acta phys. slov. 37 (1987), No. 3

QUARK-GLUON PLASMA FORMATION IN RELATIVISTIC HEAVY ION COLLISIONS WITHIN THE HYDRODYNAMICAL DESCRIPTION

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Within the one-dimensional one-component fluid-dynamical description the space-time picture of the quark-gluon plasma formation with finite rearrangement time during relativistic heavy ion collisions is studied. For the QCD-suggested conversion time scale in the order of 1 fm/c we find the maximum effect of the delayed deconfinement, manifesting itself e.g. in very broad fronts separating hadron matter and plasma, strong extra entropy production and strong time dependence of the deconfined state. The change of the flow pattern is discussed as a possible sign of the deconfinement transition.

1. INTRODUCTION

One of the primary aims in performing relativistic heavy ion collisions is to look for novel states of nuclear matter. The most interesting question concerns the possibility of achieving a transient deconfinement state [1]. Most theoretical studies deal with the scheme of ultrarelativistic heavy ion collisions which are accessible already in cosmic ray experiments. On the other hand experiments are now planned in the range of $T_{lab}/A \sim 10\,\text{GeV}$ (fixed target), which extend the present heavy ion reactions at Bevalac and Dubna energies. Recent estimates of the nuclear stopping power [2] indicate that in this energy range the transformation of kinetic incidence energy into internal excitation energy is possible, at least in some of the collisions. If the energy density achieved is sufficiently large, a baryon-rich quark gluon plasma is expected to be produced [3].

In the present paper we consider the formation of this plasma in heavy ion collisions with bombarding energies $T_{lab}/A \sim 10\,\text{GeV}$ within the one-component one-dimensional fluid-dynamical model. It is our aim to study the plasma formation characteristics in one well-defined scheme of the collision process in order to give a reference point for further work. The one-component hydrodyna-

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mical model will certainly only apply to a part of central collisions where a giant bag, created via fluctuations or seat effects, stops the incoming matter and accelerates the local equilibration. The model of interpenetrating nuclei has been considered in Ref. [4]. The application of a two-fluid model with finite stopping power will be subject of forthcoming investigations.

The main component of hydrodynamics is the equation of state. Despite of many efforts a straightforward QCD-founded equation of state does not exist at present. Obviously there is a clear difference between the characteristic structures in the hadronic and plasma states. This difference can be visualised by the fact that there are well-separated quark clusters in the hadronic state, while the plasma is a state of less-correlated quarks and gluons. Therefore, with increasing density or temperature one expects a rearrangement of quarks which is called the deconfinement transition.

According to this rearrangement one should be able to calculate the equation of state for any given density and temperature; nevertheless one cannot expect that a particular approximation is applicable in the whole relevant range. In most cases two asymptotic equations of state can be obtained for extreme stages of the transition, one for nuclear matter and the other for the plasma. These approximations will not be valid for intermediate regions and, therefore, the asymptotic equations of state do not match smoothly, but are rather a break-point in the thermodynamical potentials, thus representing a first-order phase transition somewhere in the transition region. Since there the asymptotic equations of state are not necessarily correct, this first-order phase transition may be a result of the approximations used. In fact, even for the vanishing chemical potential the order of the deconfinement transition has not been determined yet. (A tendency to believe in second-order transition can be observed at present, according to the private communication of J. Polónyi.)

While the order of the deconfinement transition may possess a high theoretical relevance, it does not necessarily have a great influence on the final result of the dynamical processes leading to deconfinement. A continuous rapid structural change may simulate a first-order transition in several aspects if the rearrangement requires both energy and time. (Note that the compressibility is not infinite during a first-order transition in a true dynamical process.) Therefore, we use here a two-phase model to study the transition dynamics, in particular non-equilibrium effects and the finite rearrangement time for changing the structure, and their influence on the space-time picture of the plasma formation during the compression stage. We tackle the question whether there are peculiarities of the plasma formation process which might in themselves signalize plasma occurrence. Some of them are the extra entropy production [5] and the shock front or flow instability [6] considered recently.

