PRODUCTION OF TRANSVERSE ENERGY IN THE Pb—FRAGMENTATION REGION OF ^{16}O + Pb COLLISIONS AT 200 GeV PER NUCLEON

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We maintain that the recent data obtained by the HELIOS collaboration at CERN SPS on E_T distribution in ^{16}O + Pb collisions at 200 GeV/nucleon can be to some extent understood within the wounded nucleon model with a simple geometrical calculation of the number of interacting nucleons in Pb. We consider the ^{16}O + Pb collision as a collision of the Pb beam with a ^{16}O target and assume that each of the interacting nucleons in Pb contributes to the total transverse energy in the Pb fragmentation region about as much as the (roughly estimated) transverse energy in a p ^{16}O interaction at the same energy and in the same rapidity region.

I. INTRODUCTION

The Helios collaboration at the CERN SPS have recently obtained data on transverse energy distributions in the Pb fragmentation region $-0.1 < y < 2.9$ for ^{16}O + Pb collisions at 200 GeV per nucleon [1]. Transverse energy distributions for p — Pb collisions have been measured under the same conditions.

The data on E_T — distributions in ^{16}O – Pb collisions at 200 GeV/nucleon in the central rapidity region $2.2 < y < 3.8$ have been recently presented by the NA-35 collaboration [2]. The 16-fold convolution of the E_T — distribution in p — Au collisions roughly reproduces [2] the high E_T tail of the ^{16}O – Pb distribution and it turns out [3] that the whole E_T — distribution of ^{16}O + Pb in the central region can be understood as the weighted convolution of the number of interacting nucleons in ^{16}O with weights calculated by a simple geometrical model. In the Pb fragmentation region the situation is different, the HELIOS collaboration have pointed out that the 16-fold convolution of the p-Pb transverse energy distribution is somewhat above the data on the ^{16}O + Pb transverse energy distributions, the tail being shifted by about 20—30 GeV.

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The explanation of this difference is an important problem. Qualitatively it is probably connected with the fact that the multiparticle production in the central rapidity region is influenced by the first two or three collisions of the nucleon from ^{16}O with the nucleons in Pb, whereas the Pb fragmentation region includes also contributions from more collisions with the Pb nucleus. In successive collisions the incoming nucleon from ^{16}O loses more and more energy and as the number of the collisions increases there is less energy spent on the multiparticle production and late collisions contribute only to lower rapidities. It seems that the first two or three collisions of each of the nucleons from ^{16}O with nucleons in Pb are not influenced by the propagation of other nucleons of ^{16}O through the Pb nucleus, whereas the higher order collisions are influenced in this way. This shows that the ^{16}O + Pb collision in the Pb rest frame is a rather complicated problem which has to be solved if we wish to understand the ^{16}O + Pb interaction.

The paper is organized as follows: In Sect. II we describe calculations of the number of interacting Pb — nucleons in p-Pb and ^{16}O – Pb collisions, Sect. III contains the discussion of the transverse energy distribution corresponding to a certain number of interacting Pb — nucleons. Our results are presented and compared with the data in Sect. IV. The last section contains concluding remarks concentrating on the not yet completely clarified points of the present model.

II. THE NUMBER OF INTERACTING NUCLEONS IN Pb NUCLEUS IN Pb-p AND Pb – ^{16}O COLLISIONS

In this paper we wish, however, to avoid temporarily the problem of 16-fold convolution overshooting the ^{16}O – Pb data and take a point of view which makes the p + Pb and ^{16}O + Pb interactions in the Pb fragmentation region reasonably simple. The point of view consists in looking at both interactions in the frame in which the Pb beam interacts with the proton or the ^{16}O target. The rapidity region for individual nucleon-nucleon interactions corresponding to the one covered by the HELIOS apparatus is then roughly $3 < y' < 6$, which represents the beam fragmentation region. Here y' is the rapidity in the frame in which ^{16}O is at rest. We assume that in this region each of the interacting nucleons in Pb fragments in the same way as the proton fragments in the p – ^{16}O collision at 200 GeV.

