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COMPARISON OF SOME METHODS FOR THE CALCULATION OF THE SAW DIFFRACTION[1]

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The application of the exact Kirchhoff theory and the angular spectrum method together with the approximate Fresnel integral and the FFT technique for the SAW diffraction is described. The computations show that the approximate FFT method can be applied in the region from the sending transducer to about two of its apertures and the Fresnel integral gives good results beyond this region.

I. INTRODUCTION

The SAW diffraction is an important natural property of this wave and usually has a negative effect on the function of the SAW devices [1]. In order to include this effect into precise design methods of the SAW devices rapid and accurate computations of the amplitude and phase of the diffracted SAW are required. One correct and two approximate methods for the numerical calculations of the SAW diffraction were presented in literature: the "*angular spectrum of waves*" method [2], the Fresnel approximation [3] and the approximate geometrical theory [4].

In this paper four basic methods for the numerical calculations of the SAW diffraction are compared: the Kirchhoff theory [5], the angular spectrum theory [6], the Fresnel approximation [3] and the fast Fourier transform (FFT) method [7]. The first two theories give exact results, the remaining techniques are approximated ones. The approximate geometrical theory is not included here because its satisfactory agreement with exact methods was presented in detail in literature [4].

The results are presented only for the isotropic medium. They can be used, however, by only changing the scale along the beam axis for an anisotropic medium if the parabolic approximation of the velocity surface around the beam

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axis is possible [6]. All the usually used substrates for SAW devices belong to this group with the exception of the YZ cut $LiNbO_3$, but in this case the SAW diffraction is negligible. In an isotropic medium [6] the beam profile very near the sending transducer has many small peaks, with an increasing distance the amount of peaks decreases and their height increases. At some position one sharp central maximum appears which decreases and broadens at greater distances.

II. DIFFRACTION THEORY

A formulation of the problem follows from Fig. 1. The complex amplitude $u(x, y)$ of the SAW in a point with coordinates x, y should be determined from the known amplitude $u(0, y_0)$ on the aperture $2a$ of the sending interdigital transducer. From the analogy with the Kirchhoff theory of optical diffraction [5] the following formula can be derived

$$u(x,y) = \frac{1}{2\sqrt{\lambda}} \int_{-a}^{a} u(0,y_0)\, \frac{1+\cos\beta}{\sqrt{s}}\, e^{jks}\, dy_0 \qquad (1)$$

where j is the imaginary unit, λ is the SAW wavelength and $k = 2\pi/\lambda$ is the wavenumber. The meaning of other symbols follows from Fig. 1.

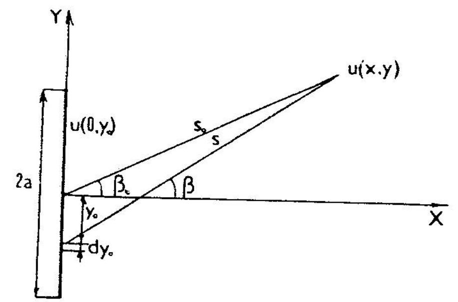

Fig. 1. Geometrical description of the SAW diffraction.

The Fresnel integral [1] is an approximation of formula (1)

$$u(x,y) = \frac{1}{\sqrt{2}} \int_{a_1}^{a_2} \exp\{\tfrac{-j\pi z^2}{2}\}\, dz \qquad (2)$$

where

$$a_1 = \frac{y-a}{q},\ a_2 = \frac{y+a}{q},\ q = \sqrt{\lambda \frac{x}{2}}. \qquad (3)$$

This assumption is valid under the condition $x \gg |y - y_0|$ for all y_0.