The plan of the paper is as follows. In Section 2 we present the generic scheme

of phase transitions in a volume-averaged hydrodynamical description. Hydrodynamical calculations of the plasma formation are reported in Section 3. The discussion of our results can be found in Section 4 and the summary is given in Section 5. The Appendix contains a presentation of our scheme for solving the relativistic hydrodynamical equations in comoving Lagrangian curvilinear coordinates, and also the derivations of some formulae too long for the main text.

2. NON—EQUILIBRIUM PHASE TRANSITIONS

The one-component fluid-dynamical model uses a unique velocity field which can be introduced if the particles motion is sufficiently correlated. According to such a four-velocity field u^i $(i = 0, \ldots, 3)$ the energy momentum tensor T^{ij} of an isotropic medium can be decomposed [7]

$$T^{ij} = (e + p)u^i u^j + pg^{ij} - 2\eta\sigma^{ij} - \zeta\Theta p^{ij} + u^i q^j + u^j q^i, \tag{2.1}$$

where g^{ij} is the metric tensor (signature $+2$ is used); η and ζ stand for the shear and bulk viscosity coefficients; σ^{ij}, p^{ij} and Θ denote the shear and projection tensor ($\sigma^{ij} = (u^{i;j} + u^{j;i})/2 - g^{ij}\Theta/3$, $p^{ij} = g^{ij} + u^i u^j$) and the expansion ($\Theta = u^i_{;i}$) and q^i describes the heat flux, respectively. The equations of motion take the form

$$T^{ij}_{;j} = 0, \tag{2.2}$$

$$(nu^i)_{;i} = 0. \tag{2.3}$$

The thermodynamic quantities n, e and p are the baryon density, energy density and pressure defined in the local rest frame. The equations of motion are manifestly covariant by using the covariant derivative [7].

Now we want to describe the evolutionary aspects of dynamical phase transitions with respect to different intensive variables of the two phases of the medium. Thus, finite relaxation times and a finite growth velocity of the new phase are taken into account.

Consider a mixture, with particle numbers N_a $(a = 1, 2)$ in the phases 1 and 2 and with occupied volumes V_a; the total particle number is N and the total volume V. Introduce the coefficients

$$x = N_1/N, \quad \hat{x} = V_1/V. \tag{2.4}$$

If the individual phases are sufficiently small, then only volume averages appear in the hydrodynamical equations. These averaged quantities can be constructed as

$$A = xA_1 + (1 - x)A_2, \quad a = \hat{x}a_1 + (1 - \hat{x})a_2, \tag{2.5}$$

where A stands for any specific quantity and a for any density. Having the averaged density n, the weight x follows from

$$x = (n_2 - n)/(n_2 - n_1) \tag{2.6}$$

and $\hat{x}$ is determined by

$$\hat{x} = x n_1/n. \tag{2.6'}$$

The energy momentum tensor of the mixture consists of densities, so it can be written as

$$T^{ij} = \hat{x} T_1^{ij} + (1 - \hat{x}) T_2^{ij}. \tag{2.7}$$

Using the equations of motion (2.2) and (2.3) one can calculate the entropy production in the mixture (2.7). Neglecting viscosity and heat conduction the specific entropy (s) increase due to the nonequilibrium nature of the transition is (cf. Appendix B)

$$\dot{s} = \frac{2}{T_1 + T_2}\left\{(\mu_2 - \mu_1)\dot{x} + ((p_1 - p_2)/n)\dot{\hat{x}} - \frac{1}{2}(T_1 - T_2) \cdot (xs_1 - (1 - x)s_2)^{\cdot}\right\} \tag{2.8}$$

(a dot means the comoving derivative, e.g. $\dot{s} = s_{;i}u^i$; μ_a, p_a and T_a denote the chemical potentials, pressures and temperatures, respectively). The physical meaning of the terms entering this equation is obvious: they represent three different equilibration processes as shown by the three intensive differences together with the entropy production of each equilibration process. The second law of thermodynamics prescribes the direction of processes:

(i) if $\mu_1 > \mu_2$, then $\dot{x} < 0$, that is there is a particle transfer into phase 2;

(ii) if $p_1 > p_2$, then $\dot{\hat{x}} > 0$, that is the first phase expands and compresses the low-pressure phase;

(iii) if $T_1 > T_2$, then there is a heat flux from phase 1 into phase 2, thus decreasing the total difference of the entropies in the phases $N(xs_1 - (1 - x)s_2)$.

