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ONE-DIMENSIONAL STEADY FLOWS IN RAIL PLASMA ACCELERATORS

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A simple procedure to determine the space distribution of density, velocity, temperature and other quantities in a current layer moving in the rail plasma accelerator is proposed in this paper. The movement at a constant velocity and the experimental-based knowledge of the current density distribution within the cluster are assumed.

I. INTRODUCTION

A gradual burning of the conductive plasma layer and its displacement toward the end of the accelerator appears in the plasma accelerator under atmospheric pressure [1]. In this way a plasma cluster or a shock wave arises, which leaves behind a partially ionized moving gas. In this study we will consider the plasma cluster with a constant velocity movement between two plane parallel electrodes. The constant movement arises either under some experimental conditions [1] or the actual time development can be approximated with the time net $t_1, \ldots, t_N$, in which case all functions are supposed to be constant in the time intervals $\Delta t = t_i - t_{i-1}$; $i = 2, \ldots, N$.

II. THE CONSERVATION LAWS IN FLOWING PLASMA

Let us consider the mass, momentum and energy conservation laws of the moving plasma and associated electromagnetic field if the heat flow and the viscosity are neglected:

$$\frac{\partial \varrho}{\partial t} + \operatorname{div} \varrho \boldsymbol{v} = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\varrho v_i + \boldsymbol{D} \times \boldsymbol{B}_i) + \frac{\partial}{\partial x_j}(\varrho v_i v_j + p\delta_{ij} + T^E_{ij} + T^M_{ij}) = 0 \tag{2}$$

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$$\frac{\partial}{\partial t}\left(\frac{1}{2}\varrho v^2 + u + \frac{\boldsymbol{E}.\boldsymbol{D}}{2} + \frac{\boldsymbol{H}.\boldsymbol{B}}{2}\right) + \operatorname{div}\left[\left(\frac{\varrho v^2}{2} + u + p\right)\boldsymbol{v} + \boldsymbol{E}\times\boldsymbol{H}\right] = 0, \quad (3)$$

where ϱ is the density, $\boldsymbol{v}$ the velocity, u the internal energy density, T^E the electric part of the Maxwell tensor and T^M the magnetic part of the Maxwell tensor, respectively, e.g.

$$T_{ij}^E = \frac{\boldsymbol{E}.\boldsymbol{D}}{2}\delta_{ij} - E_i D_j \qquad T_{ij}^M = \frac{\boldsymbol{H}.\boldsymbol{B}}{2}\delta_{ij} - H_i B_j. \quad (4)$$

The electric and the magnetic field fulfil the Maxwell equations

$$\operatorname{rot}\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (5)$$

$$\operatorname{rot}\boldsymbol{H} = \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}. \quad (6)$$

Having introduced a coordinate system moving with the current layer (see Fig. 1) the terms with the time derivatives in the system (1)—(3), (5)—(6) are zero. The orientation of the individual vectors in the rail plasma accelerator can be seen in Fig. 1.

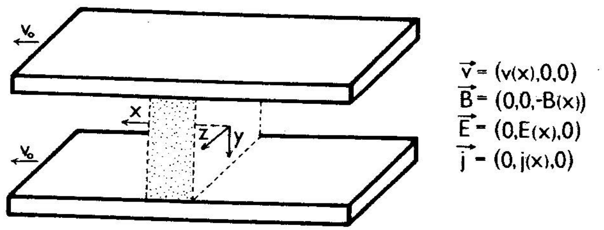

Fig. 1. The coordinate system moving with the current layer

After applying the Gauss theorem to the system of eqs. (1)—(3), we have from eqs. (5)—(6)

$$\varrho v = const \quad (7)$$

$$\varrho v^2 + p + \frac{B^2}{2\mu_0} = const \quad (8)$$

$$\left(\frac{\varrho v^2}{2} + u + p\right)v - \frac{EB}{\mu_0} = const \quad (9)$$