Thee are no data on the E_T — distribution in p – ^{16}O collisions at 200 GeV in the rapidity $3 < y' < 6$, thus we have to make some estimates based on the available data [4] on particle multiplicities in this rapidity range in pp, pAr and pXe collisions. The average numbers of hadrons produced in these interactions

for $3 < y' < 6$ are 5.5, 7.7 and 8.5 respectively (we multiplied the numbers of charged hadrons by 3/2). Describing that by CA^{a} we find that in a p ^{16}O interaction we can expect on an average about 7 particles. Assuming further that this number is Poisson distributed and that each of the hadrons contributes on an average 0.4 GeV to the transverse energy we can obtain the distribution which might roughly describe the E_T — distribution in a p ^{16}O collision in the proton fragmentation region for $3 < y' < 6$. We shall come back to this point later on in more detail. The N_w-fold convolution of this distribution is denoted as (($p(E_T;\, N_w)$. Here N_w denotes the number of interacting nucleons in the Pb – ^{16}O collision, the index w standing for "wounded", although we deviate here somewhat from the simplest version of the wounded nucleon model [5].

If the probability distribution of the number of interacting nucleons in a Pb – p or Pb – ^{16}O collisions is $P(N_w)$, the E_T distribution is given by the formula

$$P(E_T) = \sum_{N_w} P(N_w)\, P(E_T;\, N_w)\,. \qquad (1)$$

We shall now describe the calculation of $P(N_w)$ first for Pb – p and then for Pb – ^{16}O collisions.

The former is given by the standard Glauber model formula

$$P(N_w) = \frac{1}{\sigma_{p\mathrm{Pb}}} \int_0^R \mathrm{d}^2 b \begin{bmatrix} A \\ N_w \end{bmatrix} \left[\frac{N(b)}{A}\right]^{N_w} \left[1 - \frac{N(b)}{A}\right]^{A - N_w} \qquad (2)$$

where $A = 208$, $\sigma_{p\mathrm{Pb}}$ is the normalization constant, $\sigma_{p\mathrm{Pb}} = 1750$ mb, $N(b) = \sigma_{\mathrm{in}}^{pp} \int \mathrm{d}z \varrho_A(z,\, b)$ where ϱ_A is the nuclear density normalized to A and $\sigma_{in}^{PP} = 32$ mb is the proton — proton inelastic cross section. The nuclear density is taken in the Wood-Saxon form $\varrho_A(r) = \varrho_A\{1 + \exp[(r - R_A)/d]\}^{-1}$ where $R_A = (1.19 A^{1/3} - 1.61 A^{-1/3})$ fm and $d = 0.54$ fm. In this situation $\langle N_w \rangle = 3.8$.

With calculations of probabilities $P(N_w)$ for the Pb – ^{16}O collision the situation is more complicated and one can think about more ways to proceed. The standard way is probably the generalization of Eq. (2) along the lines of the Glauber model. Let R be the radius of Pb r the radius of ^{16}O, $\boldsymbol{B}$ is the impact parameter of the centre of Pb with respect to the centre of ^{16}O and $\boldsymbol{s}$ is the impact parameter distance for a particular nucleon in Pb with respect to the centre of Pb. Then $\boldsymbol{b} = \boldsymbol{B} + \boldsymbol{s}$ is the impact parameter of a particular nucleon in Pb with respect to the centre of the ^{16}O nucleus. The distance travelled by this nucleon within the ^{16}O nucleus in $t(\boldsymbol{B}, \boldsymbol{s}) = 2\sqrt{r^2 - (\boldsymbol{B} + \boldsymbol{s})^2}$ and the probability that this nucleon passes through ^{16}O without an interaction is