This shows how the system tries to achieve equilibrium. In an equilibrium transition the Gibbs conditions are fulfilled ($T_1 = T_2$, $p_1 = p_2$, $\mu_1 = \mu_2$) and the entropy production vanishes. The non-equilibrium transition should be always taken into account if the time necessary for building up the new phase is comparable to (or longer than) the characteristic time of the change of the other thermodynamic quantities. Generally speaking, in such cases the relative weight of the phases differs from the energy minimum.

To solve the hydrodynamical equations one must take into account the relaxation equations for the quantities x, $\hat{x}$ and $xs_1 - (1 - x)s_2$ determining the relaxation of chemical potential, pressure and temperature differences of both phases, respectively.

3. PLASMA FORMATION DYNAMICS

Now we apply the scheme of non-equilibrium transitions to the delayed baryon-rich plasma formation. The idealizations we introduce to make the situation tractable are the following ones:

(i) one-dimensional (plane-symmetric) flow,

(ii) thermal and mechanical (pressure) equilibrium are achieved much faster than chemical equilibrium [8],

(iii) neglect of heat conduction, i.e. $q^i = 0$,

(iv) use of a two-phase model equation of state.

According to item (i) viscosity enters the equations of motion as a modification of pressure via

$$\tilde{p} = p - (\eta + 4\xi/3)\Theta. \tag{3.1}$$

Due to item (ii) one relaxation equation for the progress variable x is needed which we take in the linearized form for the conversion rate

$$\dot{x} = -(x - x_{eq})/\tau, \tag{3.2}$$

where τ is the relaxation time scale in the order of the QCD time scale $\hbar c/B^{1/4} \sim 1$ fm/c (B is the vacuum pressure) and x_{eq} is the equilibrium weight belonging to the minimum free energy,

$$x_{eq} = \frac{\tilde{n}_1}{n}\,\frac{\tilde{n}_2 - n}{\tilde{n}_2 - \tilde{n}_1}, \tag{3.3}$$

where $\tilde{n}_a(T)$ denote the phase boundaries. (In principle the rate eq. (3.2) should be determined by a microscopic picture of droplet creation and growth, but in a non-static environment this is a tremendous task hampered, in the present case, by some unknown parameters for describing nucleation. So we use the relaxation time approximation [9] which represents a slow conversion law since spinodal decomposition is not taken into account.) Our calculations are based on a suitably parametrized nuclear matter equation of state which reads for the energy density and the pressure (some physical arguments are listed in Appendix C)

$$\begin{aligned} e &= mn + Kn(n/n_0 - 1)^2/18 + 3Tn/2 + \pi^2 T^4/10 \\ p &= K(n/n_0)^2(n - n_0)/9 + Tn + \pi^2 T^4/30, \end{aligned} \tag{3.4}$$

where $n_0 = 0.16\,\mathrm{fm}^{-3}$ is the nuclear ground state density and m denotes the nucleon mass; K stands for the nuclear incompressibility, respectively. This simple form is based on a cold parabolic compression part and a thermal Boltzmann part for the nucleons and a pion component (massless and noninteracting for convenience) as well. More sophisticated models are discussed in Ref. [10], but for our purpose eq. (3.4) is sufficient.

