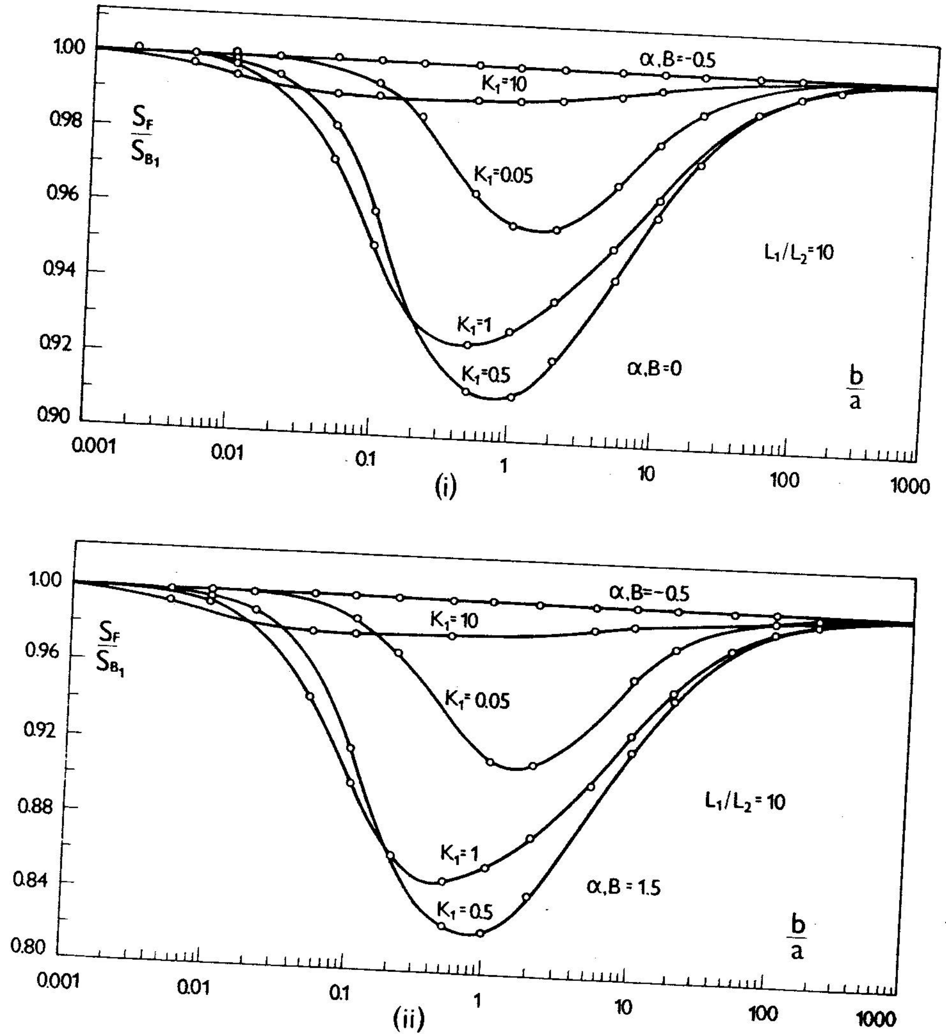

Fig. 4. Variation in S_F/S_{B1} with b/a for $K_1 = 10, 1, 0.5, 0.05$, $L_1/L_2 = 10$ and (i) $\alpha = \beta = 0$, (ii) $\alpha = \beta = 1.5$ (In both Figures, the straight line corresponds to the values $\alpha = \beta = -0.5$).