$$p(\boldsymbol{B},\, \boldsymbol{s},\, \text{through}) = \exp[-\lambda t(\boldsymbol{B},\, \boldsymbol{s})] \qquad (3)$$

where $\lambda = \sigma\varrho$ is the inverse mean free path. Note that ϱ denotes now the total density (taken as constant) of nucleons in ^{16}O normalized to $\varrho(4/3)\pi r^3 = 16$. With $\sigma = 32$ mb and $\varrho = 0.15\,\text{fm}^{-3}$ we have $\lambda = 0.48\,\text{fm}^{-1}$. We now average $p(\boldsymbol{B}, \boldsymbol{s}, \text{through})$ over $\boldsymbol{s}$ at fixed $\boldsymbol{B}$ obtaining

$$p(\boldsymbol{B}, \text{through}) = \left[\frac{4}{3}\pi R^3\right]^{-1} \int_0^R 2\sqrt{R^2 - s^2}\exp(-2\lambda\sqrt{r^2 - (\boldsymbol{B}^2 + \boldsymbol{s}^2)})\,\mathrm{d}s \quad (4)$$

The factor $2\sqrt{R^2 - s^2}$ is due to the probability of finding a particular nucleon in Pb with the impact parameter distance s from the centre of Pb.

The probabilities $P(N_w; B)$ for having N_w wounded nucleons at this value of $\boldsymbol{B}$ are

$$P(N_w; B) = \begin{bmatrix} 208 \\ N_w \end{bmatrix} [1 - p(B, \text{through})]^{N_w} [p(B, \text{through})]^{208 - N_w} \quad (5)$$

Averaging these over B we find

$$P(N_w) = \frac{1}{\pi(R + r)^2} \int_0^{R+r} \mathrm{d}^2 B P(N_w, B). \quad (6)$$

These $P(N_w)$ are substituted into Eq (1).

An alternative way is to use the Monte Carlo calculation in which nucleons are distributed at random but with constant average densities in both Pb and ^{16}O. The calculation will be described in more detail elsewhere and we shall refer to $P(N_w)$ computed in this way as to the MC-computation.

Both calculations consider nucleons in nuclei as a "gas". The third possibility is to look at a nucleon distribution as a "liquid" with constant density. In a crude way the distribution $P(N_w)$ can be estimated as follows. The first fix the impact parameter B and calculate the volume of Pb which goes through the ^{16}O during the collision. Multiplying this volume by the density we obtain the number $N_w(B)$ of the interacting nucleons in Pb. Since B is distributed with the probability density $P(B) = 2\pi B\,\mathrm{d}B/[\Pi(R + r)^2]$ we obtain $P(N_w)$ from the formula $P(B)\,\mathrm{d}B = P(N_w)\,\mathrm{d}N_w$. $P(N_w)$ calculated in this way will be referred to as LDM (liquid drop model). Note that in the LDM the maximal number of interacting nucleons in Pb is given as [6]

$$(N_w)_{max} = 208\{1 - [1 - (16/208)^{2/3}]^{3/2}\} = 56.$$

In Fg. 1. we plot the three distributions. We consider the MC as the most realistic, although the correct result lies probably somewhere on the way from MC to LDM. The tail of the MC distribution extends to the highest values of N_w. This is due to the fact that neither the LDM nor the Glauber model

calculation takes into account the density fluctuations in the ^{16}O nucleus. In the LDM the density is taken as constant whereas in the Glauber model each of the nucleons of Pb passes through the ^{16}O nucleus as if the nuclear matter in ^{16}O were distributed with the constant density.