The plazma is described as an ideal massless quark gluon gas with confinement effects solely parametrized by the vacuum pressure $B = (235\,\mathrm{MeV})$ [4, 11], i.e.

$$\begin{aligned} e &= 37\pi^2T^4/30 + 3\mu_q^2T^2 + 3\mu_q^4/2\pi^2 + B, \\ p &= (e - 4B)/3, \\ n &= 2\mu_q(T^2 + \mu_q^2/\pi^2)/3, \end{aligned} \tag{3.5}$$

where μ_q is the quark chemical potential. Correction terms should be included in eq. (3.5), but up to now there has been no well-founded (lattice) QCD confirmation (except the lowest-order perturbative corrections) hence the present simplest form should be used for clarity.

Applying the Gibbs conditions for matching eqs. (3.4) and (3.5) one gets a first order phase transition with a coexistence region as displayed in Ref. 12 (Fig. 2). According to the statements in the introduction and item (iv) we accept this first-order phase transition; thus the rapid raise of entropy and energy density in the transition zone is described in the present model, being a consequence of different degrees of freedom in both phases.

Transport coefficients in nuclear matter and in individual plasma constituents have been calculated recently [13]. However, from kinetic theory arguments it is known [14] that they are not simply additive for mixtures. Therefore, to explore viscosity effects, one may use for illustrative purposes the gas viscosity of Ref. [15] and the viscosity of Danielewicz [13], both continuing into the deconfined phase without obstacle.

The bulk viscosity coefficient ξ is here taken to be 0, but the delayed phase transition gives rise to relaxation phenomena attribute to ξ [13]. The dynamical equations (2.2), (2.3) and (3.2) are solved in Lagrangian coordinates with the method described in the Appendix.

In Fig. 1 the time evolution of density profiles is displayed for a bombarding energy of 7 GeV/nucleon and the relaxation time $\tau = 1$ fm/c. The large densities achieved are consequences of both the zero mean free path inherent in the one-component hydrodynamics (i.e. the abrupt stopping of the first cell) and the softness of the equation of state in the mixture phase. For the utilized value of τ there is a broad front separating the hadron phase and the (initially partial) deconfined state. There is no indication of a double front sometimes discussed as a possible sign of the phase transition. The different viscosity types employed give the same profiles. The collision time is large enough for the almost complete deconfinement in the central part; however, as seen in Fig. 1, the compressed matter behaves strongly time dependent, i.e. stationary conditions are not

achieved. The density remains below the value determined by the Rankine--Hugoniot-Taub equation, while the temperature exceeds the corresponding value. These results must be contrasted with those obtained for smaller relaxation times $\tau < 1$ fm/c [6, 12], where stationary conditions with a stable front are obtained and the final states satisfy the Rankine-Hugoniot-Taub equations.

The extra entropy according to eq. (2.8) has been calculated in Ref. [5] to be in the order of 1. The large entropy carried by the plasma will be accordingly enhanced, thus improving the chances for the plasma diagnostic already envisaged by entropy measurements [3]. In the limit $\tau \to 0$, where the phase mixture is determined by the Maxwell construction, one finds across the front an entropy increase as given by the shock wave model independent of the viscosity used.

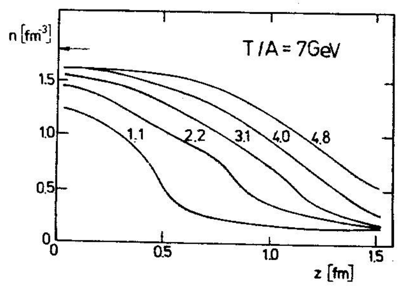

Fig. 1.: Evolution of the half density profiles for two colliding nuclear tubes with the length of 15 fm corresponding to the diameter of uranium. The value of the relaxation time τ is 1 fm/c. The profiles are labelled by the centre of mass time in fm/c. The arrow indicates the density predicted by the shock wave model.