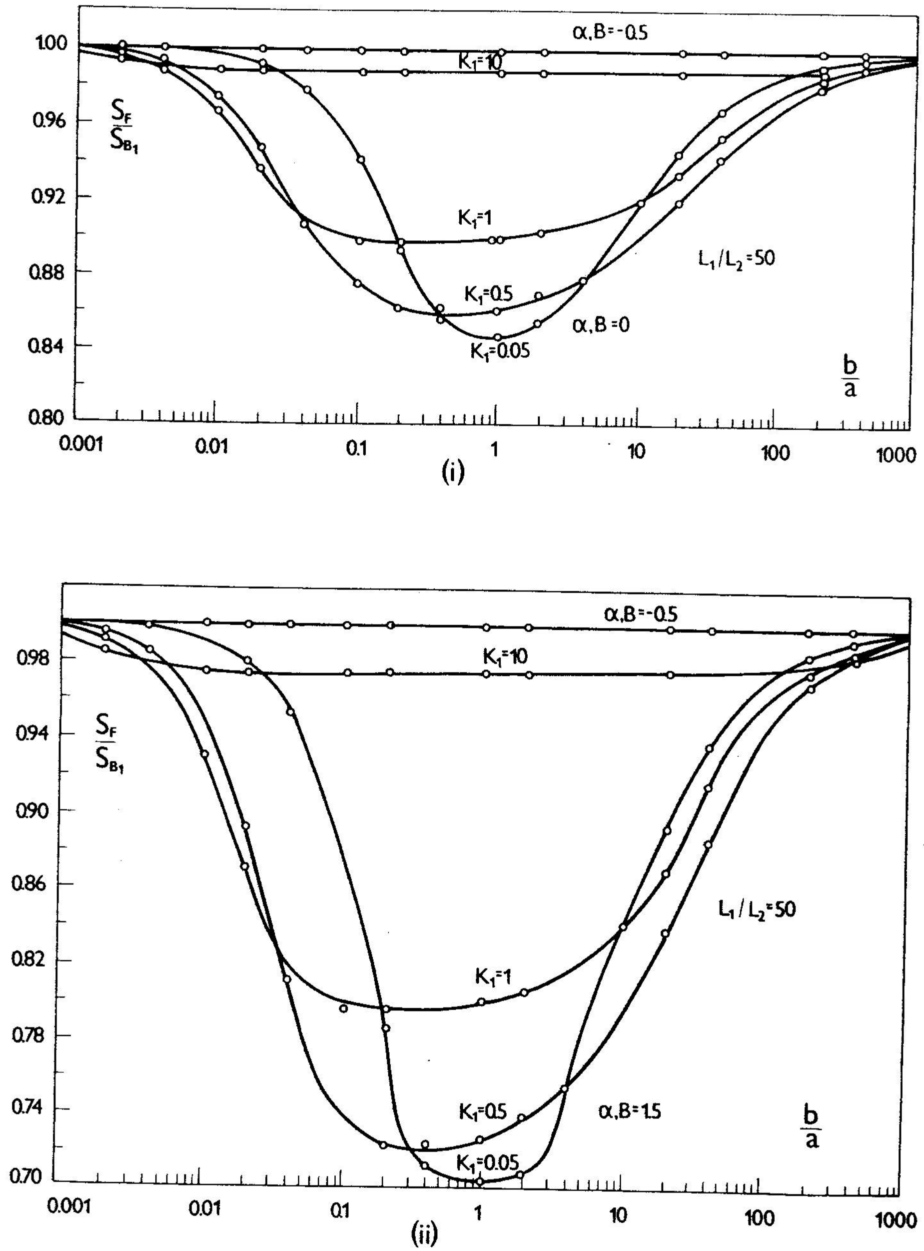

Fig. 5. Variation in S_F/S_{B1} with b/a for $K_1 = 10, 1, 0.5, 0.05$, $L_1/L_2 = 10$ and (i) $\alpha = \beta = 0$, (ii) $\alpha = \beta = 1.5$ (In both Figures, the straight line corresponds to the values $\alpha = \beta = -0.5$).

b) When the values of b/a are not very small and not very large, σ_f/σ_{B1} and S_f/S_{B1} tend to values depending on the solutions of the functions $F_1(K, P, Q)$, $F_2(K, P, Q)$, $\Psi_1(K, P, Q)$ and $\Psi_2(K, P, Q)$. However S_f/S_{B1} equals unity for all values of K_1 and L_1/L_2 when $\alpha = \beta = -0.5$

c) When $b/a \gg 1$, i.e. the thicknesses of the layers labelled by number 2 are very large. σ_f/σ_{B1} and S_f/S_{B1} tend to L_2/L_1 and unity, respectively, for all values of K_1, L_1/L_2 and α, β (except when $\alpha = \beta = 0.5$, since S_f/S_{B1} equals unity in this case).