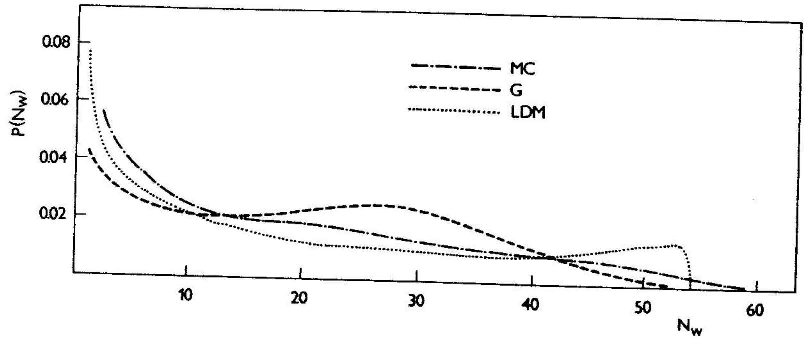

Fig. 1. The probability distribution $P(N_w)$ of the number of interacting nucleons N_w in the Pb nucleus in the Pb – ^{16}O collision. MC — Monte Carlo calculation, G — Glauber model Eq (6), LDM — liquid drop model. The long tail of MC is essential for a correct description of the E_T distribution.

III. TRANSVERSE ENERGY DISTRIBUTION FOR THE N_w INTERACTING NUCLEONS IN THE Pb NUCLEUS

We turn now to the calculation of the second quantity entering Eq. (1), namely the transverse energy distribution $p(E_T; N_w)$. As discussed above we assume that each of the N_w interacting nucleons fragments in about the same way as a proton in $p\,^{16}$O scattering in the rapidity region $3 < y' < 6$. It is natural to take the number of the produced hadrons to be Poisson distributed with the average value μ. If there are N_w fragmenting nucleons, the number of the produced pions will be a convolution of N_w Poisson distributions. This convolution is again Poisson distributed with the mean value μN_w. The distribution $P(n, N_w)$ of the produced pions is therefore given by the formula

$$P(n; N_w) = \frac{a^n}{n!} e^{-a}, \quad a = \mu N_w. \quad (7)$$

The transverse energy distribution of each of these pion will be taken as

$$P^{(1)}(E_T) = \frac{1}{T^2} e^{-E_T/T} \quad (8)$$

where T is a phenomenological parameter with the value $T = 0.2\,\text{GeV}$. A simple calculation gives $\langle E_T \rangle = 2T = 0.4\,\text{GeV}$. The convolution of n-distributions $p^{(1)}(E_T)$ gives

$$P^{(n)}(E_T) = \frac{1}{T(2n-1)!}\, e^{-E_T/T}\left[\frac{E_T}{T}\right]^{2n-1}, \quad T = 0.2\,\text{GeV}. \tag{9}$$

This can be obtained either by a straightforward calculation or simply by "dimensional arguments" [7]. The distribution $P(E_T; N_w)$ entering Eq. (1) then becomes

$$P(E_T; N_w) = \sum_n P(n; N_w)\, P^{(n)}(E_T) \tag{10}$$

where the two terms in the r.h.s. are given by Eqs. (7) and (9).

IV. COMPARISON WITH THE DATA ON $p - \text{Pb}$ AND $^{16}\text{O} - \text{Pb}$ INTERACTIONS AT 200 GeV/c

We shall now compare the results following from the model described above with the data [1] on E_T distributions in $p - \text{Pb}$ and $^{16}\text{O} - \text{Pb}$ interactions in the Pb fragmentation region.

For the $p - \text{Pb}$ collision we calculate $P(N_w)$ by Eq. (2) and $P(E_T, N_w)$ by Eq. (10). The value of the coefficient μ entering Eq. (7) is taken as $\mu = 5.5$, which corresponds [4] to the number of particles within the rapidity interval $0 < y < 3$ in a pp collisions. Note that when calculating the average value N_w from $P(N_w)$ calculated according to Eq. (2) we obtain $N_w = 3.8$. This is in contradiction with the preliminary data [8] which seemed to require [9] a higher value of $\langle N_w \rangle$ indicating probably the presence of some cascading. A possible presence of