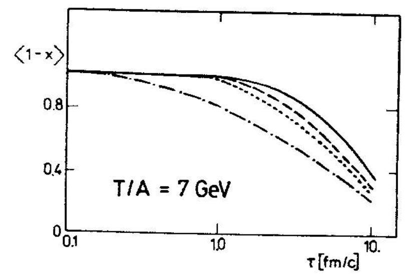

Fig. 2.: The mass fraction of the matter standing in the deconfined state as function of the relaxation time τ. Curves are given for averaging over the part containing 25% (full line), 50% (dashed line) and 75% (dotted line) of the mass of the incoming tube with the length of 15 fm. For comparison, the deconfined mass fraction is displayed for an incoming tube with the length of 8 fm corresponding to the diameter of calcium (chain line, average over 75% of the mass). The values displayed are taken at maximum transformation, i.e. at a time instant when the outermost shells are already expanding.

In Fig. 2 the mass fraction of the deconfined matter as function of the relaxation time τ is displayed. One observes that at $\tau < 1$ fm/c the pure deconfined state is reached, while for $\tau > 1$ fm/c its fraction decreases fast. When the conversion rate is too small, even in the central part the matter remains in an overheated hadronic state with plasma droplets immersed. Before the latter coalesce, the system expands again. Possible experimental signs of such droplets are discussed in Ref. [16].

Even though the pure quark-gluon phase is reached for $\tau \leq 1$ fm/c, the finite transformation rate affects the dynamics considerably. This can be seen for example in the front structures between hadron and quark matter. In Fig. 3 typical density profiles are displayed for different relaxation times. On the scale used the differences between the viscosity types utilized are not detectable. The front profiles are determined by the relaxation process. The strongest broadening happens at $\tau \sim 1$ fm/c. At smaller τ the matter is quickly transformed and typical narrow fronts appear; at larger values of τ the material conversion is too slow, compared to the dynamical time scales, to affect the front structure, and consequently rather small front widths occur separating instreaming cold nuclear matter and overheated nuclear matter. Only in the intermediate range $\tau \sim 1$ fm/c is the conversion time comparable with the dynamical time scale and a maximum affection of the front structures can be observed. Similarly, at $\tau \sim 1$ fm/c the extra entropy production, according to eq. (2.8), takes its maximum.

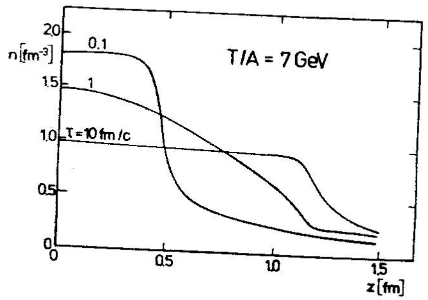

Fig. 3.: Typical density profiles for different values of the relaxation time τ (in fm/c). The centre of mass time, where the snapshots are taken, are 2.5, 2.8 and 2.3 fm/c for $\tau = 0.1$, 1.0 and 10 fm/c, respectively. The length of the incoming tubes is 15 fm.

4. DISCUSSION

Our results show that the QCD-suggested time scale for the rearrangement of hadronic matter into plasma is not ab initio small in comparison with the time of a relativistic heavy ion collision. Therefore, the stationary estimates relying on the shock wave model should be taken with caution and have to be replaced by proper dynamical calculations. Taking the proper development of the deconfined phase, we find that two regimes are possible (i) fast rearrangement (stationary conditions in the compressed zone are achieved) and (ii) slow rearrangement (the deconfined phase behaves strongly time dependent and there happens a broadening of the front separating the confined and the deconfined states). Adopting the QCD time scale $\hbar c/B^{1/4} \sim 1$ fm/c/ one expects that the second case is relevant for the relativistic heavy ion collisions. Thus the collective flow must be different from a situation where the transition does not appear and where, within the hydrodynamical model narrow fronts and stationary conditions

behind them occur. In particular the delayed phase transition changes the flow pattern not only in head-on collisions but also in noncentral and asymmetric collisions. One can identify the collective flow by measuring the $dN/d\cos\Theta_{flow}$ distribution. The present studies indicate that the clear peak observed in the global flow analysis should broaden essentially or even vanish due to the changed flow pattern above the threshold of the deconfinement transition. This discussion is in line with Ref. [6], where we argued that a particular shock front instability in a narrow band of bombarding energies above the deconfinement threshold affects the flow in the same manner. Since our present calculations indicate that the front behaves similarly as in the case of its instability, we conclude that for the finite transformation time of the deconfinement the changed flow pattern continues up to higher energies.