$$E = const \quad (10)$$

$$\frac{dB}{dx} = \mu_0 j. \quad (11)$$

This system of equations (the Rankine-Hugoniot conditions) together with Ohm's law yields a continuous solution or a shock wave solution for the quantities ϱ, v, T [2] describing the plasma. The experimental knowledge of the function $j(x)$ will be considered here instead of Ohm's law. We will also assume a partially ionized plasma with the rate of ionization α, which can be derived from Saha's equation of equilibrium ionization in first approximation [3]:

$$\frac{n_e^2}{n_n} = \frac{2g_1}{g_0}\left(\frac{m_e}{2\pi\hbar^2}\right)^{3/2}(kT)^{3/2}\exp(-I_0/kT), \quad (12)$$

where I_0 is the ionization potential. In discussing these equations we use the relations

$$u = \frac{p}{\gamma - 1} \quad (13)$$

$$p = (n_e + n_i + n_n)kT \quad (14)$$

$$n_n = \frac{\varrho}{m_i}(1 - \alpha) \quad (15)$$

$$n_e = \frac{\varrho}{m_i}\alpha. \quad (16)$$

Next we introduce non-dimensional variables according to the scheme

$$\bar\varrho = \varrho/\varrho_0 \qquad \bar v = v/v_0 \qquad \bar\alpha = \alpha$$
$$\bar T = kT/m_i v_0^2 \qquad \bar B = B/\sqrt{\mu_0\varrho_0 v_0^2} \qquad \bar E = E/\sqrt{\mu_0\varrho_0 v_0^4} \quad (17)$$
$$\bar j = l\sqrt{\mu_0/\varrho_0 v_0^2}\,j \qquad \bar x = x/l,$$

where α is the rate of ionization, ϱ_0, v_0, T_0, B_0 are the values of density, velocity, temperature and magnetic field ahead of the current layer, l is the width of the current layer, respectively. The system of eqs. (7)—(12) becomes

$$\bar\varrho\bar v = c_1 \quad ; \quad c_1 = 1 \quad (18)$$

$$\bar v + \bar\varrho(1 + \bar\alpha)\bar T + \bar B^2/2 = c_2 \quad ; \quad c_2 = 1 + (1 + \bar\alpha_0)\bar T_0 + \frac{\bar B_0^2}{2} \quad (19)$$

$$\frac{\bar v^2}{2} + \frac{\gamma}{\gamma - 1}(1 + \bar\alpha)\bar T - \bar E\bar B = c_3 \quad ; \quad c_3 = \frac{1}{2} + (1 + \bar\alpha_0)\bar T_0\frac{\gamma}{\gamma - 1} - \bar E_0\bar B_0 \quad (20)$$

$$\bar{E} = c_4 \quad ; \quad c_4 = \bar{E}_0;\ E_0 = \frac{U}{b} + v_0 B_0 \tag{21}$$

$$\frac{d\bar{B}}{d\bar{x}} = \bar{j} \tag{22}$$

$$\bar{\varrho}\frac{\bar{a}^2}{1-\bar{a}} = f(\bar{T}) \tag{23}$$

where we have denoted

$$\tau = \frac{I_0}{m_i v_0^2}$$

$$f(\xi) = A\xi^{3/2} e^{-\tau/\xi} \quad ; \quad A = \frac{2g_1}{g_0}\frac{m_i}{\varrho_0}\left(\frac{m_e m_i}{2\pi\hbar^2}\right)^{3/2} v_0^3, \tag{24}$$

b is the distance between the electrodes and U is the voltage applied on the electrodes, respectively. Next we eliminate the quantities E and ϱ from the system of Eqs. (18)—(23) and after simple algebraic manipulations we obtain the system

$$\frac{\gamma+1}{2(\gamma-1)}\bar{v}^2 + \frac{\gamma}{\gamma-1}\left(\frac{\bar{B}^2}{2} - c_2\right)\bar{v} + c_3 + c_4\bar{B} = 0 \tag{25}$$

$$(1 + \bar{a}(\bar{v}, \bar{T}))\,\bar{T} = c_2\bar{v} - \frac{\bar{v}\bar{B}^2}{2} - \bar{v}^2 \tag{26}$$

$$\frac{d\bar{B}}{d\bar{x}} = \bar{j} \tag{27}$$

$$\bar{a}(\bar{v}, \bar{T}) = \frac{2}{1 + \sqrt{(1 + 4/(\bar{v}\bar{f})}}, \tag{28}$$

which will serve as a scheme for the numerical solutions of the particular quantities.

