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# THE GRAVITATIONAL FIELD OF AN ABELIAN DYON IN THE Bi-METRIC THEORY OF GRAVITATION

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The gravitational field of an abelian dyon in the bi-metric theory of gravitation is found as a power series expansion in $r$ for large $r$. The different masses appearing in time and space components suggest a possible way to detect such dyons astrophysically if they are of macroscopic dimensions by anomalous gravitational red shifts and deviations from G.R. geodesic motion.

## THE DYON IN THE Bi-METRIC THEORY OF GRAVITATION

A dyon is a classical particle with both an electric and magnetic charge which may or may not have a non-abelian structure in its core. Many authors have investigated the solutions to $SO_3$ non-abelian gauge theory and have related the dyon mass to its electric and magnetic charge and the V.E.V. of the Higgs field at spatial $\infty$ [1, 2, 3]. The great interest in dyons was motivated by the recognition on the part of Rubakov and Callan that monopoles can catalyse proton decay with strong interaction rates by an effective transformation of a monopole to a dyon with the subsequent transformation of two $u$ quarks into a lepton-anti-quark pair [4]. Apart from the applicat on of dyon models to particle theory, Sikivie has suggested a way in which domain walls can attain both electric and magnetic charge via the interaction with a monopole [5]. Just what effect electric and magnetic charge has on the distribution of matter in the early universe is a poorly understood and much taken for granted subject. For instance, the acquisition of electric and magnetic charge can severely limit the size of black holes and thus any anomalous astrophysical structures might have some relationship to dyon configurations or monopole-like black holes [6]. Besides the profounded effects on all elementary particle processes, the existence of dyon-like objects would certainly open a fresh and unexplored field of investigation of which the student of the early universe would view with anxious

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eyes. In two previous notes, I have investigated the structure of dyon-like solutions to a non-linear abelian gauge theory in General Relativity [7, 8]. Since any non-abelian gauge theory would appear abelian for large $r$, it is well worth-while to investigate dyon-like solutions within the context of theories of gravitation other than G. R. The purpose of this note is to solve the electric, magnetic and gravitational fields in the Bi-metric theory and demonstrate how a test of such a theory can be established if electric and magnetically charged objects are found that sufficiently curve the space in which they are embedded.

## THE Bi-METRIC THEORY AND THE ABELIAN DYON

Consider the isotropic spherically symmetric metrix, $x^4 = ct$, $x^1 = r . x^2 = \Theta, x^3 = \Phi$,

$$(ds)^2 = e^{2\Phi}(dx^4)^2 - e^{2\chi}(dr^2 - e^{2X}r^2(d\Theta^2 + \sin^2\Theta(d\Phi)^2), \qquad (1.1)$$

for the field $g_{\mu\nu}$ and the metric $\gamma_{\mu\nu}$,

$(ds)^2 = (dx^4) - dr^2 - r^2(d\Theta)^2 - r^2\sin^2\Theta(d\Phi)^2$, for the flat background metric. Now consider a radial electric and magnetic field $F_{14} = E(r)$, $E_{23} = r^2\text{-}\sin\Theta B_r$ where $F_{\mu\nu}$ is the E.M. field tensor. Now Maxwell's equations read

$$\frac{d}{dx^\nu}(\sqrt{-g}\,F^{\mu\nu}) = 0 \qquad (1.2)$$

with

$$F^{41} = g^{44}g^{11}F_{41}, \sqrt{-g} = e^{\Phi+\chi+2X}r^2\sin\Theta$$

substituting in Equation (1.2) we have

$$\frac{\partial}{\partial r}\left[\frac{r^2e^{\Phi+\chi+2X}E}{e^{-2\Phi}e^{-2\chi}}\right] = 0,\ E = \frac{e}{r^2}e^{\Phi+\chi-2X} \qquad (1.3)$$

Now from the existence of a potential outside the dyon core we have

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0$$

or

$$\frac{\partial}{\partial r}(r^2\sin\Theta B_r) = 0,\ B_r = \frac{q}{r^2}$$

$e$ = electric charge, $q$ = magnetic charge. Using the E. M. lagrangian



$$L = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\sqrt{-g},$$

and the expression for $T_{\mu\nu} = \dfrac{2}{\sqrt{-g}}\dfrac{\partial L}{\partial g^{\mu\nu}}$ [8], we have

$$T_{\mu\nu} = \frac{1}{4\pi}\left(\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} - F_{\mu\alpha}F_{\nu\beta}g^{\beta\alpha}\right), \qquad (1.4)$$

$$T^\mu_\nu = \frac{1}{4\pi}(\delta^\mu_\nu F_{\alpha\beta}F^{\alpha\beta} - F^{\mu\nu}F_{\nu\beta}). \qquad (1.5)$$