In considering the dynamical path through the phase diagram (cf. Ref. [12]) one finds that during the compression a fluid element is rather hot in entering the coexistence region. Then latent heat is needed for melting the hadrons, and consequently the matter cools being strongly compressed due to the softness of the equation of state in the mixture phase. After reaching the pure deconfined state it is heated up again accompanied by relatively small compression. Up to $T_{lab}/A \approx 7$ GeV the intermediate state is hotter than the final state. Therefore, a possible signal of plasma formation would be the detection of two thermal sources of directly emitted lepton pairs and gammas to be attributed to the hotter intermediate state and the cooler final state. The finite transformation time enhances this effect since the matter stays longer in the superheated intermediate state; still it may be hard to filter out such a double source effect from disturbing signals.

Some criticism of the present model might arise from the neglect of sidewards flow and finite stopping power. Sidewards flow will decrease the density. Quantitative estimates must rely on the threedimensional hydrodynamical model. A comparison of the present one-component one-dimensional model with the two-fluid model with finite stopping power is in preparation.

5. SUMMARY

In the present paper we have considered one well-defined scheme of plasma formation in relativistic heavy ion collisions relying on the assumption of rapid stopping of incoming nuclear matter and subsequent fast thermalization. We have found a strong broadening of the front between confined and deconfined matter as well as a non-stationary behaviour of the deconfined state caused by the finite rearrangement time. It is suggested that these peculiarities might enable to detect the deconfinement transition experimentally via collective flow analysis. The effects of the finite transformation time turn out to be much more important than viscosity effects in considering the front structure and entropy production.

At the considered bombarding energies the final plasma is cooler than the intermediate superheated nuclear matter; so two thermal sources of directly emitted lepton pairs and photons should be observable in sophisticated measurements.

ACKNOWLEDGEMENT

Valuable discussions with L. P. Csernai are gratefully acknowledged.

APPENDIX

A. Lagrangian coordinates

The Minkowski metric is described by the line element

$$ds^2 = -dT^2 + dY^2 + dx^2 + dz^2. \quad (A.1)$$

In a plane-symmetric and one-dimensional flow the velocity u^i possesses only T and Y components being independent of x and z. Then there are two-dimensional shells containing particles of the same velocity. One can introduce some continuous and monotonous numbering of the shells. A specific particle can be labelled by r belonging to the shell and by two coordinate values of x and z on the shell. None of them change during the evolution of the system. Then the motion of the particle can be given in parametric form

$$T = T(t,r), \quad Y = Y(t,r), \quad x = const, \quad z = const, \quad (A.2)$$

where t is some parameter of the evolution.

Now introduce t and r as new coordinates being Lagrangian comoving coordinates. Observe that the pair (t,r) is not unique; r is simply a numbering of the shells and t is still an undetermined evolution parameter. Therefore, the new pair

$$\tilde{t} = \tilde{t}(t,r), \quad \tilde{r} = \tilde{r}(r) \quad (A.3)$$

can also be used if the matrix of transformation is positive definite. Choosing a particular pair the two-dimensional part of the line element (A.1) in them obtains the form

$$ds^2 = -(T_{,t}^2 - Y_{,t}^2)\,dt^2 + 2(T_{,t}T_{,r} - Y_{,t}Y_{,r})\,dt\,dr + (T_{,r}^2 - Y_{,r}^2)\,dr^2. \quad (A.4)$$