III. THE RESULTS OF NUMERICAL SOLUTIONS

Knowing the function $j(x)$, we can derive the $B(x)$dependence from (27). The velocity distribution $v(x)$ can be determined from the quadratic eq. (25). As the determinant of this equation takes the form $D = [\bar{v} - \gamma(1 + \bar{a})\,\bar{\varrho}\bar{T}]^2/(\gamma - 1)^2 \geq 0$, we always have a real solution. Using some iterative method the $T(x)$ dependence can be obtained from eq. (26) and the $a(x)$ dependence can be

directly computed from eq. (28). Let us remark that the system of eqs. (18)—(23) does not depend on the width of the current layer l, which takes place only in the transformation (17).

The numerical scheme was tested on the current density dependences

$$j(x) = j_0\left[1 - \cos\left(\frac{\pi x}{l}\right)\right]^2. \tag{29}$$

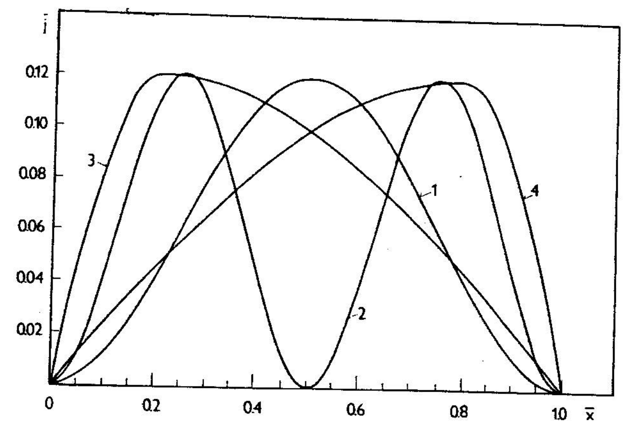

Fig. 2. The current density distributions used in the numerical test:
1 — symmetric layer
2 — symmetric "double" layer
3 — non-symmetric layer with $\xi = 0.2$
4 — non-symmetric layer with $\xi = 0.8$

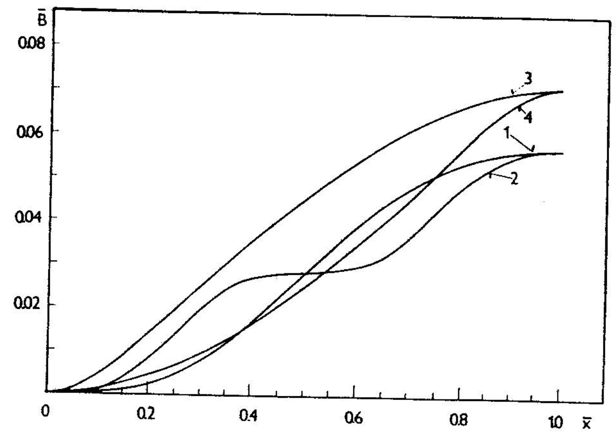

Fig. 3. Magnetic field distribution for current densities from Fig. 1.

$$j(x) = j_0\left[1 - \cos\left(\frac{2\pi x}{l}\right)\right]^2 \qquad (30)$$

$$j(x) = j_0 \begin{cases} \sin\left(\dfrac{\pi x}{2l\xi}\right) & \dfrac{x}{l} \in \langle 0, \xi) \\ \sin\left(\dfrac{\pi}{2(\xi - 1)}\left(2\xi - 1 - \dfrac{x}{l}\right)\right) & \dfrac{x}{l} \in \langle \xi, 1 \rangle . \end{cases} \qquad (31)$$

The relation (29) describes a symmetric current layer, (30) is a symmetric current „double" layer and (31) represents a non-symmetric current layer, where $\xi \in (0, 1)$ — see Fig. 2. The responding magnetic fields are plotted in Fig. 3, the velocity distributions in Fig. 4, the temperatures in Fig. 5 and the rates of ionization in Fig. 6. The calculations were performed for the nitrogen plasma under the