The angular spectrum theory for an isotropic medium [6] is based on the formulae

$$u(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k_y) \exp(j(x\sqrt{k^2 - k_y^2} + yk_y))\, dk_y \qquad (4a)$$

$$F(k_y) = \int_{-\infty}^{\infty} u(0,y_0)\, e^{-jk_y y_0} dy_0 \qquad (4b)$$

where k_x, k_y are the components of the wave vector k. The meaning of other symbols follows from Fig. 1. These formulae are the Fourier transform pair. If the amplitude of the SAW on the aperture is constant, $u(0,y_0) = A$, then the formula (4b) has the form

$$F(k_y) = A\frac{a}{\pi}\,\frac{\sin k_y a}{k_y a}. \qquad (5)$$

The FFT technique [7] is an approximation of the angular spectrum theory. According to the definition of the FFT

$$u_i = u(x, y_i) = \frac{1}{N} \sum_{k=0}^{N-1} B_k \exp\left\{j\frac{2\pi i k}{N}\right\} \qquad (6)$$

where for the case of the constant amplitude of the SAW on the aperture

$$B_k = Aa\,\frac{\sin \pi a n_k}{\pi a n_k} \exp\left(j2\pi\frac{x}{\lambda}\sqrt{1-n_k^2}\right) \qquad (7)$$

the complex amplitude of the SAW can be computed in points with the y coordinate

$$y_i = i\frac{\lambda}{2} \qquad i = 0, 1, \ldots, N-1. \qquad (8)$$

The y coordinate n_k of the unit vector has these values

$$n_k = \frac{k - \frac{N}{2}}{\frac{N}{2}} \qquad k = 0, 1, \ldots, N-1. \qquad (9)$$

In practice the numerical integration of the equation (4a) uses a formula similar to that of the equation (6). Nevertheless there are important differences between these two techniques. The sum (6) requires the N to be a power of 2, the interval between the neighbouring values of the independent variable n_k should be $N/2$ and the minimal distance between points at which the SAW amplitude is computed should be $\lambda/2$. The numerical integration has no limitation. The function computed by both methods is periodic, however, the increase of the number of steps N in the case of the FFT causes the increase of the period only, while the accuracy of the numerical integration improves too.

III. COMPARISON OF METHODS

The complex amplitude of the diffracted SAW can be obtained only by the numerical computations. The numerical integration was applied to formulae (1) and (4a) of the exact theories while the standard subprogram was used for the computation of the Fresnel integral (2) and the FFT (6). It was found that the accuracy of the numerical integration of the integral (1) derived from the Kirchhoff theory was satisfactory at all distances when minimally 100 steps were used. On the other hand the number of steps for the same accuracy in the angular spectrum theory increases with the increasing distance. When comparing the methods the numerical integration was always made with 100 steps and the results from the Kirchhoff theory was taken as a standard.

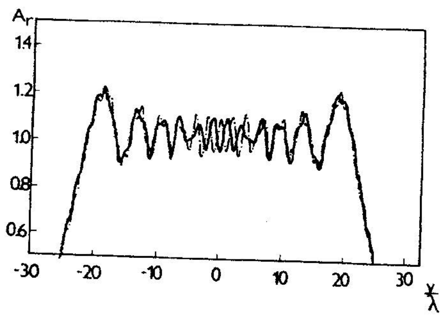

Fig. 2. Comparison of the exact Kirchhoff theory (full curve) and the approximate Fresnel integral (dot and dashed curve) at the distance equal to the aperture.

The approximate results of the Fresnel integral (2) are compared with the exact theory in Fig. 2. In this and the following figures the relative SAW amplitude A_r is shown. The distance from the sending transducer for Fig. 2 is equal to the aperture. The agreement in the central part of the beam is not good. At this distance the results of the angular spectrum method are close to the results of the exact theory.

The beam profile at the distance equal to three apertures is in Fig. 3. The angular spectrum theory shows some deviations in all parts of the profile. On the other hand the Fresnel integral is very close to the exact theory. The differences between the exact theory and both above mentioned methods on the beam profile at two apertures from the sending transducer are roughly equal.

The less known beam profiles on lines parallel to the beam axis are in Fig. 4. The scale on the horizontal axis is logarithmic. The curves oscillate with an increasing amplitude and period.