By means of eq. (A.3) the g_{tr} term can be removed. Namely,

$$\tilde{g}^{tr} = (d\tilde{r}/dr)\{(\partial\tilde{t}/\partial t)\,g^{tr} + (\partial\tilde{t}/\partial r)\,g^{rr}\},$$

and if $g^{rr} \neq 0$, the bracketed term can be made zero by choosing a proper new evolution parameter $\tilde{t}(t,r)$. The remaining transformations are

$$\tilde{t} = \tilde{t}(t), \quad \tilde{r} = \tilde{r}(r). \quad (A.5)$$

We write the obtained metric in the form

$$ds^2 = -e^{2\Phi(t,r)}dt^2 + e^{2\Lambda(t,r)}dr^2 + dx^2 + dz^2. \quad (A.6)$$

Since the r coordinate of the particles is constant, the velocity possesses only a t component, and it is normalized, therefore

$$u^i = (e^{-\Phi}, 0, 0, 0). \quad (A.7)$$

Consider now the details of the transformation. Obviously

$$e^{2\Phi}dt^2 - e^{2\Lambda}dr^2 = dT^2 - dY^2 \quad (A.8)$$

and, $u_i dx^i$ being a scalar,

$$-e^{\Phi}dt = -\Gamma dT + u\,dY. \quad (A.9)$$

One can express the derivatives of T and Y from eqs. (A.8) and (A.9)

$$Y_{,t} = u e^{\Phi}, \; Y_{,r} = \Gamma e^{\Lambda}, \; T_{,t} = Y_{,t}/v, \; T_{,r} = v Y_{,r} v = u/\Gamma, \; \Gamma^2 = 1 + u^2. \quad (A.10)$$

The proper clock time $\hat{t}$ and the coordinate time t are related via $d\hat{t} = e^{\Phi}dt$.

The projection of equation (2.2) onto u_i yields with $q^i = 0$

$$(e/n)^{\cdot} - (\tilde{p}/n^2)\,\dot{n} = 0, \quad (A.11)$$

while the projection onto p_{ik} results for $k = 1$ in

$$\Phi_{,r} = -\tilde{p}_{,r}/(e + \tilde{p}). \quad (A.12)$$

The metric (A.6) describes a flat space in curvilinear coordinates. The flatness condition means the vanishing of the curvature tensor, $R_{ijkl} = 0$ [7]. The only non-trivial equation expressing the flatness of the plane-symmetric metric (A.6) reads [17]

$$(\Phi_{,rr} + \Phi_{,r}^2 - \Phi_{,r}\Lambda_{,r})\,e^{2(\Phi-\Lambda)} - (\Lambda_{,tt} + \Lambda_{,t}^2 - \Phi_{,t}\Lambda_{,t}) = 0. \quad (A.13)$$

This equation and the integrability condition for Y, stemming from eqs. (A.10), can be rewritten to get

$$u\Phi_{,r}e^{\Phi} = \Gamma_{,t}e^{\Lambda}, \quad u_{,r}e^{\Phi} = e^{\Lambda}\Lambda_{,t}\Gamma. \quad (A.14)$$

Using eq. (A.12) and the baryon conservation eq. (2.3) in the form

$$n = N_0 e^{-\Lambda}/F$$

with N_0 and F as normalization factors, we find the remaining dynamical equations

$$u_{,t} e^{-\Phi} = F\Gamma \tilde{p}_{,r}/(e + \tilde{p}), \tag{A.15}$$

$$n = \Gamma N_0/FY_{,r}. \tag{A.16}$$

The whole set of equations to be solved consists of the thermodynamic equations relating e, p and n, equations for η and ζ as function of n and T and the dynamical equations (A.10—16). Observe the great and unexpected similarity with the spherically symmetrical motion in comoving coordinates (cf. refs [7, 18]). Thus we use the same difference scheme as in ref. [18] to solve the set of dynamical equations.

A more detailed derivation including the initial and the boundary conditions can be found in ref. [19]. The advantage of the comoving coordinates is that one follows the fate of fluid elements in their rest frame and, even during large compressions, no rezoning is necessary.

Note that in the coordinate system (A.6) the nuclear tubes are stretched by the Lorentz factor Γ due to the use of the internal coordinate time instead of being contracted when using the observer time.