We have

$$F_{14} = \frac{e}{r^2}e^{\Phi+\chi-2X},\ F_{23} = q\sin\Theta,$$

$$F^{14} = \frac{-e}{r^2}e^{-\Phi-\chi-2X},\ F^{23} = \frac{q}{r^4\sin^1\Theta}e^{-4},$$

$$F_{\alpha\beta}F^{\alpha\beta} = 2F_{14}F^{14} + 2F_{23}F^{23} = \frac{-2e^2}{r^4}e^{-4\chi} + \frac{2q^2}{r^4}e^{-4X},$$

$$T^1_1 = \frac{1}{8\pi}\left(\frac{e^2}{r^4} + \frac{q^2}{r^4}\right)e^{-4X}, \qquad (1.6)$$

$$T^2_2 = -\frac{1}{8\pi}\left(\frac{e^2}{r^4} + \frac{q^2}{r^4}\right)e^{-4X},$$

$$T^3_3 = -\frac{1}{8\pi}\left(\frac{e^2}{r^4} + \frac{q^2}{r^4}\right)e^{-4X},$$

$$T^4_4 = \frac{1}{8\pi}\left(\frac{e^2}{r^4} + \frac{q^2}{r^4}\right)e^{-4X}.$$

The field equations of the Bi-Metric theory are given by Rosen [9]

$$N_{\mu\nu} = -\frac{8\pi kG}{C^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \qquad (1.7)$$

$$N^\mu_\nu = -8\pi KG\left(T^\mu_\nu - \frac{1}{2}\delta^\mu_\nu T\right), \quad \text{where } C = 1, \qquad (1.8)$$

$G$ = gravitational constant, $K = (g/\gamma)^{1/2}$ and where

$$N_{\mu\nu} = \frac{1}{2}\gamma^{\alpha\beta}g_{\mu\nu/\alpha\beta} - \frac{1}{2}\gamma^{\alpha\beta}g^{\lambda\sigma}g_{\lambda\mu/\alpha}g_{\sigma\nu/\beta}. \qquad (1.9)$$

The $(g_{\lambda\mu/\alpha})$ bar in Equation (1.9) means covariant differentiation with respect to the flat background metric.
The equations are using Equation (1.1) and Equation (1.8)

$$-\Phi'' - \frac{2}{r}\Phi' = -8\pi G e^{\Phi+\chi+2X}\left(\frac{1}{8\pi}\right)\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)e^{-4X}, \qquad (1.10)$$

$$-\chi'' - \frac{2}{r}\chi' - \frac{2}{r^2}\sinh Z = -8\pi G e^{\Phi+\chi+2X}\left(\frac{1}{8\pi}\right)\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)e^{-4\chi}, \qquad (1.11)$$

$$-X'' - \frac{2}{r}\chi' + \frac{1}{r^2}\sinh Z = -8\pi G e^{\Phi+\chi+2X}\left(\frac{1}{8\pi}\right)\left(-\frac{e^2}{r^4}-\frac{q^2}{r^4}\right)e^{-4X}, \qquad (1.12)$$

where $Z = 2(X-\chi)$. Rewriting Equation (1.10), Equation (1.11) and Equation (1.12) in terms of $W = \Phi+\chi-2X$, $Z = 2X-2\chi$, $X = \Phi+\chi+2X$ we have

$$W'' + \frac{2}{r}W' + \frac{4}{r^2}\sinh Z = 4G\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)e^{W}, \qquad (1.13)$$

$$Z'' + \frac{2}{r}Z' - \frac{6}{r^2}\sinh Z = -4G\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)e^{W}, \qquad (1.14)$$

$$X'' + \frac{2}{r}X' = 0. \qquad (1.15)$$

Linearizing Equation (1.13), Equation (1.14) for large $r$ we have

$$W'' + \frac{2}{r}W' + \frac{4}{r^2}Z = 4G\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)(1+W), \qquad (1.16)$$

$$Z'' + \frac{2}{r}Z' - \frac{6}{r^2}Z = -4G\left(\frac{e^2}{r^4}+\frac{q^2}{r^4}\right)(1+W), \qquad (1.17)$$

$$X'' + \frac{2}{r}X' = 0. \qquad (1.18)$$

Equation (1.18) yields the solution $X(r) = \frac{A}{r}$. We set

$$W(r) = \sum_{i=1}^{\infty}\frac{B_i}{r^i}, \qquad (1.19)$$

$$Z(r) = \sum_{i=1}^{\infty}\frac{C_i}{r^i}. \qquad (1.20)$$



Inserting Equation (1.19) and Equation (1.20) into Equation (1.16) and Equation (1.17) we have

$$C_1 = 0 \qquad B_1 = \text{arb.},$$
$$C_2 = Ge^2 + Gq^2, \qquad B_2 = 0. \qquad (1.21)$$