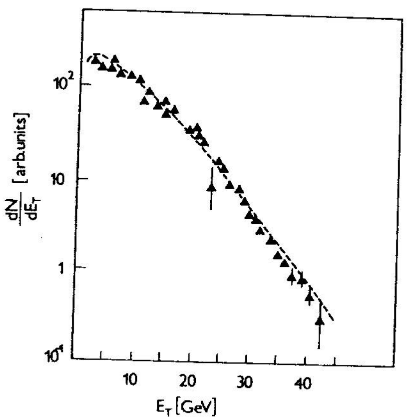

Fig. 2. Transverse energy distribution in $p - \text{Pb}$ collisions for $0 < y < 3$. Our calculations (dashed line) were obtained by Eq. (2) and Eq. (10) with $\mu = 5.5$ in Eq. (7).

cascading has been recently discussed by Baym et al. [10]. The authors of Ref. [10] find a strong evidence for cascading. It seems to us that their results are based on the preliminary data [9]. According to our results the new data [1] do not require an important contribution of cascading to E_T distributions in $p - \text{Pb}$ collisions. This follows from Fig. 2. It should be noted however, that a more detailed analysis can change this statement. The results presented in Fig. 2 were obtained under the simplifying assumption that each for the N_w interacting nucleons in Pb contributes roughly the same amount of E_T to the total transverse energy. This need not be true for higher values of N_w and then cascading should be responsible for covering this overestimate.

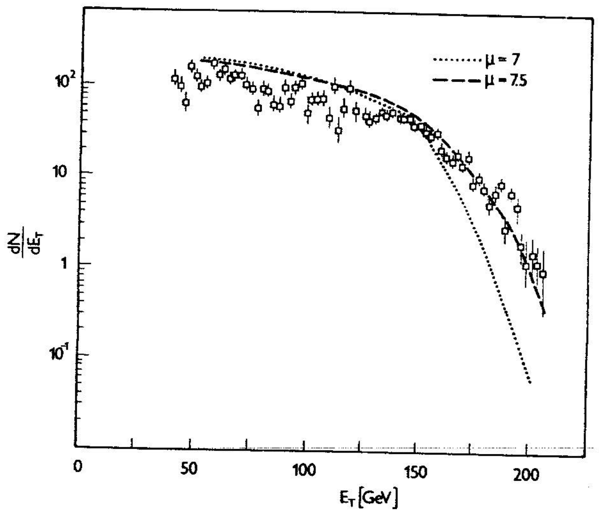

Fig. 3. Comparison of our results on E_T — distribution in $^{16}\text{O} - \text{Pb}$ collisions for $0 < y < 3$.

In calculating E_T distributions for $^{16}\text{O} - \text{Pb}$ collilsions we take the distribution $P(N_w)$ of the number of wounded nucleons in Pb from MC calculations in Fig. 1 and $P(E_T; N_w)$ is obtained from Eq. (10) with $P(n; N_w)$ given by Eq. (7), where $\mu = 7$ corresponds to the number of particles produced in a $p\ ^{16}\text{O}$ interaction in the rapidity interval $3 < y' < 6$ as estimated above on the basis of data in [4]. Our results on E_T distributions for $0 < y < 3$ in $^{16}\text{O} - \text{Pb}$ collisions are shown in Fig. 3. We have presented there two curves, the former (dot.) corresponds to $\mu = 7$, the latter (dash) to $\mu = 7.5$. The data on the average number of hadrons produced in $p\ ^{16}\text{O}$ collision for $3 < y' < 6$ would provide the correct value of μ. Our — not very reliable — estimate presented above would prefer $\mu = 7$ and for the difference between the data and the calculations cascading should be responsible. Even if that were the case cascading would be responsible only for about 7% of particles seen in the final state.

In Fig. 3 we do not present calculations of $P(N_w)$ following from $P(N_w)$ distributions corresponding to the Glauber or the liquid drop model (LDM). These E_T distributions decrease much earlier than the MC ones and cannot describe the data. This point just emphasizes the necessity to understand $P(N_w)$ distributions if detailed comparisons with data on E_T distributions should be performed.