IV. CONCLUSION

By the above analysis we have obtained general, as well as asymptotic expressions for the transport coefficients of a multilayer metallic film taking into account that the film is subjected to an external electric field E_x and a parallel temperature gradient ∇T. The analysis has been carried out using a general energy dependence of the relaxation time in the two layers of the film. The results show that for a multilayer metallic film the transport coefficients exhibit size effects, except for the thermoelectric power and the Peltier coefficient if the predominant scattering of the conduction electrons is by lattice phonons. This has a simple physical explanation; namely, if $\alpha = \beta = -0.5$, we have a constant bulk mean free path independent of energy. The analysis has shown that the ratio of the film to bulk electronic thermal conductivity in metals behaves in the same manner as the analogical ratio for the electrical conductivity. They are independent of the type of scattering which is dominating. The same is valid for the Peltier coefficient and thermoelectric power, but they have a strong dependency on the type of the dominating scattering.

Finally, it should be pointed out that the analysis can also be applied to metal-semiconductor or semiconductor-semiconductor multilayer films which may occur in microelectronic applications. In case of semiconducting films, however, the theory will become somewhat more complicated because of the usual presence of some surface change at the interfaces, the bending of the bands and the possibility of strong charge fluctuations.

REFERENCES

[1] Fuchs, K.: Proc. Cambridge Philos. Soc. *34* (1983), 100.
[2] Sondheimer, E. H.: Adv. Phys. *1* (1952), 1.
[3] Lucas, M. S. P.: Thin Solid Films *2* (1968), 337.
[4] Lucas, M. S. P.: Thin Solid Films *7* (1971), 435.
[5] Bezák, V., Kedro, M., Pevala, A.: Thin Solid Films *23* (1974), 305.
[6] Bergmann, G.: Phys. Rev. *19* ('979), 3933.
[7] Khater, F.: Acta Phys. Slov. *33* (1983), 43.
[8] Dimmich, R., Warkus, F.: Thin Solid Dilms *109* (1983), 103.

[9] Schumacher, D., Stark, D.: Thin Solid Films *116* (1984), 199.
[10] Shishido, F.: Japan. J. Appl. Phys. *11* (1972), 790.
[11] Chopra, K. L., Nath, P.: J. Appl. Phys. *45* (1974), 192
[12] Chopra, K. L, Sharma, S. K., Yadava, V. N.: Thin Solid Films *20* (1974), 209.
[13] Yadava, V. N., Sharma, S. K., Chopra, K. L.: Thin Solid Films *21* (1974), 297.
[14] Kimura, m., Okahara, K., Miyamoto, T.: J. Appl. Phys. *50* (1979), 1222.
[15] Carcia, P. F., Suna, A.: J. Appl. Phys. *54* (1983), 2000.
[16] Bezák, V., Krempaský, J.: Czech. J. Phys. *B18* (1968), 1264.

Received December 16th, 1986
Revised version received March 11th, 1987

МАСШТАБНЫЕ ЭФФЕКТЫ В МНОГОСЛОЙНЫХ МЕТАЛЛИШЕСКИХ ПЛЕНКАХ

В работе на основе принципа Фукса—Зондгеймера приводится теоретический анализ коэффициентов переноса в многослойных металлических пленках. Используемая тонкая пленка состоит из большого числа чередующихся слоев двух различных металлов. Предполагается, что два различных времени релаксации для объемного рассеяния электронов проводимости являются функциями энергии. Кроме параметра отражения Фукса P, введен еще параметр Q, который соответствует доли электронов проводимости, отраженных на границах раздела слоев. Найдены численные значения электропроводности и коэффициента термоэлектродвижущей силы для случая полного прохождения электронов проводимости через границу раздела между слоями металла.