Now using

$$\frac{B_1}{r} = \Phi + \chi - 2X, \qquad (1.22)$$

$$\frac{Ge^2+Gq^2}{r^2} = 2X - 2\chi, \; \frac{A}{r} = \Phi + \chi + 2X,$$

we have

$$X = \frac{A}{4r} - \frac{B_1}{4r},$$
$$\Phi = \frac{A+3B_1}{4r} + \left(\frac{Ge^2+Gq^2}{2r^2}\right), \; \chi = \frac{A-B_1}{4r} - \left(\frac{Ge^2+Gq^2}{2r^2}\right) \qquad (1.23)$$
$$(ds)^2 = e^{\frac{A+3B_1}{2r}+\frac{Ge^2+Gq^2}{r^2}}(dx^4)^2 - e^{\frac{A-B_1}{2r}-\frac{Ge^2+Gq^2}{r^2}}(dr)^2 - e^{\frac{A-B_1}{2r}}r^2(d\Theta^2 + \sin^2\Theta(d\Phi)^2).$$

Calling $\frac{A+3B_1}{2} = -2Gm$, $\frac{A-B_1}{2} = 2Gm'$ we have

$$(ds)^2 = e^{\frac{-2Gm}{r}}e^{\frac{Ge^2+Gq^2}{r^2}}(dx^4)^2 - e^{\frac{2Gm'}{r}}e^{\frac{-Ge^2-Gq^2}{r^2}}(dr)^2 - e^{\frac{2Gm'}{r}}r^2(d\Theta^2 + \sin^2\Theta(d\Phi)^2). \qquad (1.24)$$

In order to interpret the metric in Equation (1.24) we must go back to the analysis of Rosen for both a point particle and an extended object in the bi-metric theory [10]. In that analysis it was shown in the absence of the electric and the magnetic charge, that the metric for a point particle has the form

$$(ds)^2 = e^{\frac{-2Gm}{r}}(dx^4)^2 - e^{\frac{2Gm}{r}}(dr^2 + r^2 d\Theta^2 + r^2\sin^2\Theta(d\Phi)^2) \qquad (1.25)$$

if the space outside the particle is such that $T_{\mu\nu} = 0$ and the condition $(Kg^{\mu\nu})_{|\nu} = 0$, which is the analogue of the De Donder condition of General Relativity, can be consistently imposed so as not to contradict the field equations. It was also shown in that discussion that Equation (1.25) emerges from the bi-metric theory outside an extended mass distribution if the extended material object could be represented by a distribution of matter with $p = 0$. In our solution, since $T_{\mu\nu} \neq 0$ in the space surrounding the dyon, it is clear that no condition can be imposed consistent with the field equations. The solution also suggests here that the notion of a point dyon-like object breaks down and suggests that we condiser the dyon as an extended object with the two mass parameters given by Equation (1.24). Since we have not analysed the internal structure of the dyon, we interpret the two mass parameters $m, m'$ in Equation (1.24) as related by a knowledge of the internal structure of the dyon together with the field equations that yield a solution to be matched with Equation (1.24) at the outer radius of the dyon. From the point of view of a distant observer, this theory will differ from the Reissner Nordstrom solution and any anomalous redshifts or deviations from General Relativistic geodesic paths in the field of such a dyon might provide evidence for the validity of Equation (1.24). Of course, the whole question of the meaning of two different masses becomes an important issue here if the red-shift measurements are made for such a compact object and a mass is determined from the shift of various lines and this mass is compared with $m'$ measured by the motion of an object in the field of the dyon when the path deviates from the General Relativistic result. A comparison might be made to test the above theory.


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## LIST OF SYMBOLS

$g_{44} = e^{2\Phi}$
$g_{11} = e^2$

$\frac{g_{33}}{\sin^2\Theta} = g_{22} = e^2 X r^2$

$\gamma_{\mu\nu}$ = Flat background metric.
$W = \Phi + \chi - 2X$
$Z = 2X - 2\chi$
$X = \Phi + \chi + 2X$
Note $\chi$ is different than $X$.
Also $x$ is different than $X$.

$g_{\mu\nu/a}$ = Covariant differentiation with respect to
$m, m'$ are masses associated with the time-like and space-like coordinates.
$G$ = Gravitational constant.

$g_{\mu\nu}$ = Dynamical metric.
$e$ = electric charge
$q$ = magnetic charge

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## ГРАВИТАЦИОННОЕ ПОЛЕ АБЕЛЕВСКОГО ДИОНА В БИМЕТРИЧЕСКОЙ ТЕОРИИ ГРАВИТАЦИИ


Для случая больших $r$ найдено разложение по степеням $r$ гравитационного поля абелеского диона в биметрической теории гравитации. Различные массы, появляющиеся во временных и пространственных компонентах, подсказывают способ, как можно астрофизически детектировать такие дионы, если они имеют макроскопические размеры. Это можно сделать при помощи аномального гравитационного красного смещения и отклонений от движения по геодезической линии согласно общей теории относительности.