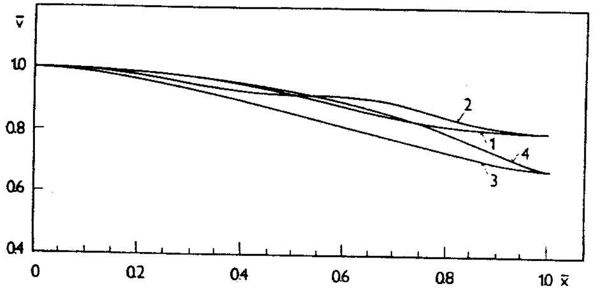

Fig. 4. Velocity field distributions for current densities from Fig. 1.

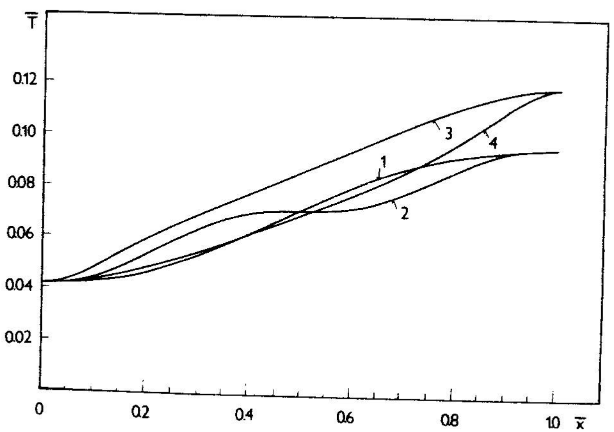

Fig. 5. Temperature field distributions for current densities from Fig. 1.

atmospheric pressure, the voltage applied on the electrodes of the accelerator was $U = 12000\,V$ and the distance of the electrodes $b = 2\,cm$. The remaining input values are in Tab. 1.

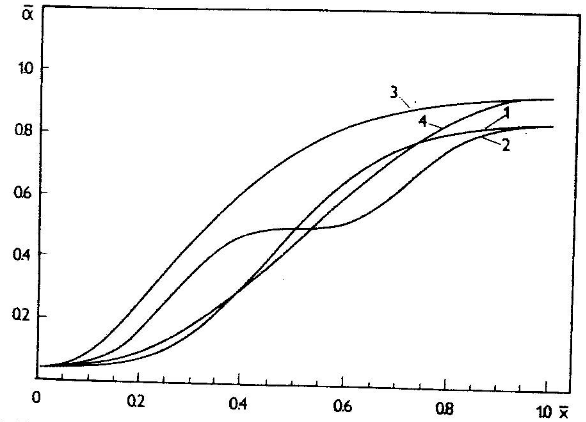

Fig. 6. Rate of ionization distributions for current densities from Fig. 1.

Table 1

Values of quantities ahead of the current layer

v_0	T_0	ϱ_0	B_0
$1.3 \times 10^4\,ms^{-1}$	12000 K	$1\,kgm^{-3}$	0

IV. CONCLUSION

The above mentioned procedure can serve as a simple numerical estimate of density, velocity, temperature, rate of ionization and other quantity distributions in the cluster, assuming we know the space current density distributions.

Ahead of and behind the current layer we find the areas of weakly ionized gas, which together with the current layer form the plasma cluster. The only remaining problem is the estimate of the temperature T_0 ahead of the current layer, the other values v_0, ϱ_0 and B_0 are well known from experiments.

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ОДНОМЕРНЫЕ УСТАНОВИВШИЕСЯ ПОТОКИ В ПЛАЗМЕННЫХ УСКОРИТЕЛЯХ РЕЛЬСОТРОННОГО ТИПА

В работе описан простой метод для определения пространственного распределения плотности, скорости, температуры и других величин в проводящем слое, который движется в плазменном ускорителе рельсотронного типа. Предполагается, что движение происходит с постоянной скоростью и экспериментальные данные о распределении плотности тока в кластерах известны.