B. The entropy production

The energy and momentum balance of a system is governed by the vanishing of the divergence of the energy-momentum tensor, which is eq. (2.2). This is a vectorial equation of four components, while the four-velocity u^i possesses only three independent components. Therefore the equation

$$T^{ij}_{;j} u_i = 0 \tag{B.1}$$

is independent of the equation of motion; it carries thermodynamic meaning [20]. Now, for a one-phase simple fluid the local state is characterized by two thermodynamic variables, the particle number density n and the entropy density ns; since eq. (B.1) contains derivatives along the velocity field, it can give an equation for the combination of $\dot{n}$ and $\dot{s}$, whence, by means of eq. (2.3), $\dot{n}$ can be removed, so the final result is a source equation for s (cf. Ref. [21], but note the difference between the definitions of s there and here).

In our case the local state is characterized by five thermodynamic data, which may be chosen, e.g., as n_1, n_2, s_1, s_2 and x. (The second weight factor $\hat{x}$ can be expressed by them too.) Now, eq. (2.2) yields an equation for the combination

of the dot derivatives of these five data. Hence, using also the thermodynamic relations

$$de_i = T_i d(n_i s_i) + \mu_i dn_i$$
$$p_i = T_i n_i s_i + \mu_i n_i - e_i \tag{B.2}$$

($i = 1, 2$) valid for each individual state, by rearranging the terms, one can directly obtain eq. (2.8) for the specific entropy s

$$s = xs_1 + (1 - x)s_2 \tag{B.3}$$

of the mixture. For more details see Ref. [22].

C. The equations of state

The most informative form of the equations of state is a thermodynamic potential expressed by its own variables, e.g. the energy density e as a function of the particle number density n and entropy density ns; as shown by eq. (B.2), the temperature T, chemical potential μ and pressure p can be given by derivatives. If e is given in any other form, e.g. by n and T, then eq. (B.2) shows that partial differential equations are obtained, hence extra information is needed. Using the variables n and T, both $e(n, T)$ and $p(n, t)$ are necessary, but they are not independent; eq. (B.2) leads to the integrability condition [19]

$$n\frac{\partial e}{\partial n} + T\frac{\partial p}{\partial T} = e + p. \tag{C.1}$$

Now, the energy density in eq. (3.4) contains four different term. The first is the energy equivalent of the rest mass. The second is the simplest approximation for the compresion energy; we know that the cold nuclear matter possesses an energy minimum at n_0. The third is the thermal energy for a classical Boltzmann gas; these three terms were used in Refs. [5] and [12]. The last term is a blackbody radiation energy for spin 0 and isospin 1; this is meant as the pion contribution if the mass is neglected. Therefore this approximation is decent at moderate compression, and at temperatures above 100 MeV but well below the nucleon mass. Even there eq. (3.4) is a simplification, but this fact is counterbalanced by the benefits of an analytic form. The pressure function is taken according to eq. (C.1).

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Received May 13th, 1986
Revised version received July 18th, 1986

ОБРАЗОВАНИЕ КВАРК—ГЛЮОННОЙ ПЛАЗМЫ В СТОЛКНОВЕНИЯХ ТЯЖЕЛЫХ РЕЛЯТИВИСТСКИХ ИОНОВ В РАМКАХ ГИДРОДИНАМИЧЕСКОЙ МОДЕЛИ

В работе в рамках одномерной однокомпонентной гидродинамической модели изучается пространственно-временная картина образования кварк-глюонной плазмы с конечным временем перегруппировки при столкновениях тяжелых релятивистских ионов. Для предложенного КХД преобразования масштаба времени порядка 1 фм/с обнаружен максимальный еффект замедленного деконфайнмента, т. е. явное его выражение в очень широкой области раздела адронной материи и плазмы, в интенсивном образовании дополнительной энтропии и в сильной временной зависимости этого состояния. Обсуждается изменение картины как возможного характерного признака деконфайнментного перехода.