Forgetting the details a rough and qualitative explanation of the data is rather simple. Suppose that the $P(N_w)$ distribution is flat[1] and extends up to about $N_w = 60$. Then we expect a flat E_T distribution extending to about $60 \times \mu \times 0.4$ GeV where μ is the average number of hadrons produced per interacting nucleon. For $\mu = 7$ we obtain 168 GeV and for $\mu = 7.5$ we have 180 GeV. After this value of E_T we expect a decrease which is essentially regulated by the tail in $P(N_w)$, and the fluctuations in μ and in the single particle E_T distributions. We do not claim we have a very realistic description of these fluctuations so some discrepancies in Fig. 3 are expected.

V. COMMENTS AND CONCLUSIONS

In contradiction to the preliminary data [10] on E_T distributions in p—Pb collisions the recent data [1] are well described by the model with $P(N_w)$ calculated by Eq. (2) with $\langle N_w \rangle = 3.8$. The shape of the E_T distributions in ^{16}O – Pb collisions is qualitatively understood as following from the $P(N_w)$ distribution extending to about $N_w = 60$. Here N_w is the number of interacting nucleons in the Pb – ^{16}O collisions with Pb imagined as a beam particle and E_T calculated as the transverse energy in the Pb fragmentation region. Looking at the Pb – ^{16}O collision in this way we have changed the complicated picture of the target fragmentation region for an equivalent but simpler of the beam fragmentation region.

The problem which still remains to be solved is to understand quantitatively and in detail why the 16-fold convolution of the p – Pb distribution in the Pb fragmentation region is considerably above the ^{16}O – Pb data. A rough qualitative argument goes as follows. In a ^{16}O – Pb collision some of the nucleons in Pb are kicked twice but, in the Pb fragmentation region they fragment independently of how many times they have been kicked. This assumption might however turn to be only approximate correct and then the agreement with the data in Fig. 3 may be due to a slight increase of the value of μ with respect to the — at present non available — data on the proton fragmentation in p ^{16}O collisions.

Another point indicates a possible need for a better understanding. The dotted curve in Fig. 3 corresponding to a more realistic value of μ decreases in the region 150—200 GeV considerably faster than the data. This may perhaps be caused by the longer tail of the multiplicity distributions than in the Poisson distributions we have systematically used above. This point of view seems to be indicated also by the recent data [11] on multiplicity fluctuations in the rapidity windows in pp, pAr and pXe collisions at 200 GeV/c. These data are inconsistent with the Poisson distributions and are well fitted by the negative binomial ones.

We hope to discuss these problematic points in more detail in the near future.

[1] These flat distributions cannot be realistically described by the Poisson distribution as assumed in ref. [7]. Because of that the predictions for the E_T distributions in ion-ion collisions presented in [7] are too narow.

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ПРОДУКЦИЯ ПОПЕРЕЧНОЙ ЭНЕРГИИ В ОБЛАСТИ ФРАГМЕНТАЦИИ Pb ДЛЯ РЕАКЦИИ $^{16}O + Pb$ ПРИ ЭНЕРГИЯХ 200 ГэВ/НУКЛОН

В работе показано, что последние данные по распределениям поперечной энергии в соударениях $^{16}O + Pb$ при энергии 200 ГэВ/нуклон, полученные группой «ГЕЛИОС» в ЦЕРНе, можно в какой-то степени объяснить врамках модели раненых нуклонов с простым геометрическим расчетом числа взаимодействующих нуклонов в ядре Pb. Столкновение $^{16}O + Pb$ рассматривается как взаимодействие пучка Pb с мишенью ^{16}O и при этом предполагается, что вклад каждого из взаимодействующих нуклонов ядра свинца в полную поперечную энергию в области его фрагментации примерно такой же, как поперечная энергия во взаимодействии протона с ^{16}O при той же энергии и в том же интервале быстрот.