The results of the FFT method are close to the values obtained from the angular spectrum theory when roughly the same number of steps is used. In both cases the agreement with the exact theory improves when the number of steps used in computations increases. The computation time of the angular spectrum method is very long when the number of steps is great, while the computer time of the FFT algorithm increases only slightly when the number of points increase significantly.

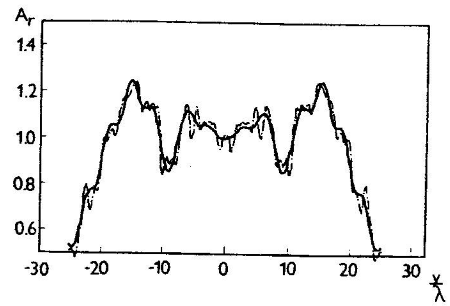

Fig. 3. Comparison of the exact Kirchhoff theory (full curve) and the angular spectrum method (dot and dashed curve) on the beam profile distant three apertures from the sending transducer.

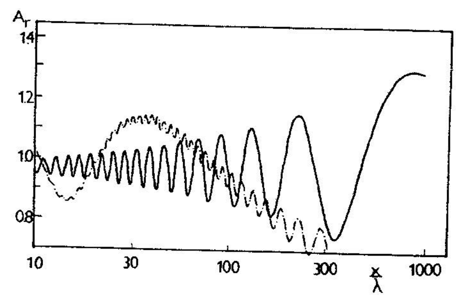

Fig. 4. The beam profile along the beam axis (full curve) and on the line parallel to the beam axis near the beam edge (dot and dashed curve). The distance of the second line from the theoretical beam edge is 10% of the aperture.

The curves obtained from the FFT method with 1024 points agree well with the exact theory for distances between transducers ranging from zero to about 20 apertures. On the other hand, the agreement between the exact theory and the Fresnel integral method is satisfactory for distances greater than about two apertures. In the SAW devices the distance between the transducers is usually of the order of the aperture.

IV. CONCLUSIONS

The rapid computation of the SAW diffraction in the analysis of the SAW devices requires the use of some approximate method. Two approximate methods for this computation, the Fresnel integral and the FFT method, were compared with the exact theories. It was found that the FFT method with 1024 points was applicable in the region from the sending transducer up to about 20 apertures, the Fresnel integral method should be used at distances greater than about 2 apertures. Both approximate techniques exist in the form of standard subprograms and the required time of their computation is of the same order. The FFT method is therefore to be preferred, because the distance between the transducers is usually small. The disadvantage of this method is that it gives the values only at points with the coordinate, which is the multiple of the half of the SAW wavelength.

REFERENCES

[1] Szabo, T. L., Slobodnik, A. J.: IEEE Trans. Son. Ultrason. *SU-21* (1974), 114.
[2] Szabo, T. L., Slobodnik, A. J.: IEEE Trans. Son. Ultrason. *SU-29* (1973), 240.
[3] Penunuri, D.: IEE Trans. Microwave Theory Tech. MTT-*26* (1978), 288.
[4] Radasky, A. V., Matthaei, G. L.: IEEE Trans. Son. Ultrason. *SU-39* (1983), 78.
[5] Born, M., Wolf, E.: *Principles of Optics*. Pergamon Press, London 1966.
[6] Matthews, M.: *Surface Wave Filters*. John Wiley and Sons, New York 1977.
[7] Brigham, E. O.: *The Fast Fourier Transform*. Prentice Hall, New Jersey 1974.

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СРАВНЕНИЕ НЕКОТОРЫХ МЕТОДОВ, ПРИМЕНЯЕМЫХ В РАСЧЕТАХ ДИФРАКЦИИ ПОВЕРХНОСТНЫХ АКУСТИЧЕСКИХ ВОЛН

В работе описывается применение точной теории Кирхгофа и метода углового спектра вместе с приближенным интегралом Френеля, а также метода БПФ для поверхностных акустических волн. Вычисления показывают, что приближенный метод БПФ может быть применен в области от преобразователя приблизительно до двух его апертур. Интеграл Френеля дает хорошие результаты вне этой области.